Quantum probes for complex classical systems

Claudia Benedetti

@ Dipartimento di Fisica, Università degli Studi di Milano
http://users.unimi.it/aqm/

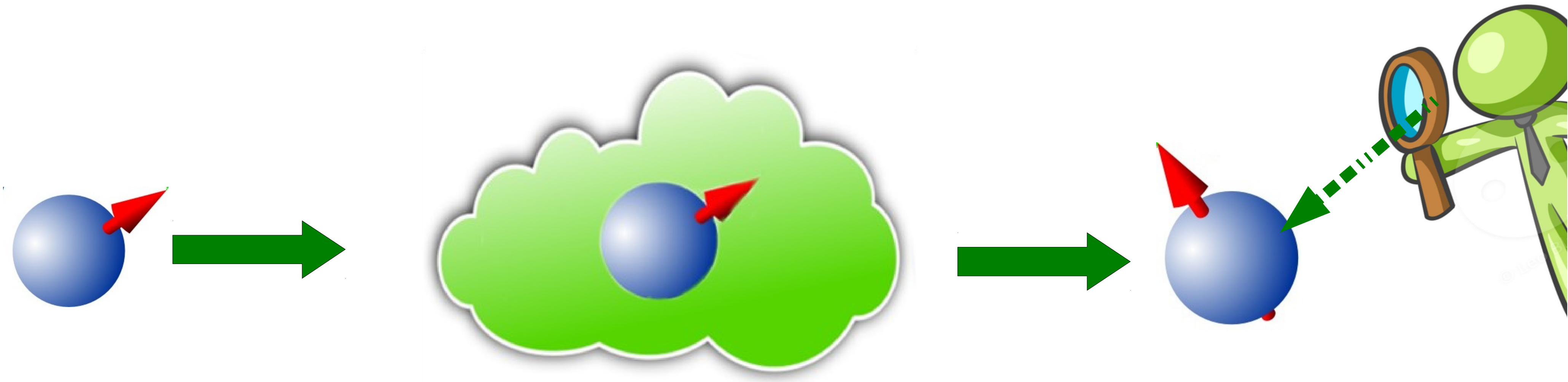
Outline

- **Motivations**
  -Classical vs Quantum Environments

- **Random flucutating fields and stochastic processes**
  - OU process
  - RTN noise
  - 1/f noise

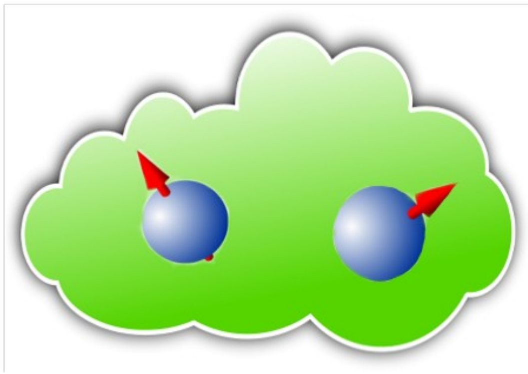
- **Quantum estimation theory for the spectral properties of a classical noise**
  -Quantum Fisher information and SNR

Motivations



- Open quantum systems are usually immersed in complex **noisy** environments
- This interaction destroys **coherence** and **quantumness**
- **"What can we learn from the dynamics of an open quantum system about its complex environment?"**

System+environment



Quantum system + quantum bath

Quantum system acted on by random fluctuating field

- $H = H_S + H_B + H_{SB}$

  - Unitary global evolution
  - Trace out the B degrees of freedom
  - CP map / Kraus operator representation
  - Master equation

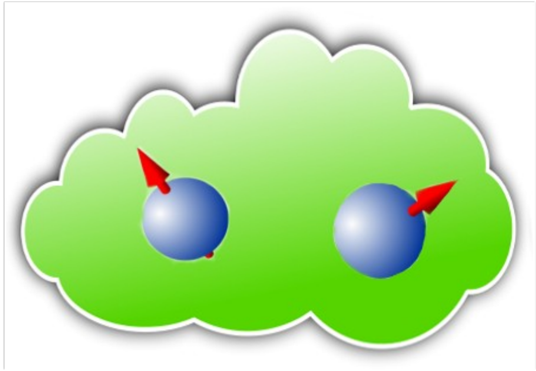
- Challenging or inappropriate (e.g. strong coupling..)
  Many degrees of freedom
  Interaction with a classical fluctuating field

- SB models with dephasing dynamics can always be written using classical models

Helm *et al*. PRA 2011
Crow & Joynt PRA 2014

Random classical fields

→ Quantum system acted on by random fluctuating field



- Semi-classical approach: quantum system + classical field
- Stochastic modeling

- Solid state systems and nanodevices → **Ornestein-Uhlenbeck process**
  RT noise
  1/f noise

Dynamics under classical noise

$$H(t) = \omega_0 \sigma_z + \nu \underline{B(t)} \sigma_z$$

To completely specify the model we need a probability functional $p[B(t)]$

Global unitary evolution $U(t) = e^{-i \int_0^t H(s) ds} = e^{-i[\omega_0 t + \int B(s) ds]\sigma_z}$

Global density matrix at time t for one realization of the process $\rho_G(t) = U(t) \rho_0 U^\dagger(t)$

System density matrix $\rho(t) = \langle \rho_G(t) \rangle_B = \int \rho_G(t) p[B(t)] dB$

Classical noise: Stochastic modeling

X: random variable

P(X): probability distribution

$\varphi_X(u)=\mathcal{E}\,[e^{iuX}]$: characteristic function

$\{X(t),\ t \in T\}$: stochastic process

$C(t)=\mathcal{E}\,[X(t_1)X(t_2)]$: autocorrelation function

$S(\omega)=\int C(s)e^{-i\omega s}\,ds$: spectral density

$P[X(t)]$: probability functional

Classical noise: Gaussian vs non-Gaussian

Gaussian processes

$\{X(t),\ t \in T\}$

Gaussian if: $\forall$ n integer and $\forall$ subset $\{t_1, .., t_n\}$, the RVs $X_1,..,X_n$ are jointly normally distributed

$$\left[\exp\left(i\sum_{j=1}^{n} u_j X(t_j)\right)\right]=$$

$$\exp\left[i\sum_j^n u_j \mu_j(t_j)-\frac{1}{2}\sum_{j,k} u_j u_k K(t_j,t_k)\right]$$

$$\mu(t_j)=\mathcal{E}[X(t_j)]$$

Mean

$$K(t_j,t_k)=\mathcal{E}\left[X(t_j)X(t_k)\right]-\mathcal{E}\left[X(t_j)\right]\left[X(t_k)\right]$$

Covariance kernel

Non-Gaussian processes

No complete information from mean and covariance function.

Cannot be mimicked by any Gaussian model

Microscopic structure of the environment plays a key role

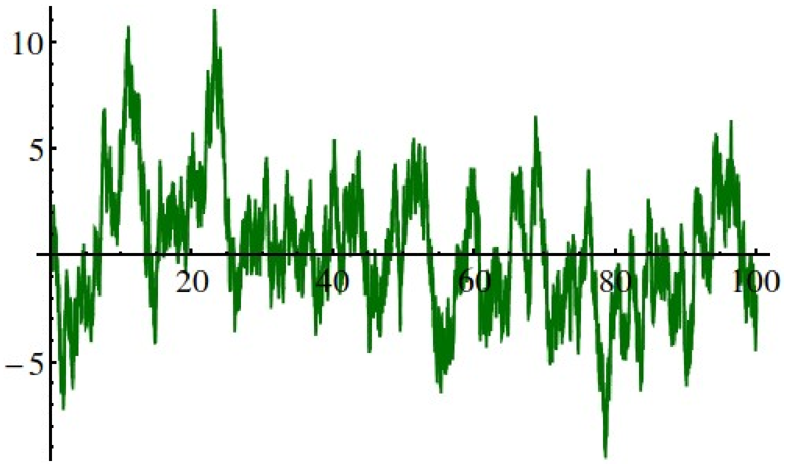
Bistable fluctuators

Gaussian noise: Ornstein–Uhlenbeck process

Process which describes the stochastic behavior of the velocity of a massive Brownian particle under the influence of friction.

$$dX(t) = \theta(\mu - X(t))\,dt + \sigma dW(t)$$

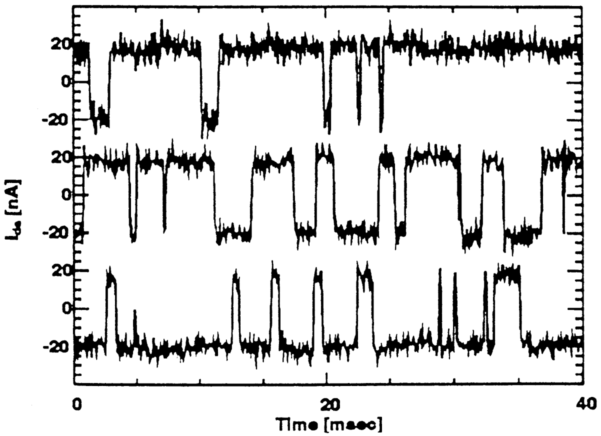
where θ is the rate of mean reversion, μ represents the long-term mean and σ_G the volatility or average magnitude per square-root time.



