Local versus Global Strategies in Multi-parameter Estimation

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Overview

- The role of inter- and intra-path correlations in interferometry
- Scheme for exploiting the "local" effects on each path
- Extension to multi-parameter estimation and local versus global strategies
- Examples: multipath interferometry and imaging
- Interpretation and caveats

Quantum Fisher Information

Cramer- Rao bound $$\Delta\phi \geq \frac{1}{\sqrt{\mu\mathcal{F}}}$$

Pure state Ψ

(assuming path symmetry)

$$\mathcal{F}(\Psi) = 2(\mathrm{Var} - \mathrm{Cov})$$

Photon number covariance between two modes of input state

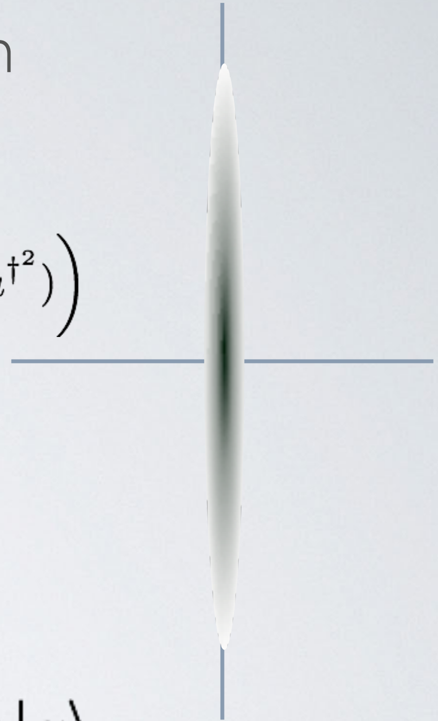
Photon number variance of input state

Depends on number variance on each path as well as correlations between paths

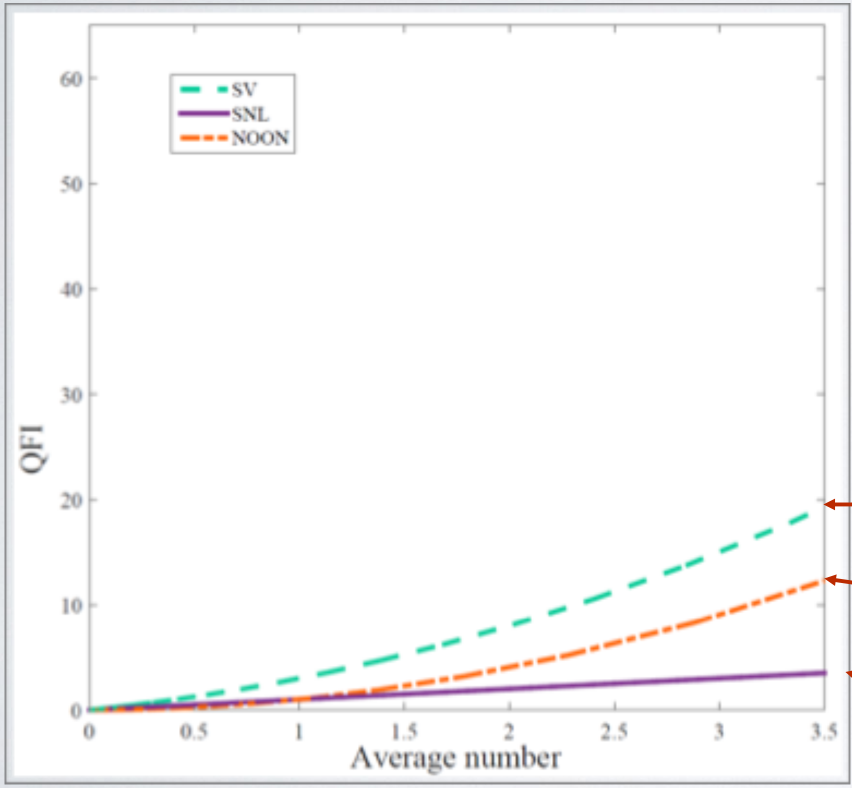
$$\mathcal{F}(\Psi) = 2(\mathrm{Var} - \mathrm{Cov})$$

