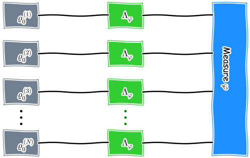
Parameter estimation in presence of collective phase noise

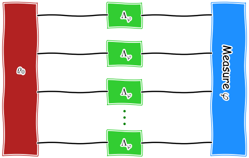
Sanah Altenburg, S. Wölk and O. Gühne

Department Physik, Universität Siegen, Siegen, Germany

Warsaw, 3. March 2016

Remember: Quantum Metrology





Standard Quantum Limit (SQL):

$$(\Delta\varphi)^2 \propto 1/N$$

Heisenberg Limit (HL):

$$(\Delta\varphi)^2 \propto 1/N^2$$

- **But noise destroys the advantage of using quantum states.**

The Question:

For a given set-up, with a given noise model:
Which quantum state should be used?
Are there better strategies?

Which quantum state should be used?

- Product states:

$$|\Psi\rangle = \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right)^{\otimes N}, \quad F_Q \leq N$$

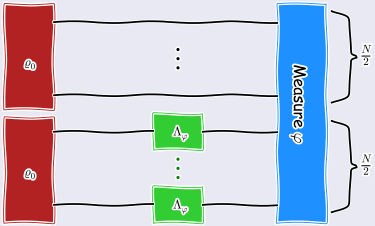
- GHZ states:

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + |1\rangle^{\otimes N}), \quad F_Q \leq N^2$$

- Symmetric Dicke states:

$$|D_N^{N/2}\rangle \propto \sum_j P_j\{|0\rangle^{\otimes N/2} \otimes |1\rangle^{\otimes N/2}\}, \quad F_Q \leq N(N+2)/2$$

Are there better strategies?

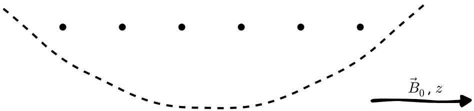


Differential Interferometry, introduced by M. Landini et al.: *Phase-noise protection in quantum-enhanced differential interferometry*, New Journal of Physics **11**, 113074 (2014)

- HL can be reached even with phase noise.

The Setup

- N trapped ions in a chain.
- Magnetic field $\vec{B_0} = B_0 \vec{e}_z$



Dynamics:

- Ideal dynamics

$$U = \exp\left(-i \underbrace{\gamma B_0 t}_{\varphi} S_z\right), \quad S_z = \sum_i \sigma^{(i)}/2$$

- Noise: Magnetic field fluctuations $B = B_0 + \Delta B(t)$

$$\varphi \to \varphi + \underbrace{\gamma \int_0^t \mathrm{d}\tau \Delta B(\tau)}_{\delta\varphi(t)}$$

Target: Estimate φ for a given time $t = T$.

The noise model[1]

- Dynamics

$$U = \underbrace{\exp[-i\varphi S_z]}_{U_\varphi}\underbrace{\exp[-i\gamma \int_0^T \mathrm{d}\tau \Delta B(\tau) S_z]}_{U_{noise}}$$

- The state ϱ_0 evolves in a noisy state $\varrho_T = U_{noise}\varrho_0 U_{noise}^\dagger$
- Time correlation function for the magnetic field fluctuations $\Delta B(\tau)$

$$\langle \Delta B(\tau)\Delta B(0)\rangle = \exp[-\frac{\tau}{\tau_c}]\langle \Delta B^2\rangle$$

- Assume Gaussian phase fluctuations with $\langle \delta\varphi(T)\rangle = 0$
- Average $\langle \varrho_T \rangle$ and calculate the QFI $F_Q^\varphi[\langle \varrho_T\rangle, S_z]$

$\gamma \Delta B = (2\pi)\,50\,\mathrm{Hz}$ and $\tau_c = 1\,\mathrm{s}$

[1]T. Monz et al.: *14-Qubit Entanglement: Creation and Coherence*, PRL **106** (2011)

Optimization over global rotations

Global rotations with the unitary

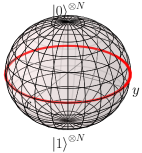
$$U_x(\alpha) = \exp\left[-i\alpha S_x\right].$$

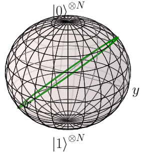
The initial states changes

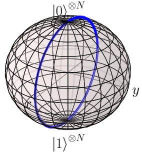
$$|GHZ(\alpha)\rangle = U_x(\alpha)\,|GHZ\rangle$$

and

$$|D_N^{N/2}(\alpha)\rangle = U_x(\frac{\pi}{2} + \alpha)\,|D_N^{N/2}\rangle\,.$$

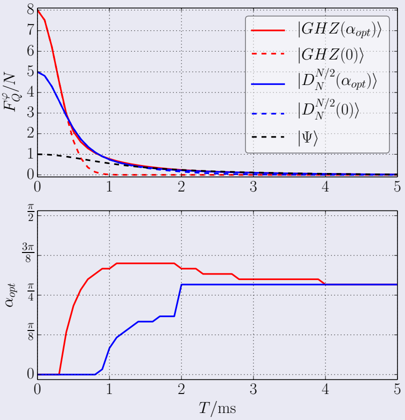




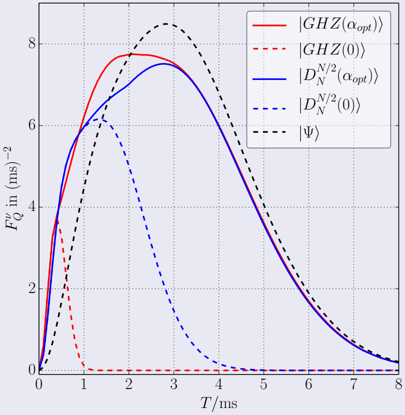


Phase and frequency estimation for $N = 8$

Phase φ estimation



Frequency ν estimation



Differential Interferometry[1]