- $\mu = 0$
- $C_{OU}(t) = \dfrac{\sigma^2}{2\theta} e^{-\theta t}$

Non-Gaussian noise: Random telegraph noise

Bistable fluctuator: It can flip between two opposite values:
X*(t)*= ± *a* with **switching** rate γ .



- $P_n(t) = \frac{(\gamma t)^n}{n!} e^{-\gamma t}$
- $C(t) = a^2 e^{-2\gamma t}$
- $P[X(t)] \to P[\varphi(t)]$

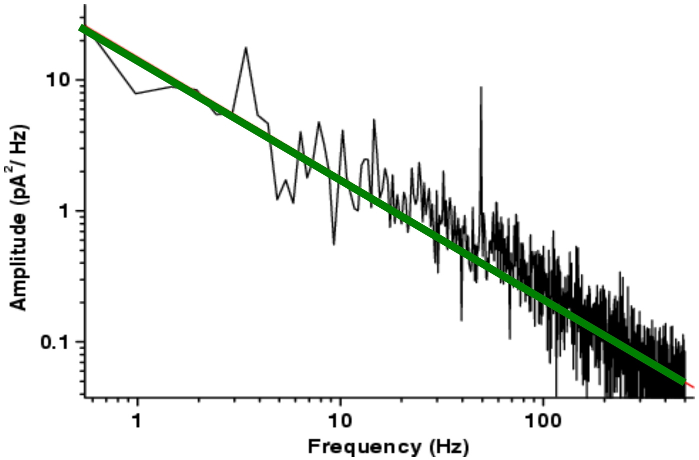
$$\varphi(t) = \int_0^t X(s) ds$$

$$p(\varphi, t) = \frac{1}{2} e^{-\gamma t}$$
$$\times \left\{ [\delta(\varphi + \nu t) + \delta(\varphi - \nu t)] + \frac{\gamma}{\nu} [\Theta(\varphi + \nu t) + \Theta(\varphi - \nu t)] \right\}$$
$$\times \left[\frac{I_1\left(\gamma t \sqrt{1 - (\varphi/\nu t)^2}\right)}{\sqrt{1 - (\varphi/\nu t)^2}} + I_0\left(\gamma t \sqrt{1 - (\varphi/\nu t)^2}\right) \right]$$

Bergli, Galperin, Altshuler PRB 2006

Non-Gaussian noise: 1/f$^{\alpha}$ noise

$$S_{1/f^{\alpha}}(f) = \int_{\gamma_1}^{\gamma_2} S_{RTN}(f,\gamma)p_{\alpha}(\gamma)d\gamma$$



Monte Carlo sampling...

$$S_{1/f}(f) = \sum_{j=1}^{N_f} S_j(f,\gamma_j) \propto \frac{1}{f^{\alpha}}$$

Linear superposition of bistable fluctuators

$$\boxed{X_{1/f}(t) = \sum_{j=1}^{N_f} X_j(t)}$$

$$p_{\alpha}(\gamma_j) = \begin{cases} \frac{1}{\gamma \ln(\gamma_2/\gamma_1)} & \alpha = 1 \\ \frac{1}{\gamma^{\alpha}(\alpha-1)\left[\frac{(\gamma_2\gamma_1)^{\alpha-1}}{\gamma_2^{\alpha-1}-\gamma_1^{\alpha-1}}\right]} & \alpha \neq 1 \end{cases}$$

$$X_j(t) = \pm 1$$

1/f noise

Dephasing of a Superconducting Flux Qubit

K. Kakuyanagi,[1] T. Meno,[2] S. Saito,[1] H. Nakano,[1] K. Semba,[1] H. Takayanagi,[3] F. Deppe,[4] and A. Shnirman[5]
[1]*NTT Basic Research Laboratories, NTT Corporation, Kanagawa, 243-0198, Japan*
[2]*NTT Advanced Technology, NTT Corporation, Kanagawa, 243-0198, Japan*
[3]*Tokyo University of Science, 1-3 Kagurazaka, Sh*
[4]*Walther-Meißner-Institut, Walther-Meißner-Strass*
[5]*Institut für Theoretische Festkörperphysik, Universität K*
(Received 22 August 2006; publishe

In order to gain a better understanding of the origin of dec
have measured the magnetic field dependence of the charact
phase relaxation time (T_2^{echo}) near the optimal operating point
means of the phase cycling method. At the optimal point, we
that the echo decay time is limited by the energy relaxation (
point, we observe a *linear* increase of the phase relaxatio
magnetic flux. This behavior can be well explained by the i

Model for $1/f$ Flux Noise in SQUIDs and Qubits

Roger H. Koch,[1] David P. DiVincenzo,[1] and John Clarke[2]
[1]*IBM Research Division, Thomas J. Watson Research Center, Yorktown Heights, New York 10598, U*
[2]*Department of Physics, University of California, Berkeley, California 94720-7300*
and Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720,
(Received 22 January 2007; published 27 June 2007)

We propose a model for $1/f$ flux noise in superconducting devices (f is frequency). The noise
generated by the magnetic moments of electrons in defect states which they occupy for a wide distributi
e of the two Kramers-degenerate ground stat
emperature. As a result, the magnetic mome
by randomly oriented defects with a density
ment with experiments.

Decoherence and $1/f$ Noise in Josephson Qubits

E. Paladino,[1] L. Faoro,[2] G. Falci,[1] and Rosario Fazio[3]
[1]*NEST-INFM & Dipartimento di Metodologie Fisiche e Chimiche (DMFCI), Università di Catania,*
[2]*Institute for Scientifi*
[3]*NES*

We propose and study
our analysis to the deco
substrate, which are also
a number of new features
degeneracy this model fo

Decoherence of Flux Qubits due to $1/f$ Flux Noise

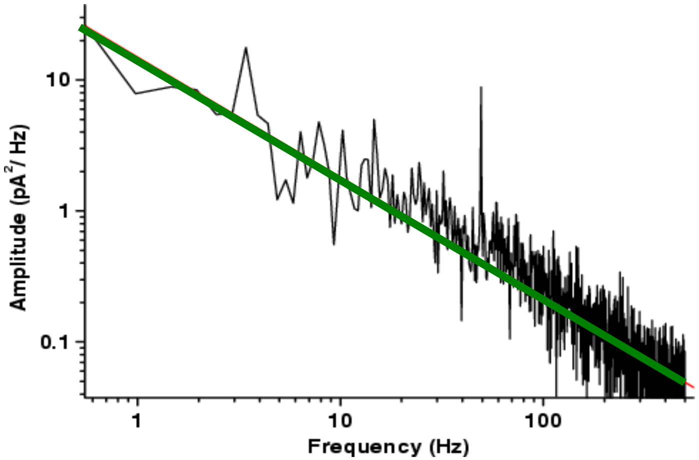
F. Yoshihara,[1] K. Harrabi,[2] A. O. Niskanen,[2,3] Y. Nakamura,[1,2,4] and J. S. Tsai[1,2,4]
[1]*The Institute of Physical and Chemical Research (RIKEN), Wako, Saitama 351-0198, Japan*
[2]*CREST-JST, Kawaguchi, Saitama 332-0012, Japan*
[3]*VTT Technical Research Centre of Finland, Sensors, P.O. Box 1000, 02044 VTT, Finland*
[4]*NEC Fundamental and Environmental Research Laboratories, Tsukuba, Ibaraki 305-8501, Japan*
(Received 16 June 2006; published 17 October 2006)

We have investigated decoherence in Josephson-junction flux qubits. Based on the measurements of decoherence at various bias conditions, we discriminate contributions of different noise sources. We present a Gaussian decay function extracted from the echo signal as evidence of dephasing due to $1/f$ flux noise whose spectral density is evaluated to be about $(10^{-6}\Phi_0)^2/\mathrm{Hz}$ at 1 Hz. We also demonstrate that, at an optimal bias condition where the noise sources are well decoupled, the coherence observed in the echo measurement is limited mainly by energy relaxation of the qubit.