The Squeezed vacuum: $|z\rangle = S(z)|0\rangle$ ← Vacuum

$$S(z) = \exp\left(\frac{1}{2}(z^*\hat{a}^2 - z\hat{a}^{\dagger 2})\right)$$



$$|\Psi_{\mathrm{SV}}\rangle = |z\rangle|z\rangle$$



Squeezed vacuums $\mathcal{F} = \bar{n}^2 + 2\bar{n}$

NOON state $\mathcal{F} = \bar{n}^2$

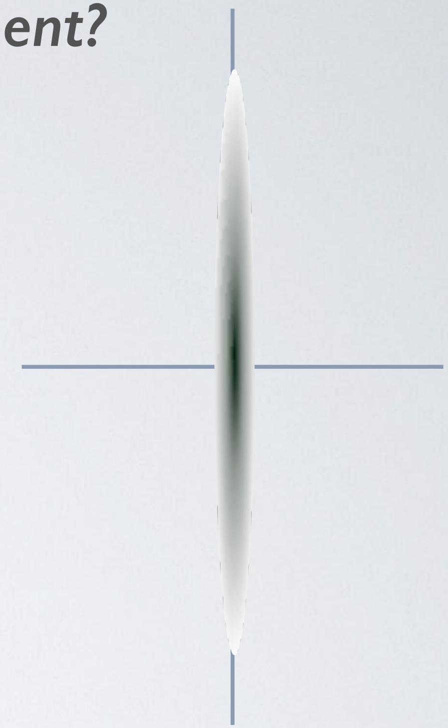
Shot noise limit $\mathcal{F} = \bar{n}$

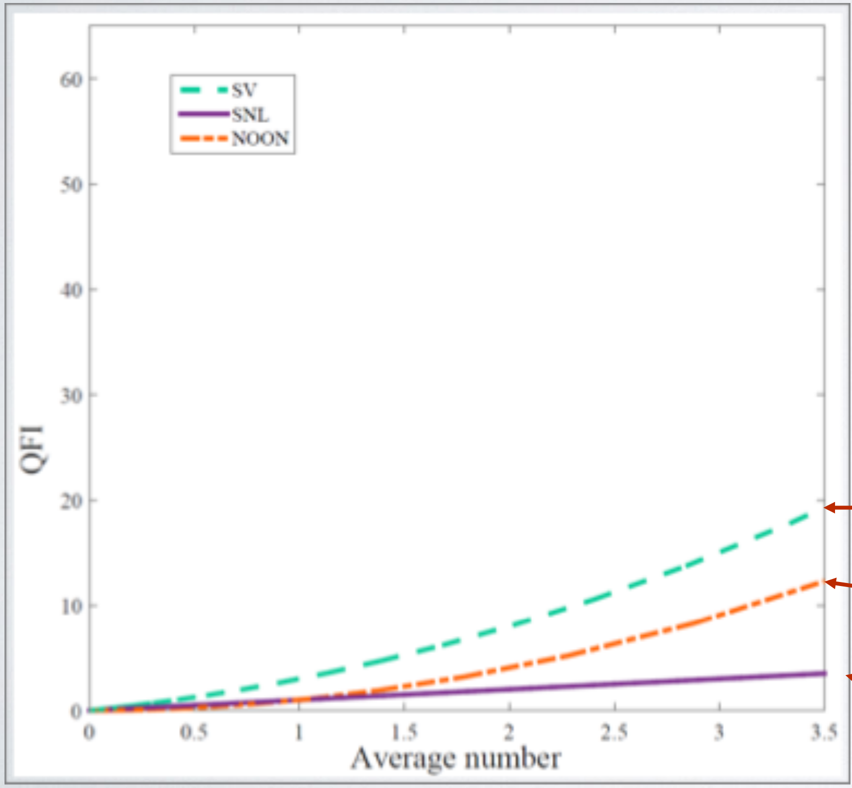
A.N. Boto, et al., Phys. Rev. Lett. 85, 2733 (2000); L. Pezzé, A. Smerzi, Phys. Rev. Lett. 100, 073601 (2008)
Petr M. Anisimov et al., Phys. Rev. Lett. 104, 103602 (2010);
R. Demkowicz-Dobrzanski, M. Jarzyna, J. Kolodynski arXiv:1405.7703 (2014)

The Squeezed entangled state

$$\mathcal{F}(\Psi) = 2(\mathrm{Var} - \mathrm{Cov})$$

Squeezing + path entanglement?





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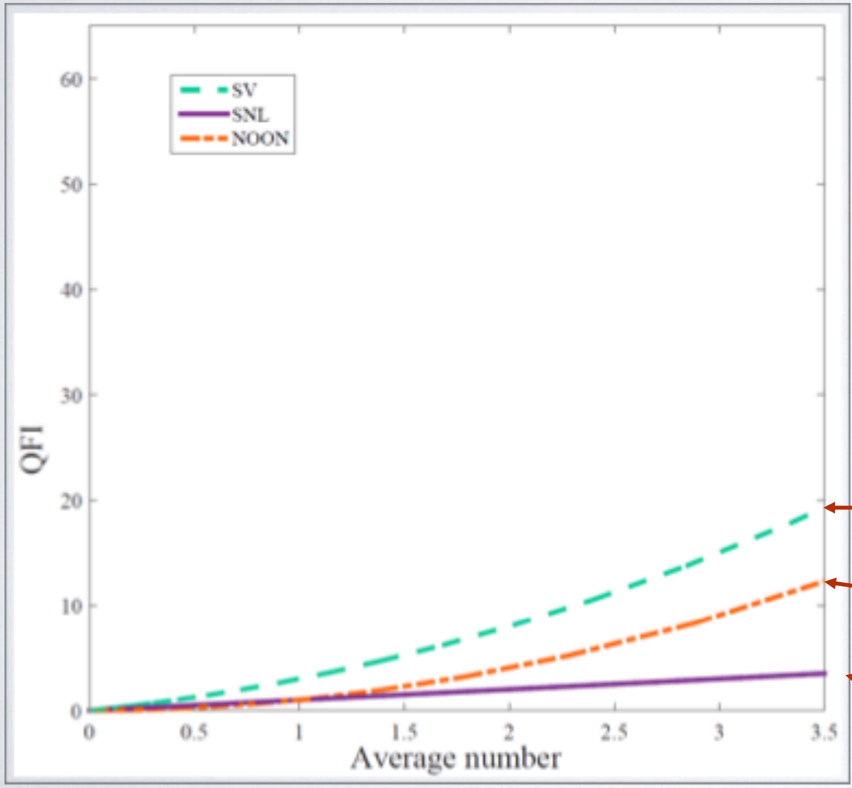
P. A. Knott, T.J. Proctor, A. Hayes, J. P. Cooling and J. Dunningham arXiv:1505.04011 (2015)