- Split the system in two equal sized parts.
- Change the initial states like

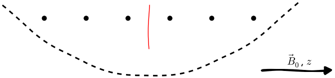
$$|GHZ\rangle \rightarrow |GHZ\rangle \otimes |GHZ\rangle$$

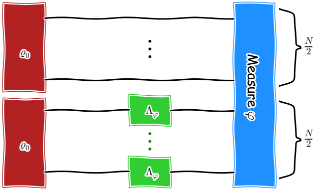
and

$$|D_N^{N/2}\rangle \rightarrow |D_{N/2}^{N/4}\rangle \otimes |D_{N/2}^{N/4}\rangle \,.$$

- Do nothing on one part of the system and let the linear map Λ_φ act on the other part:

$$U = \exp[-i\varphi(\mathbb{1}_{N/2} \otimes S_z^{N/2})] \cdot U_{noise}$$

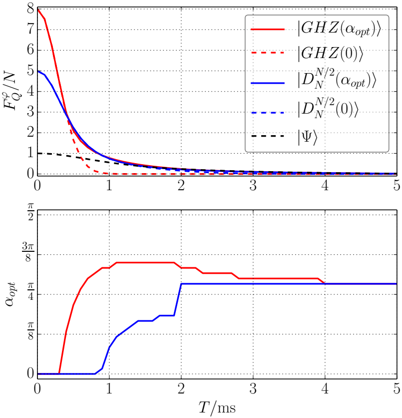




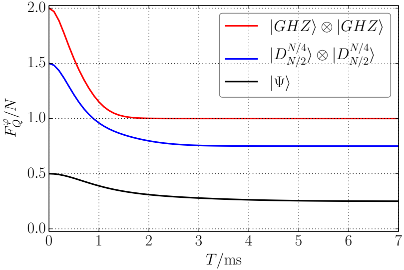
[1] M. Landini et al.: *Phase-noise protection in quantum-enhanced differential interferometry*, New Journal of Physics **11**, 113074 (2014)

DI Results: Phase estimation

Usual strategy



Differential Interferometry

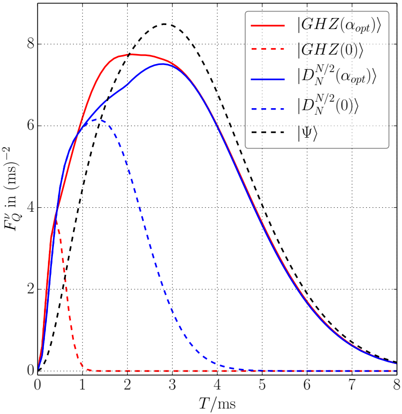


- No optimization over rotation angles necessary.

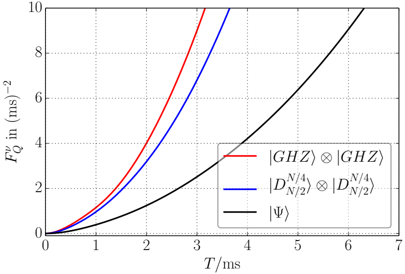
For large T: $F_Q^\varphi(GHZ) = N^2/8$, $F_Q^\varphi(D) = N(N+4)/16$ and $F_Q^\varphi(\Psi) = N/4$

DI Results: Frequency estimation

Usual strategy



Differential Interferometry



- No optimal measurement time.

For large T: $F_Q^\nu \propto T^2$

Open questions

- Experimental realization: Spin flip at $T/2$?
  - Effective hamiltonian $\mathbb{1} \otimes S_z$
  - But noise changes to

$$\exp\left[-i\gamma \int_0^T \mathrm{d}\tau \Delta B(\tau)(S_z \otimes \mathbb{1})\right]$$
$$\cdot \exp\left[-i\gamma \left(\int_0^{T/2} \mathrm{d}\tau \Delta B(\tau) - \int_{T/2}^{T} \mathrm{d}\tau \Delta B(\tau)\right)(\mathbb{1} \otimes S_z)\right]$$

  - It's even worse than the usual metrological scheme!
- For a input state of the form $|D_{N_1}^{k_1}\rangle_x \otimes |D_{N-N_1}^{k-k_1}\rangle_x$ and for large T:
  - For a product state ($k_1 = k = 0$): What is the optimal splitting $N_1 =?$ or how many ancilla qubits should I use?
  - For a given total number of excitations k, how should I distribute the excitations $k_1 =?$

Next steps

Collaboration with Ch. Wunderlich

I. Baumgart et al., arXiv:1411.7893 (2014)

- Trapped ions with time evolution

$$U = \exp\left(\gamma \int_0^T \mathrm{d}\tau \Delta B(\tau) S_z + \gamma(\Omega + \epsilon) t S_x.\right)$$

- Optimization for the estimation of ϵ.
- Differential interferometry for that dynamic.

Collaboration with M. Oszmaniec (ICFO)

- Gradient measurement

$$H = \gamma \sum_i B(z_i) \sigma_z^{(i)}$$

Thank you for your Attention! Questions?

This work was founded by the Friedrich-Ebert-Foundation.