PRL 97, 167001 (2006)

Non-Gaussian noise: 1/f$^\alpha$ noise - another microscopic model

$$S_{1/f^\alpha}(f) = \int_{\gamma_1}^{\gamma_2} S_{RTN}(f,\gamma) p_\alpha(\gamma) d\gamma$$



Statistical mixture

Ensemble $\{\gamma,\, p_\alpha(\gamma)\}$

- 1/f noise can be ascribed to a single random bistable fluctuator or to a collection of them.
- Stochasticity arise both from the process X(t) and the randomness in the switching rate.

The fluctuators have unknown switching rates, so they are described by a statistical mixture $\{\gamma,\, p_\alpha(\gamma)\}$

$$p_\alpha(\gamma_j) = \begin{cases} \dfrac{1}{\gamma \ln(\gamma_2/\gamma_1)} & \alpha = 1 \\ \dfrac{1}{\gamma^\alpha(\alpha-1)\left[\frac{(\gamma_2\gamma_1)^{\alpha-1}}{\gamma_2^{\alpha-1}-\gamma_1^{\alpha-1}}\right]} & \alpha \neq 1 \end{cases}$$

Dynamics under classical noise

$$H(t) = \omega_0 \sigma_z + \nu \underline{B(t)} \sigma_z$$

To completely specify the model we need a probability functional $p[B(t)]$

Global unitary evolution $U(t) = e^{-i \int_0^t H(s) ds} = e^{-i[\omega_0 t + \int B(s) ds]\sigma_z}$

Global density matrix at time t for one realization of the process $\rho_G(t) = U(t) \rho_0 U^\dagger(t)$

System density matrix $\rho(t) = \langle \rho_G(t) \rangle_B = \int \rho_G(t) p[B(t)] dB$

Gaussian vs non-Gaussian noise for exponential C(t)

$$H(t) = \omega_0 \sigma_z + \nu B(t) \sigma_z$$

$$\rho(t) = \frac{1}{2} \begin{pmatrix} 1 + \cos\theta & e^{-2i\omega_0} \beta(t,\gamma) \sin\theta \\ e^{2i\omega_0} \beta(t,\gamma) \sin\theta & 1 - \cos\theta \end{pmatrix}$$

OU process

$$C_{OU}(\tau) = \frac{\sigma^2}{2\theta} e^{-\theta\tau} \qquad \beta_{OU}(\theta, \tau; \sigma) = \exp\left[-\frac{\sigma^2 \left(\theta\tau - 1 + e^{-\theta\tau}\right)}{\theta^3} \right]$$

RTN

$$C_{RTN}(\tau) = a^2 e^{-2\gamma\tau} \qquad \beta_{RTN}(\gamma, \tau) = e^{-\gamma\tau} \left[\cosh\delta\tau + \frac{\gamma \sinh\delta\tau}{\delta} \right]$$

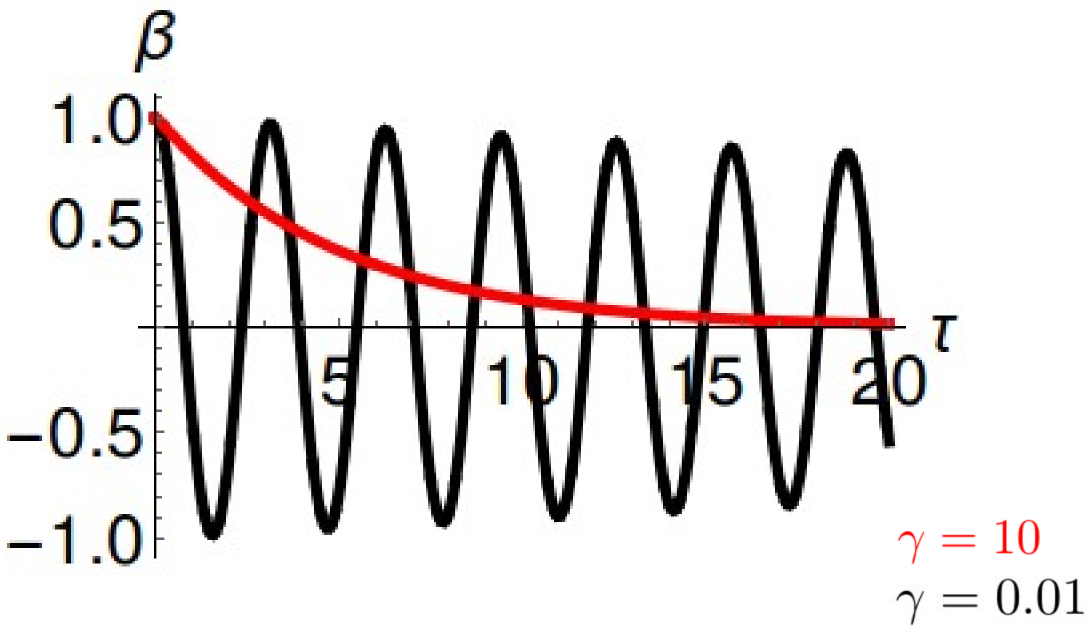
$$\delta = \sqrt{\gamma^2 - 4}$$

Dynamics under classical noise: Gaussian vs non-Gaussian

$$\rho(t) = \frac{1}{2}\begin{pmatrix} 1+\cos\theta & e^{-2i\omega_0}\beta(t,\gamma)\sin\theta \\ e^{2i\omega_0}\beta(t,\gamma)\sin\theta & 1-\cos\theta \end{pmatrix}$$

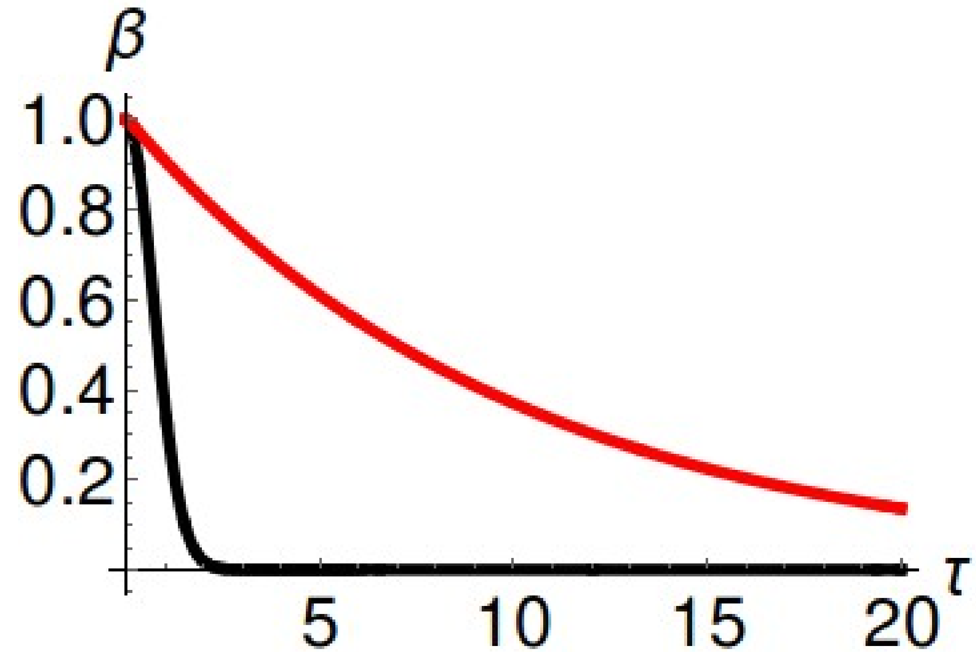
RTN

$$\beta_{RTN}(\gamma,\tau) = e^{-\gamma\tau}\left[\cosh\delta\tau + \frac{\gamma\sinh\delta\tau}{\delta}\right]$$