The Squeezed entangled state

$$\mathcal{F}(\Psi) = 2(\text{Var} - \text{Cov})$$

Squeezing + path entanglement?

The squeezed entangled state: $|\Psi_{\text{SES}}\rangle \propto |z, 0\rangle + |0, z\rangle$



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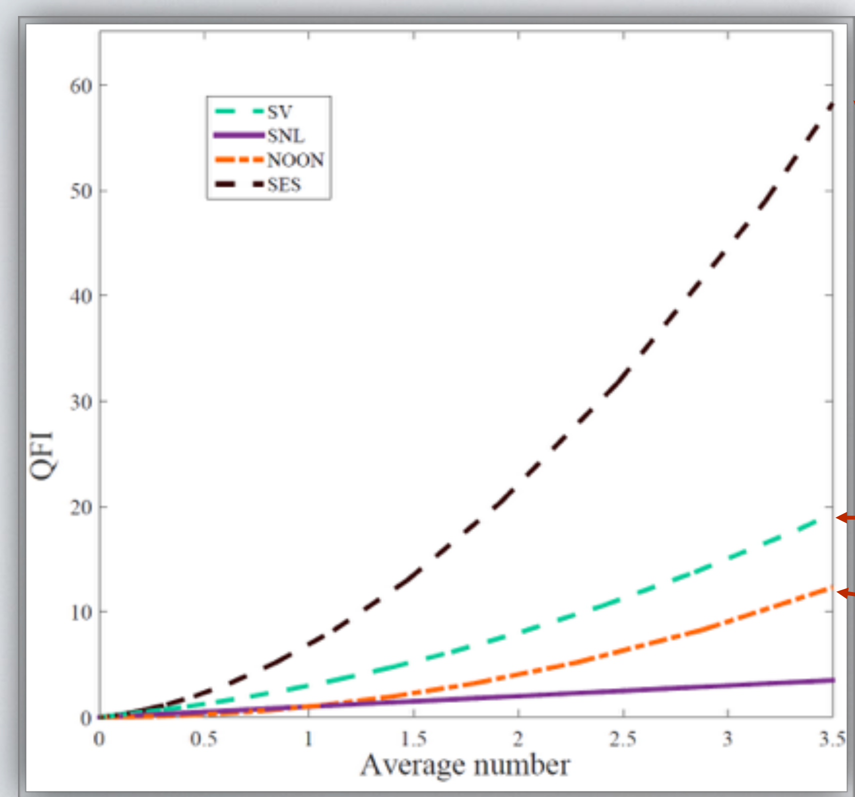

Squeezed entangled state

$$\mathcal{F} \approx 3\bar{n}^2 + 2\bar{n}$$

Squeezed vacuums $\mathcal{F} = \bar{n}^2 + 2\bar{n}$

NOON state $\mathcal{F} = \bar{n}^2$

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Squeezing cats

Entanglement between paths is **not** needed for quantum metrology.

$$\mathcal{F}(\Psi) = \bar{n}(1 + \mathcal{Q})(1 - \mathcal{J})$$

J. Sahota and N. Quesada, Phys. Rev. A 91, 013808 (2015)

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Mandel Q parameter

$$\mathcal{Q}_a = \frac{\mathrm{Var}(\hat{n}_a) - \bar{n}_a}{\bar{n}_a}$$

Measure of multi-mode correlations:

(Pearson's correlation coefficient)

$$\mathcal{J}_{ab} = \frac{\mathrm{Cov}(\hat{n}_a, \hat{n}_b)}{\sqrt{\mathrm{Var}(\hat{n}_a)\mathrm{Var}(\hat{n}_b)}}$$

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Bounded: $-1 \leq \mathcal{J} \leq 1$

Path entanglement at most factor of 2 enhancement

J. Sahota and N. Quesada, Phys. Rev. A 91, 013808 (2015)

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Squeezing cats

Cat states:

$$|\psi_{\mathrm{CAT}}\rangle \propto |\alpha\rangle + |-\alpha\rangle$$

Squeezed cat states:

$$|\psi_{\mathrm{SCS}}\rangle \propto S(z)(|\alpha\rangle + |-\alpha\rangle)$$

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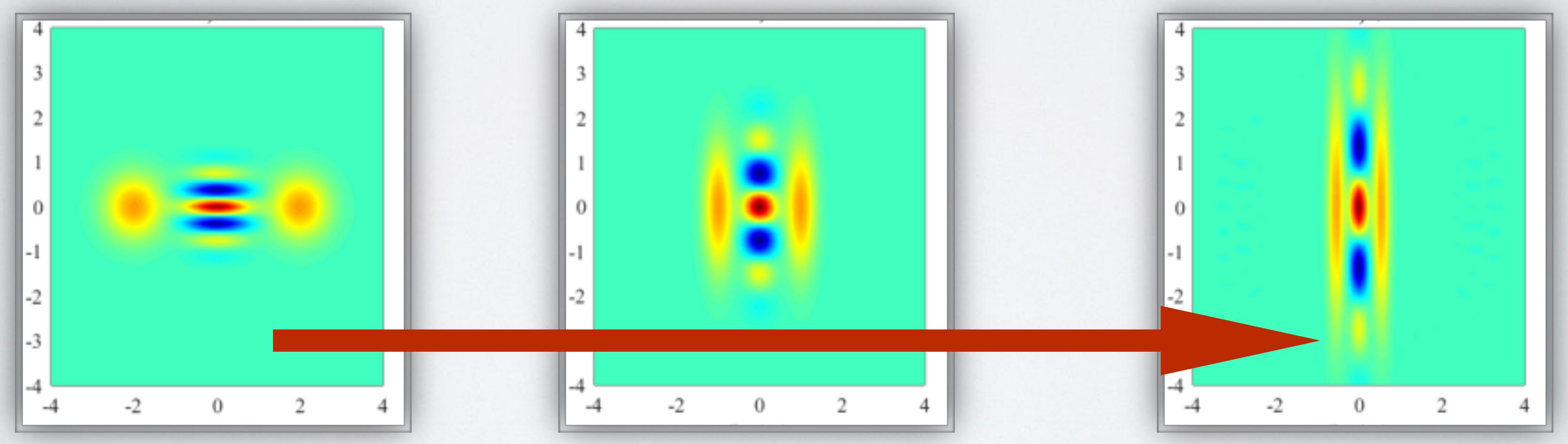
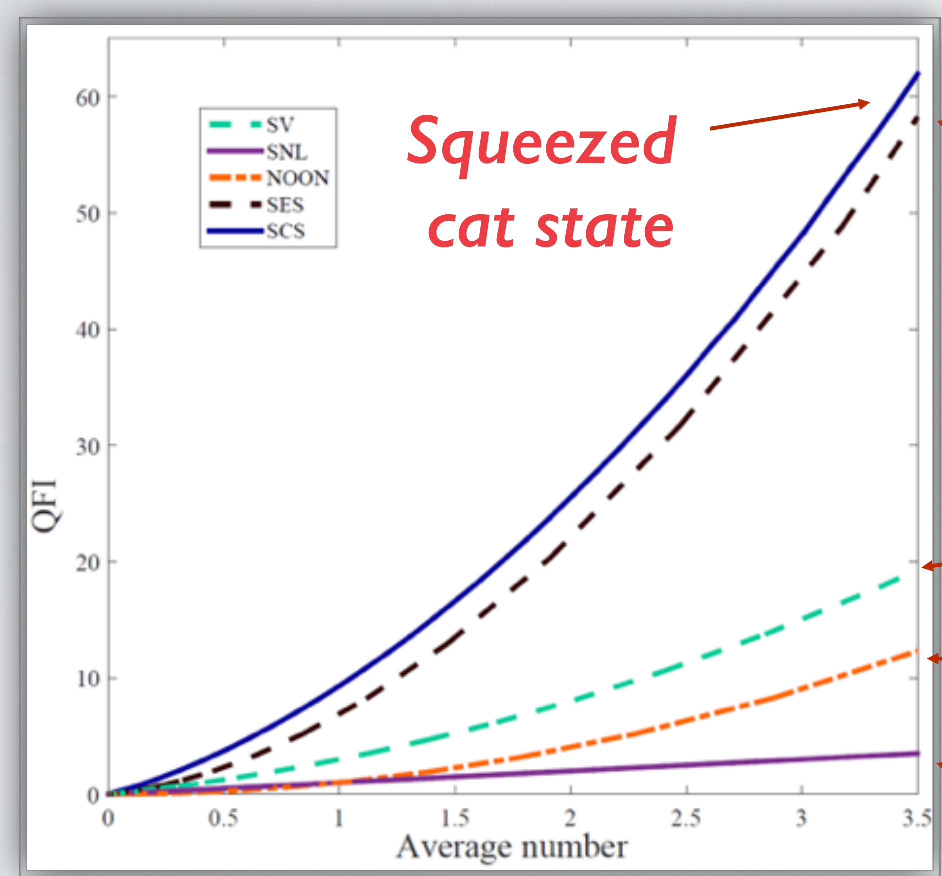