OU process

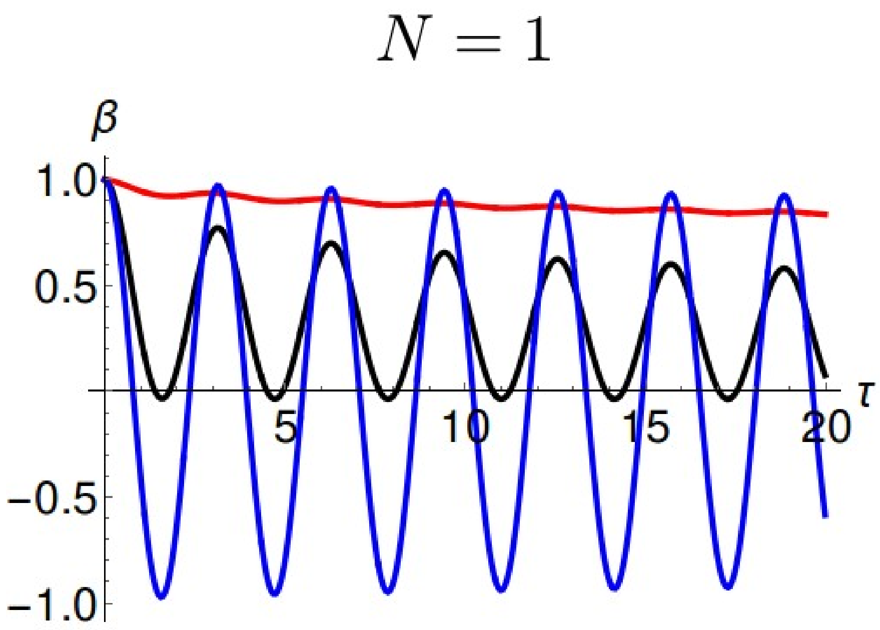
$$\beta_{OU}(\theta,\tau;\sigma) = \exp\left[-\frac{\sigma^2\left(\theta\tau - 1 + e^{-\theta\tau}\right)}{\theta^3}\right]$$

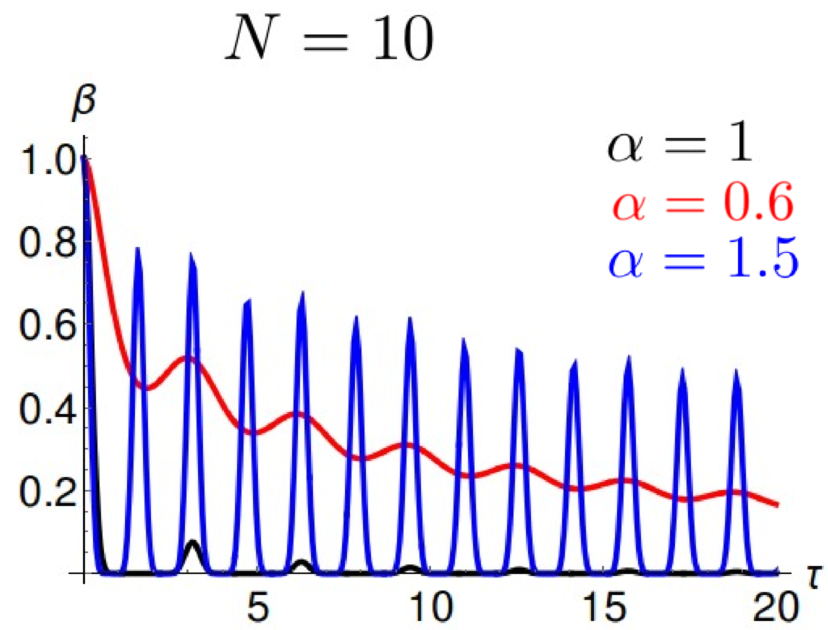


Dynamics under classical noise: $1/f^{\alpha}$

$$\rho(t) = \frac{1}{2}\begin{pmatrix} 1+\cos\theta & e^{-2i\omega_0}\beta(t,\alpha,N)\sin\theta \\ e^{2i\omega_0}\beta(t,\alpha,N)\sin\theta & 1-\cos\theta \end{pmatrix}$$

$$\beta(\tau,\alpha,N) = \int_{\gamma_1}^{\gamma_2} \beta_{RTN}(\{\gamma\},\tau)p_\alpha(\{\gamma\})d\{\gamma\}$$





Quantum estimation theory

$$\alpha \longrightarrow \rho_\alpha \longrightarrow \{x_i\} \longrightarrow \hat{\alpha} = \hat{\alpha}(x_1, x_{2,..}, x_M)$$

Not accessible

- Cramér-Rao Bound $$Var[\hat{\alpha}] \geq \frac{1}{M\,F(\alpha)}$$

- Fisher information $$F[\alpha] = \int dx\, p(x|\alpha)[\partial_\alpha \ln p(x|\alpha)]^2$$

Quantum estimation theory

$$\rho_\alpha \longrightarrow \alpha \longrightarrow \{x_i\} \longrightarrow \hat{\alpha} = \hat{\alpha}(x_1, x_2, .., x_M)$$

- Maximize the Fisher over the quantum measurements

$$F(\alpha) \leq G(\alpha)$$

- Precision of the optimal measurement

$$Var[\hat{\alpha}] \geq \frac{1}{M\, G(\alpha)}$$

- Quantum Fisher information

$$G(\alpha) = \sum_{n=1,2} \frac{(\partial_\alpha \rho_n)^2}{\rho_n} + 2 \sum_{n \neq m} \frac{(\rho_n - \rho_m)^2}{\rho_n + \rho_m} |\langle \phi_m | \partial_\alpha \phi_n \rangle|^2$$

- Quantum signal-to-noise ratio

$$R(\alpha) = \alpha^2 G(\alpha)$$

Our strategy

Maximize the extraction of information by optimizing:

- the <u>preparation of the probe</u>
- the <u>interaction time</u>
- the <u>measurement</u> at the output

The physical system

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)|1\rangle$$

$$\rho(t) = \frac{1}{2}\begin{pmatrix} 1+\cos\theta & e^{-2i\omega_0}\beta(t,\gamma)\sin\theta \\ e^{2i\omega_0}\beta(t,\gamma)\sin\theta & 1-\cos\theta \end{pmatrix}$$

$$\beta(t,\gamma) = E\left[e^{-2i\int_0^t B_\gamma(s)ds}\right]_B$$

$$p_\pm = \frac{1}{2}\left(1 \pm \sqrt{\cos^2\theta + \sin^2\theta\beta(t,\gamma)}\right)$$

$$|\pm\rangle = \ldots$$

$$\boxed{G(t,\gamma) = \frac{\left[\partial_\gamma\beta(t,\gamma)\right]^2 \sin^2\theta}{1-\beta^2(t,\gamma)}}$$