Increasing squeezing

P. A. Knott, T.J. Proctor, A. Hayes, J. P. Cooling and J. Dunningham arXiv:1505.04011 (2015)

Squeezing cats

Probe state: $$|\Psi_{\text{SCS}}\rangle = |\psi_{\text{SCS}}\rangle \otimes |\psi_{\text{SCS}}\rangle$$



Squeezed entangled state

Squeezed vacuums

NOON States

Shot noise limit

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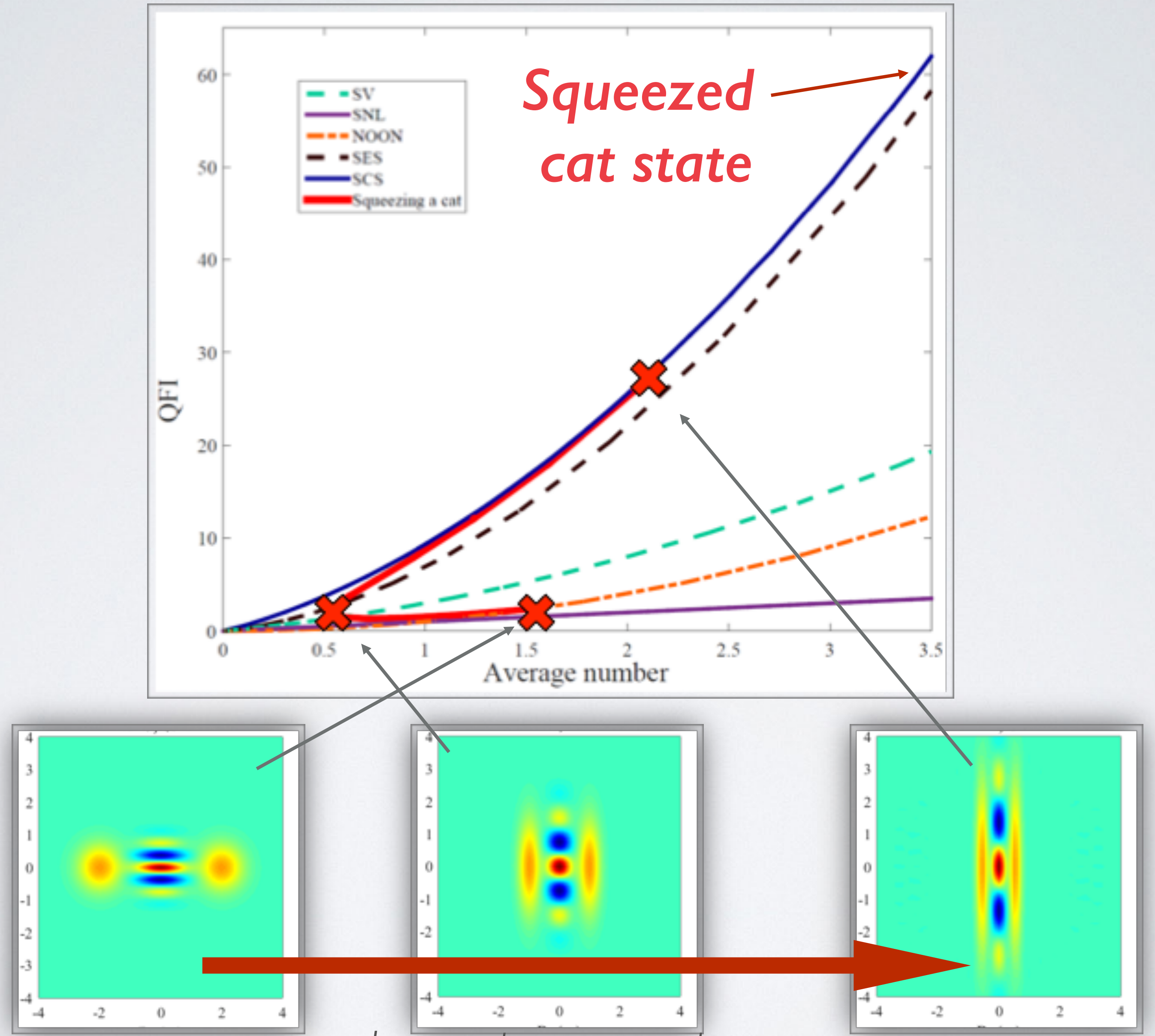
Squeezing cats