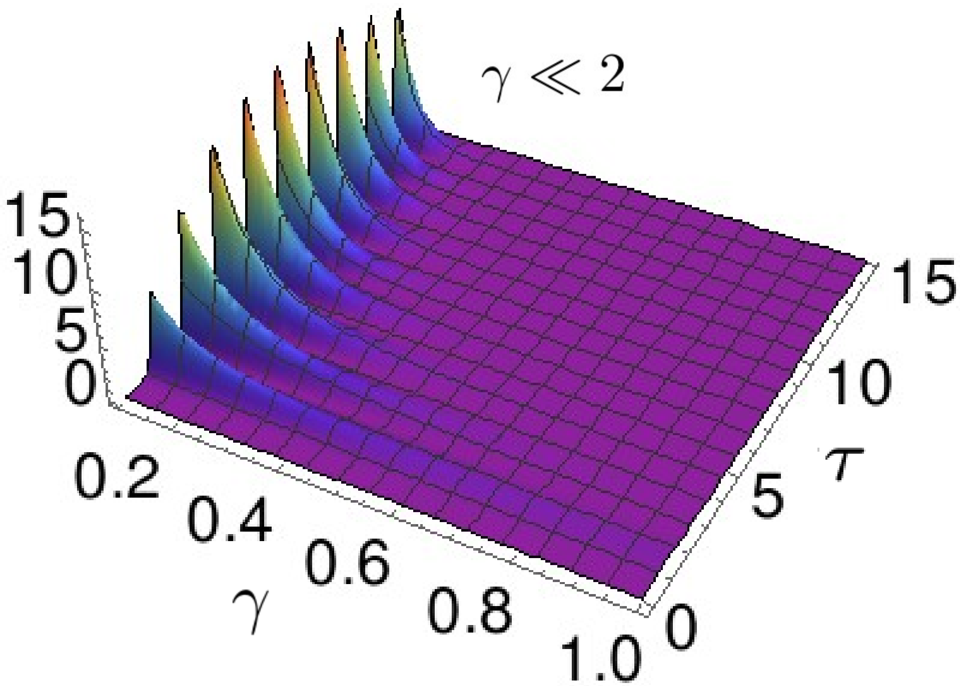
$$|\psi_{opt}\rangle = |+\rangle$$

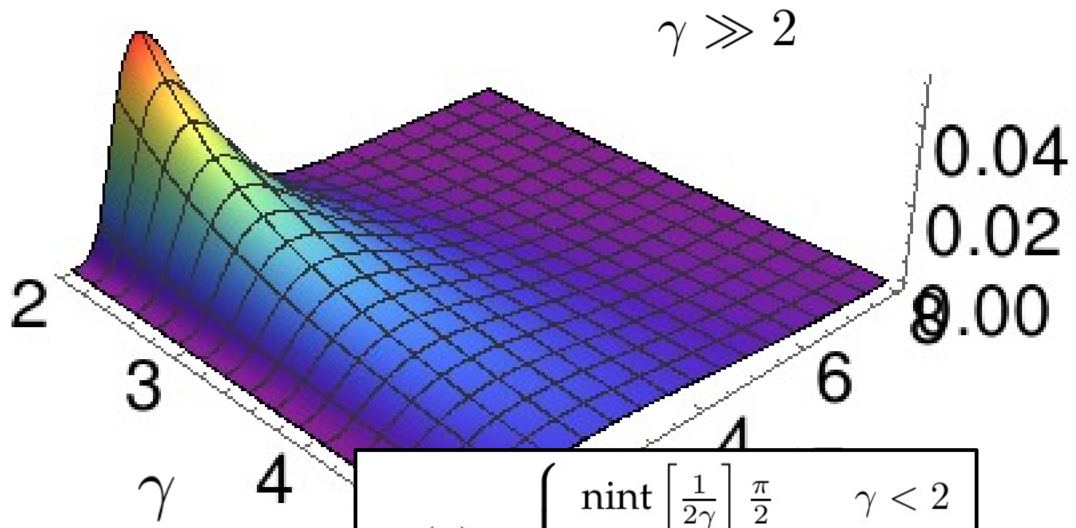
$$\Pi_{opt} = e^{i\omega_0 t\sigma_z}\sigma_x e^{-i\omega_0 t\sigma_z}$$

Quantum Fisher Information - RTN

$$G(t,\gamma) = \frac{\left[\partial_\gamma \beta(t,\gamma)\right]^2}{1-\beta^2(t,\gamma)}$$

$$\beta_{RTN}(\gamma,\tau) = e^{-\gamma\tau}\left[\cosh\delta\tau + \frac{\gamma\sinh\delta\tau}{\delta}\right]$$





$$\tau_M(\gamma) \simeq \begin{cases} \text{nint}\left[\frac{1}{2\gamma}\right]\frac{\pi}{2} & \gamma < 2 \\ \frac{2}{5}\gamma & \gamma > 2 \end{cases}$$

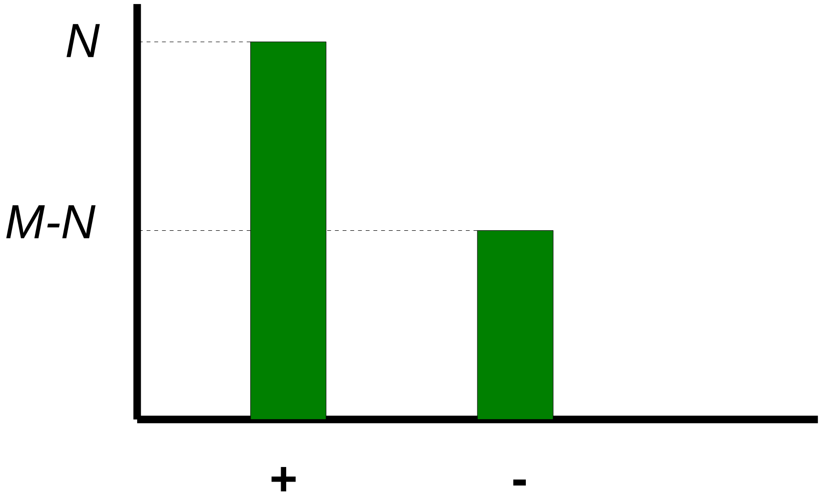
$$\boxed{G(\tau_{opt},\gamma) = \frac{a}{\gamma^2}} \longrightarrow \boxed{R(\tau_{opt},\gamma) = \text{constant}}$$

Bayesian estimator for the switching rate

Simulated experiment where we performed M repeated optimal measurements on a qubit and used the collected outputs to built an estimator

$$\rho(t) = \frac{1}{2}\begin{pmatrix} 1 & e^{-2i\omega_0}\beta(\gamma,t) \\ e^{2i\omega_0}\beta(\gamma,t) & 1 \end{pmatrix}$$

$$p_0 = p[|+\rangle] = \frac{1}{2}(1+\beta(t,\gamma))$$



N out M we measure the state $|+\rangle$

$$p(N|\gamma) \propto p_0(\gamma,t)^N (1-p_0(\gamma,t))^{M-N}$$

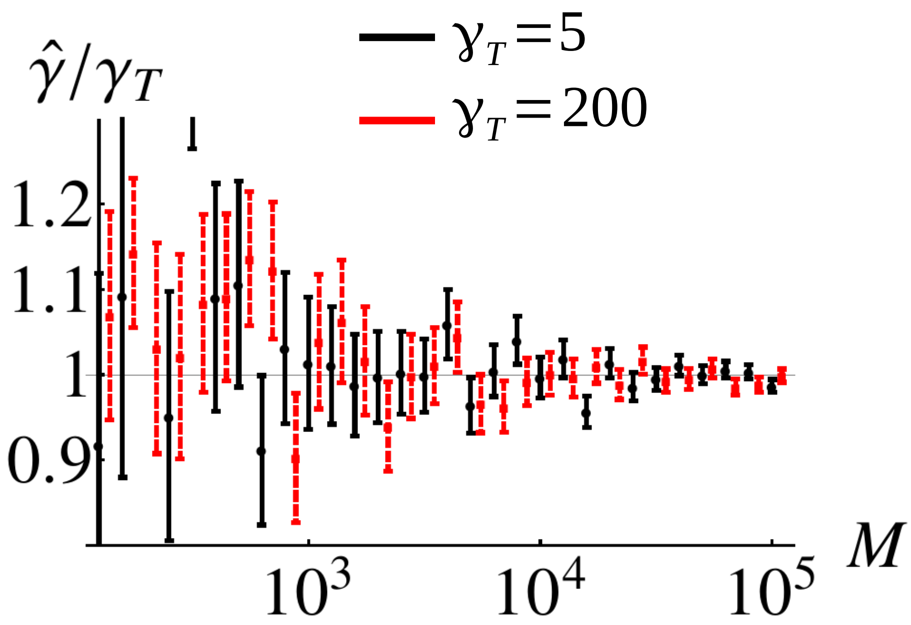
$$\underline{p(\gamma|N)}p(N) = p(N|\gamma)p(\gamma)$$

$$p(\gamma|N) \propto p_0(\gamma,t)^N (1-p_0(\gamma,t))^{M-N}$$

$$\boxed{\begin{aligned} \hat{\gamma} &= \int \gamma p(\gamma|N) d\gamma \\ Var[\hat{\gamma}] &= \int (\gamma-\hat{\gamma})^2 p(\gamma|N) d\gamma \end{aligned}}$$