Increasing squeezing

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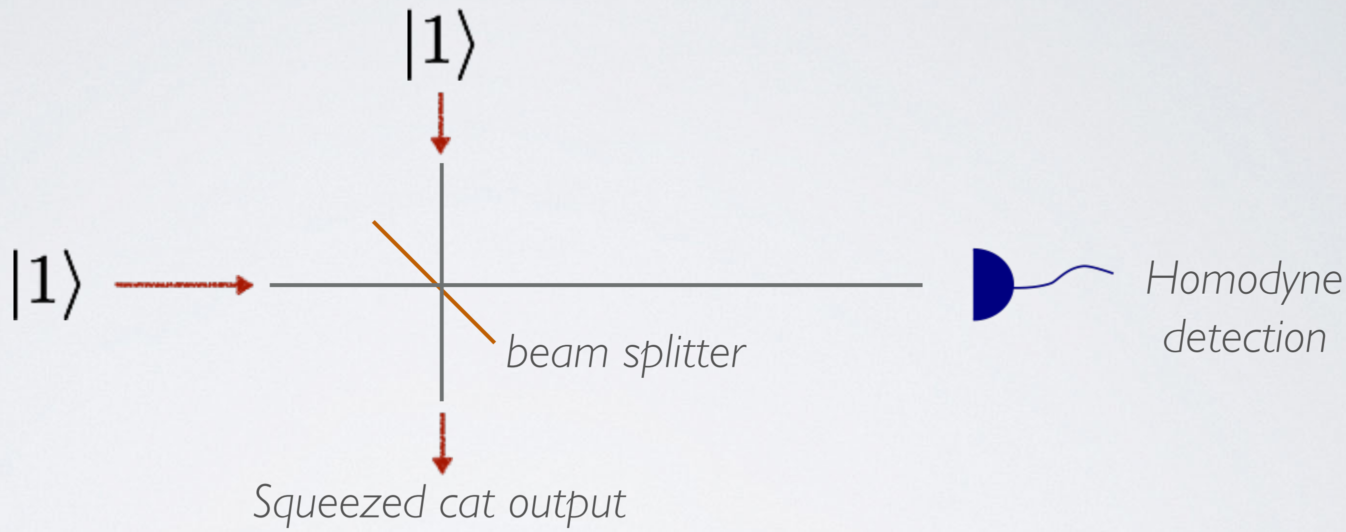
Squeezed cats have been made

Method of Etesse *et. al:*



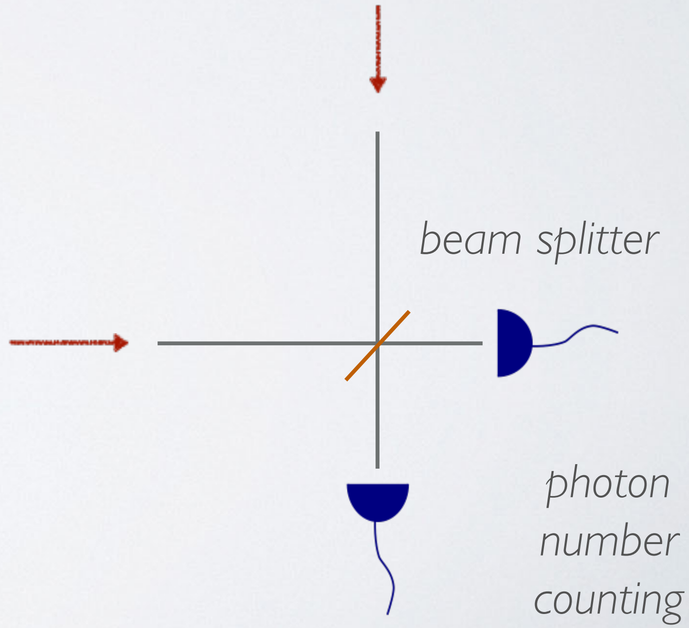
J. Etesse, M. Bouillard, B. Kanseri, and R. Tualle-Brouri, Phys. Rev. Lett. 114, 193602 (2015).

K. Huang, H. L. Jeannic, J. Ruaudel, V. Verma, M. Shaw, F. Marsili, S. Nam, E. Wu, H. Zeng, Y.-C. Jeong, et al., arXiv preprint arXiv:1503.08970 (2015).