Bayes estimator for the switching rate II

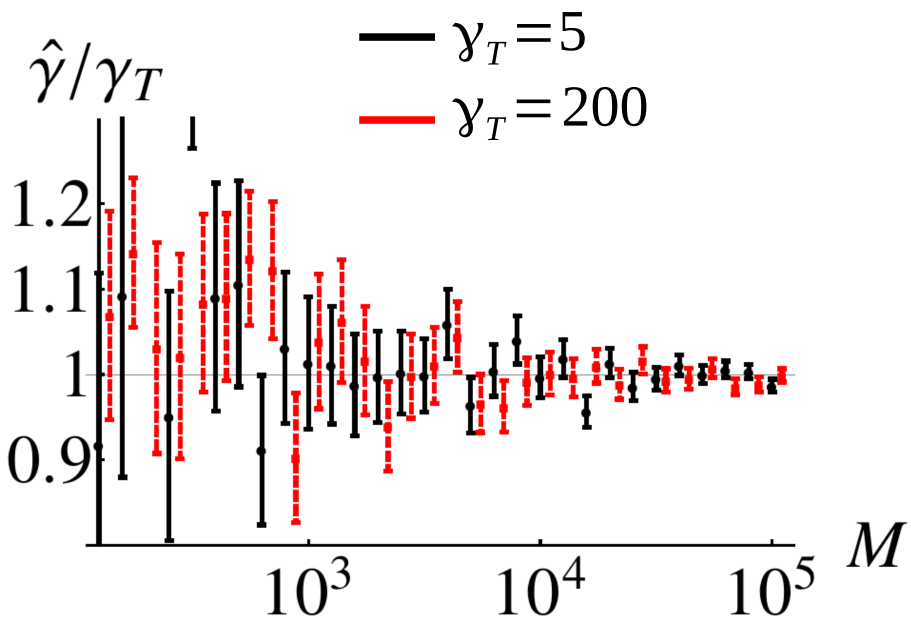
Simulated repeated measurements on a qubit

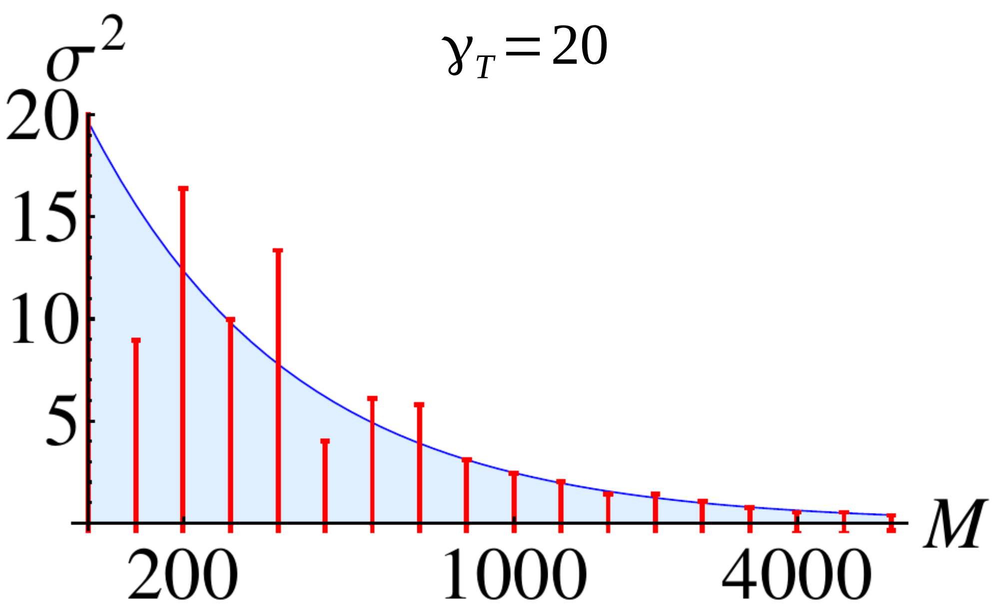


$$R(\tau_{opt}, \gamma) = \text{constant}$$

Bayes estimator for the switching rate II

Simulated repeated measurements on a qubit



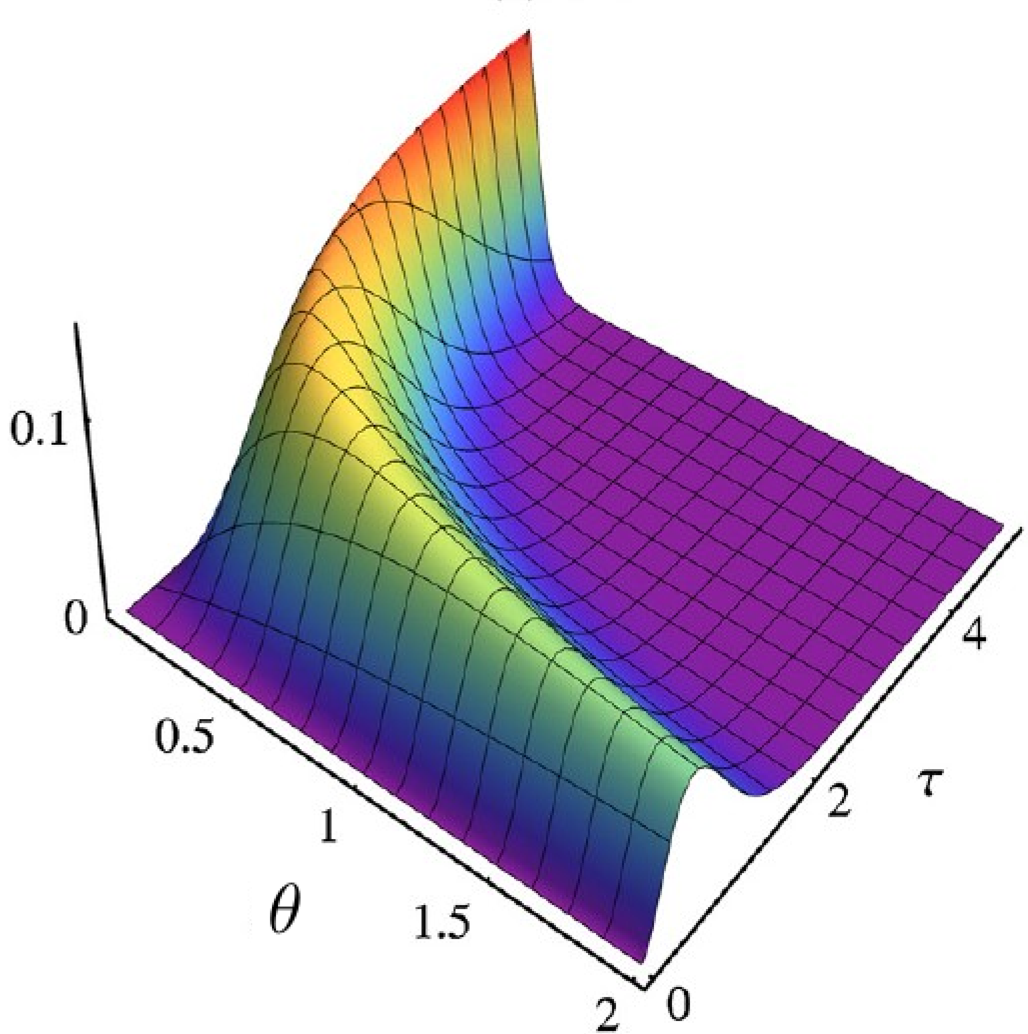


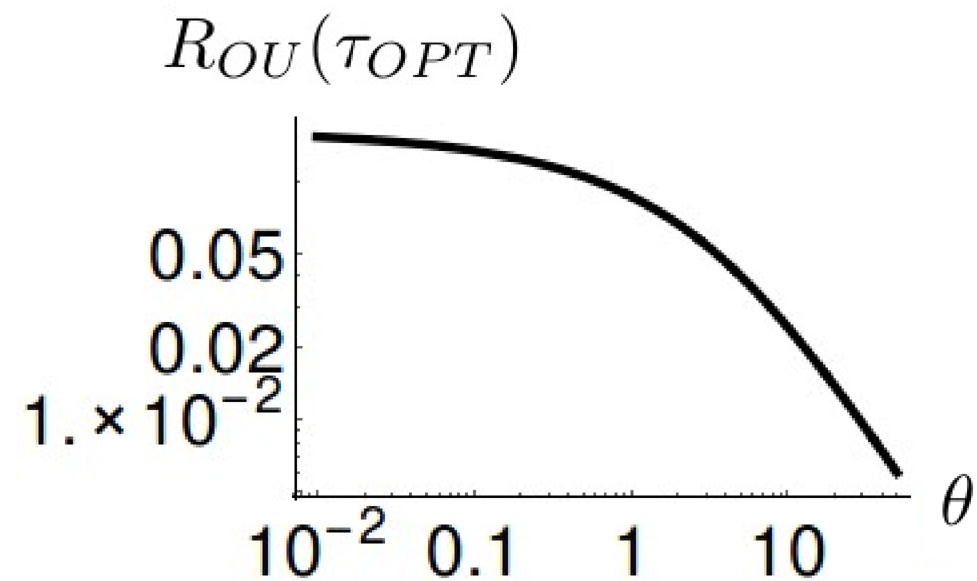
$$R(\tau_{opt}, \gamma) = \text{constant}$$

$$Var[\hat{\gamma}] \geq \frac{1}{M\,G(\gamma)}$$

OU process

$$R_{OU}(\theta,\tau) = \frac{4e^{-2\theta\tau}}{\theta^2}\left[\frac{(1-e^{-\theta\tau}+\theta\tau)^2}{e^{4\left(\tau+\frac{e^{-\theta\tau}-1}{g}\right)}-1}\right]$$





ML estimator for the OU noise

Simulated repeated measurements on a qubit

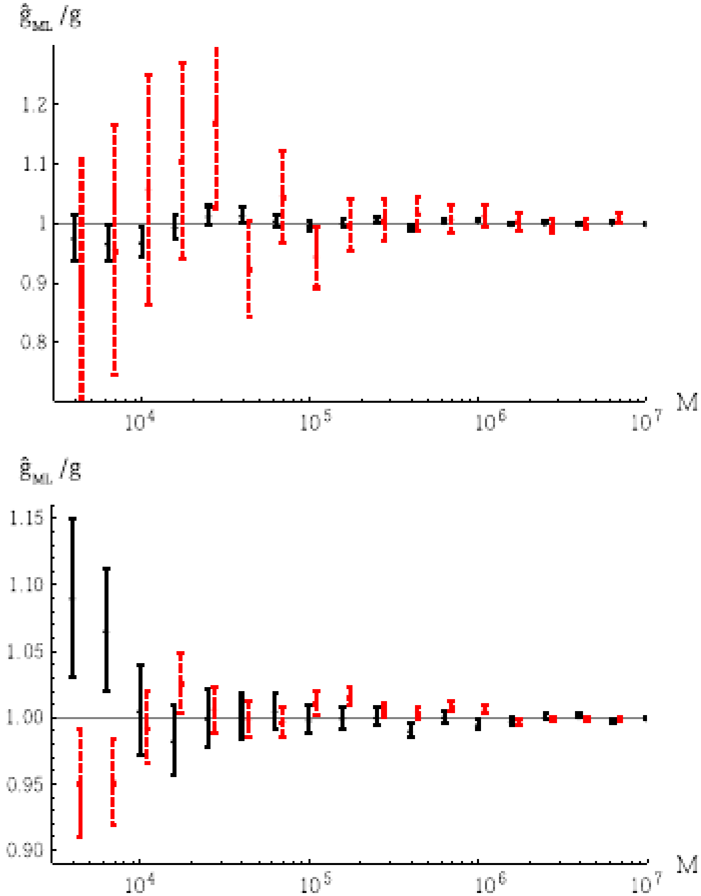


Fig. 3. (Color online.) The two panels show the ratio g_{ML}/g between the ML estimated value of g and the true value, together with the corresponding error bars, as a function of the number of repeated (simulated) measurements M. In the upper panel the results for the true values $g = 0.01$ (solid black) and $g = 100$ (red dashed) are compared. Larger values of the parameter are better estimated. In the lower panel the considered values are $g = 0.1$ (solid black) and $g = 1$ (red dashed). Notice that the simulated data in both panels are computed for the same values of M and then the red points are slightly shifted along the x-axes for the sake of clarity.

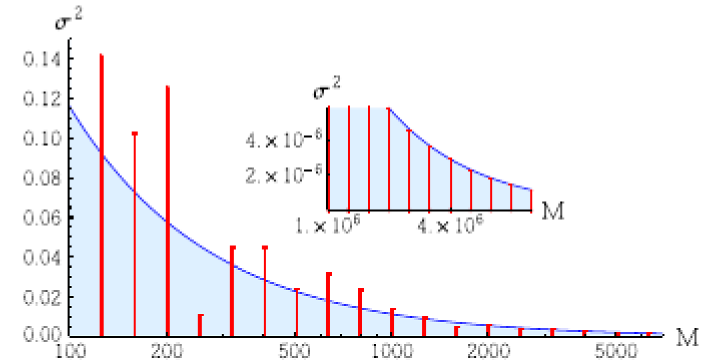


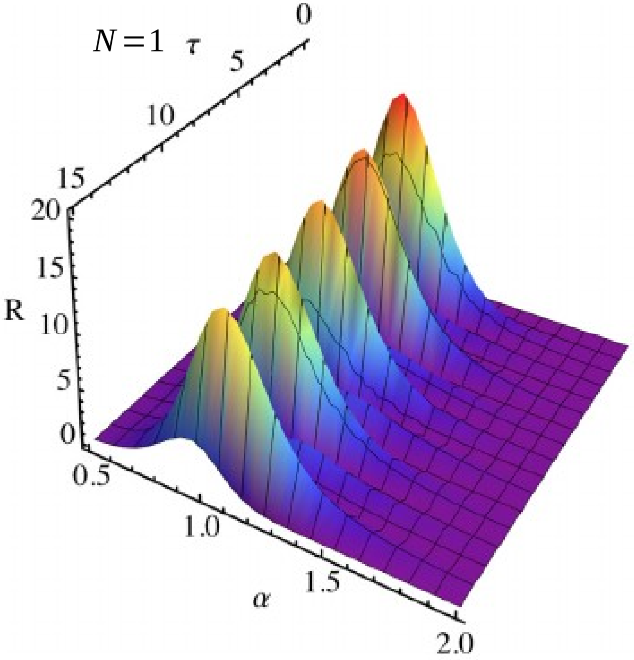
Fig. 4. (Color online.) The variance (red line) of the ML estimator as a function of the number of measurements for the case $g = 1$. The light blue area illustrates the QCR bound. Variances below the quantum bound mean that the estimator has a bias. Inset: The same as in the main frame but for a large number of measurements: the bias is no longer present.

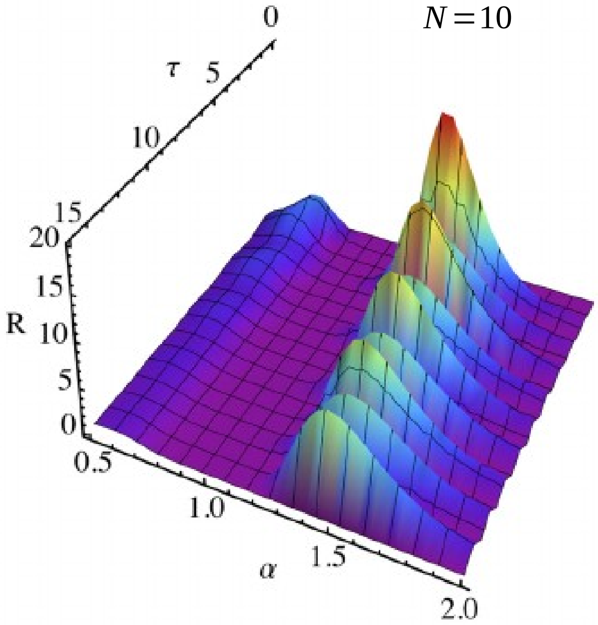
CB, M. Paris, Phys. Lett. A 378, 2495 (2014)

QET – 1/f $^{\alpha}$ noise

$$H(\tau,\alpha,N) = N^2 \frac{\beta(\tau,\alpha)^{2N-2}}{1-\beta(\tau,\alpha)^{2N}}[\partial_\alpha \beta(t,\alpha)]^2$$