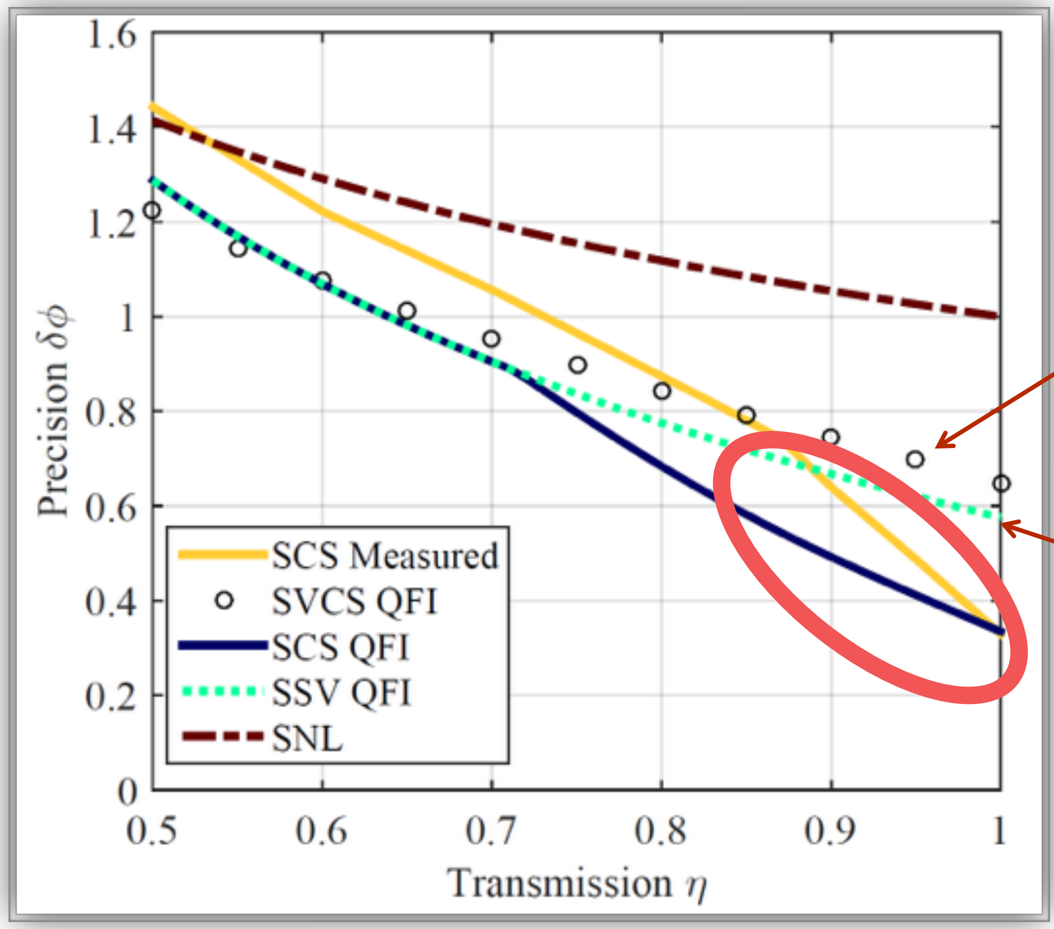
A. Ourjoumtsev, H. Jeong, R. Tualle-Brouri, and P. Grangier, Nature 448, 784 (2007)

Realistic interferometry

Measurement scheme:
saturates the CR-bound.