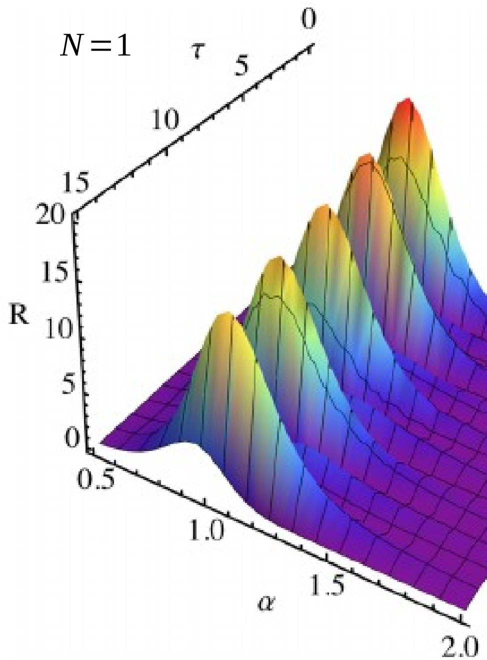
$$\beta(\tau,\alpha,N) = \int_{\gamma_1}^{\gamma_2} \beta_{RTN}(\{\gamma\},\tau)p_\alpha(\{\gamma\})d\{\gamma\}$$

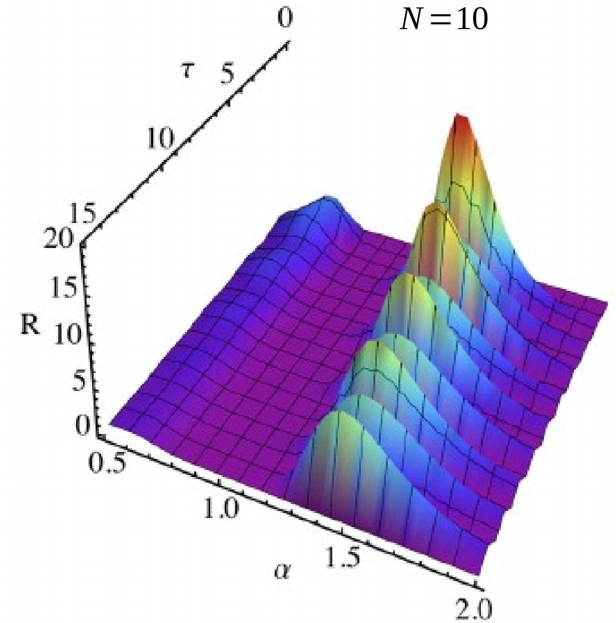


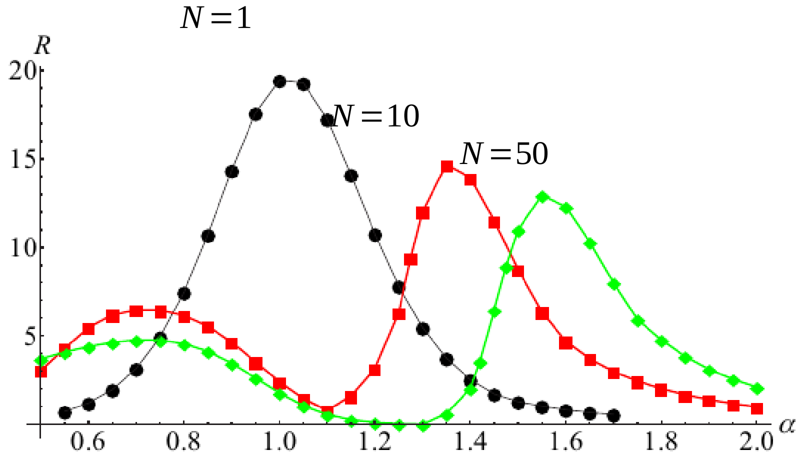


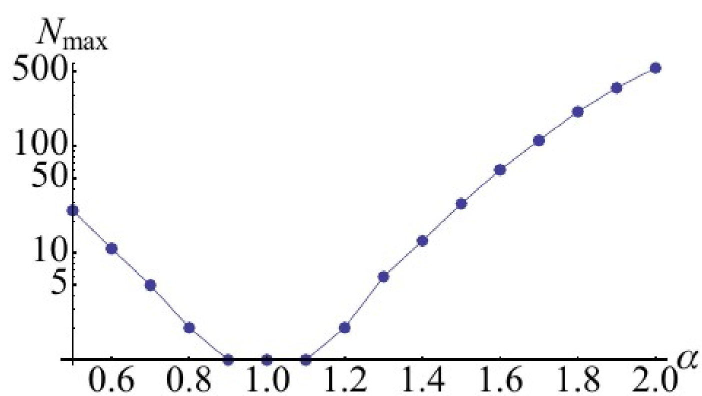
QET - Results

$$H(\tau,\alpha,N) = N^2\ \frac{\beta(\tau,\alpha)^{2N-2}}{1-\beta(\tau,\alpha)^{2N}}[\partial_\alpha\,\beta(t,\alpha)]^2$$









[redacted]

PHYSICAL REVIEW A **92**, 010302(R) (2015)

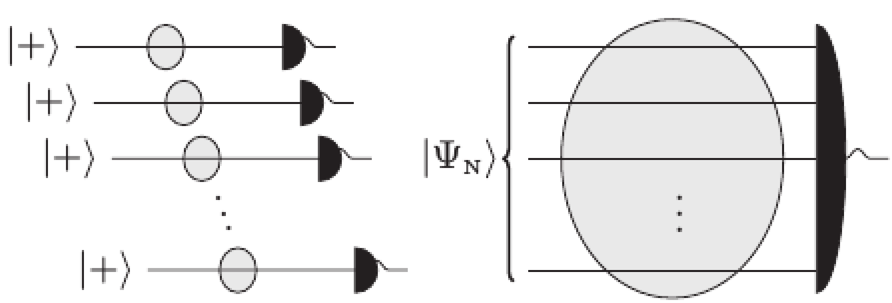
Entangled quantum probes for dynamical environmental noise

Matteo A. C. Rossi* and Matteo G. A. Paris†
Dipartimento di Fisica, Università degli Studi di Milano, 20133 Milano, Italy

(Received 17 March 2015; published 28 July 2015)

We address the use of entangled qubits as quantum probes to characterize the noise induced by complex environments. In particular, we show that a joint measurement on entangled probes can improve estimation of the correlation time for a broad class of environmental noises compared to sequential strategies involving single-qubit preparation. The enhancement appears when the noise is faster than a threshold value, a regime which may always be achieved by tuning the coupling between the quantum probe and the environment inducing the noise. Our scheme exploits time-dependent sensitivity of quantum systems to decoherence and does not require dynamical control on the probes. We derive the optimal interaction time and the optimal probe preparation, showing that it corresponds to multiqubit Greenberger-Horne-Zeilinger states when entanglement is useful. We also show the robustness of the scheme against depolarization or dephasing of the probe, and discuss simple measurements approaching optimal precision.

DOI: 10.1103/PhysRevA.92.010302 PACS number(s): 03.67.Bg, 03.65.Yz, 03.67.−a, 05.40.−a



$$H_{\mathrm{N}}^{\mathrm{SEP}}(\gamma,t) = \frac{4N}{e^{4\beta_\gamma(t)}-1}[\partial_\gamma \beta_\gamma(t)]^2.$$

$$H_{\mathrm{N}}^{\mathrm{GHZ}}(\gamma,t) = \frac{4N^4}{e^{4N^2\beta(t,\gamma)}-1}[\partial_\gamma \beta_\gamma(t)]^2.$$

Concluding remarks

- Stochastic modeling is a convenient choice to describe the interaction of a quantum system with its environment

- Quantum probes represent a resource to characterize classical environments (without collecting time series)
  Using the tools of QET, we are able to maximize the information gained by optimizing the preparation of the probe, interaction time and measurement

- Qualitative study, Need for more quntitative and comparisons

- C. Benedetti, F. Buscemi, P.Bordone, M. G. A. Paris, **Phys. Rev. A 87, 052328** (2013)
- C. Benedetti, M. G. A. Paris, S. Maniscalco, **Phys. Rev. A 89, 012114** (2014)
- C. Benedetti, F. Buscemi, P.Bordone, M. G. A. Paris**, Phy. Rev. A 89, 032114** (2014)
- C. Benedetti, M. G. A. Paris **Phys. Lett. *A* 378, 2495** (2014)
- D. Tamascelli, C. Benedetti, S. Olivares, M. G. A Paris, (2016) In preparation (precision b
  Feynman probes..)