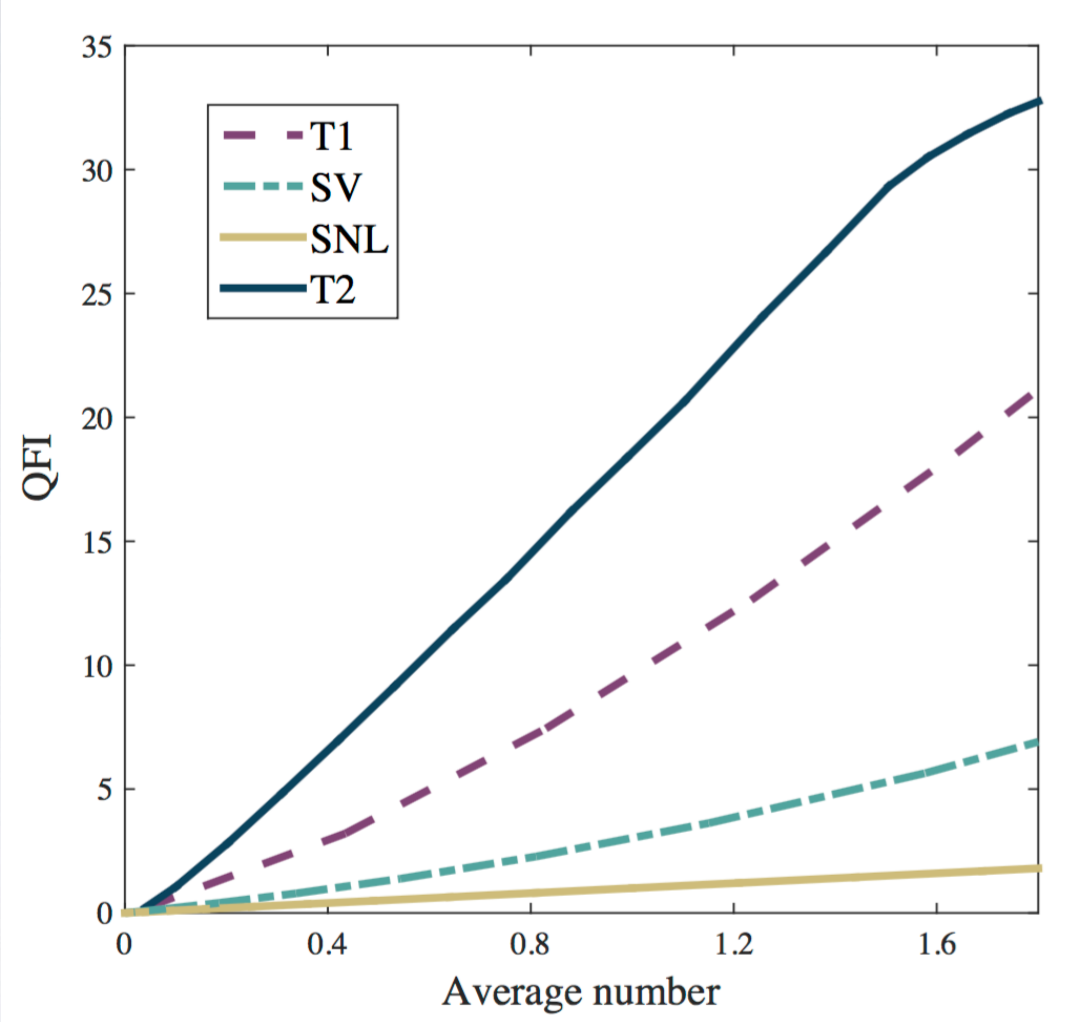


Results with loss



Measurement results are for a Bayesian simulation with around 100 repeats

Evolutionary algorithms



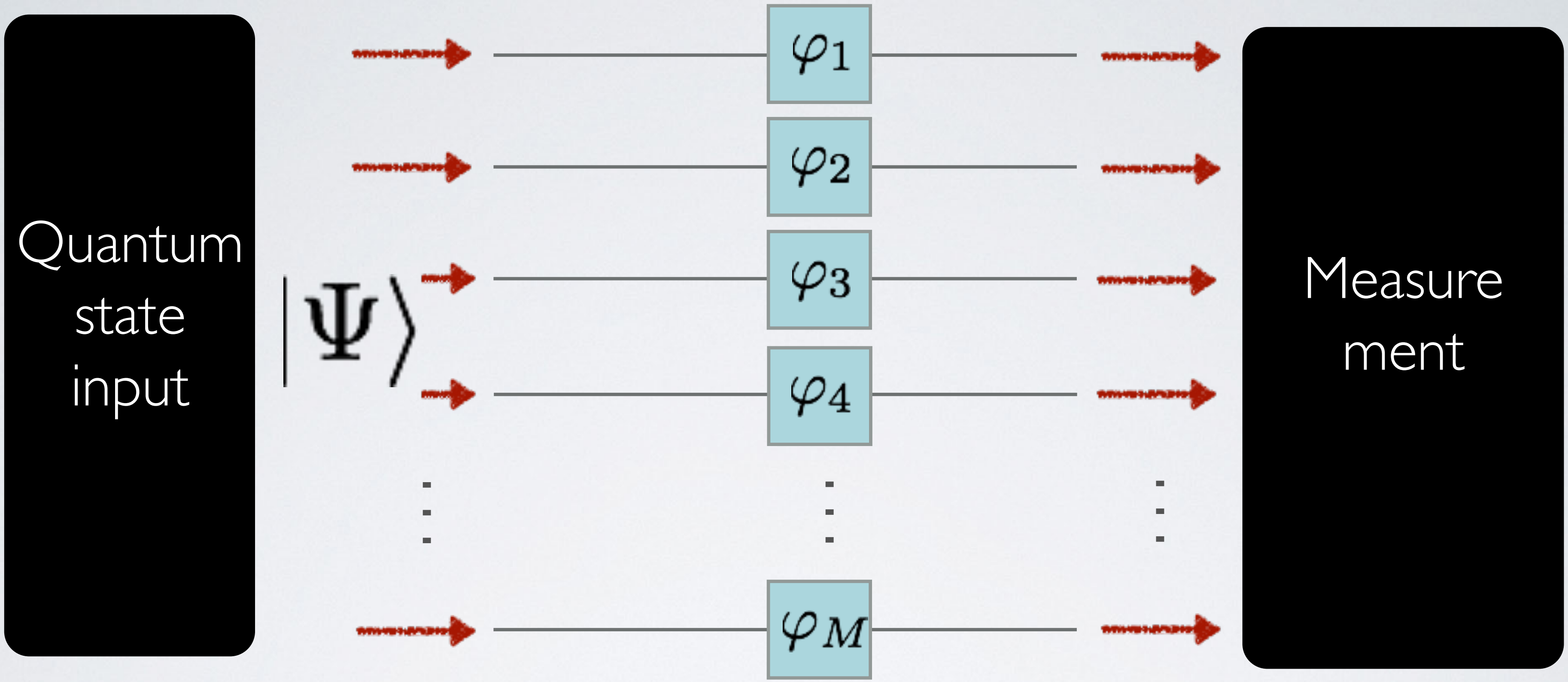
Evolutionary algorithms

Squeezed vacuums

Shot noise limit

P. A. Knott arXiv:1511.05327

Multi-parameter optical metrology

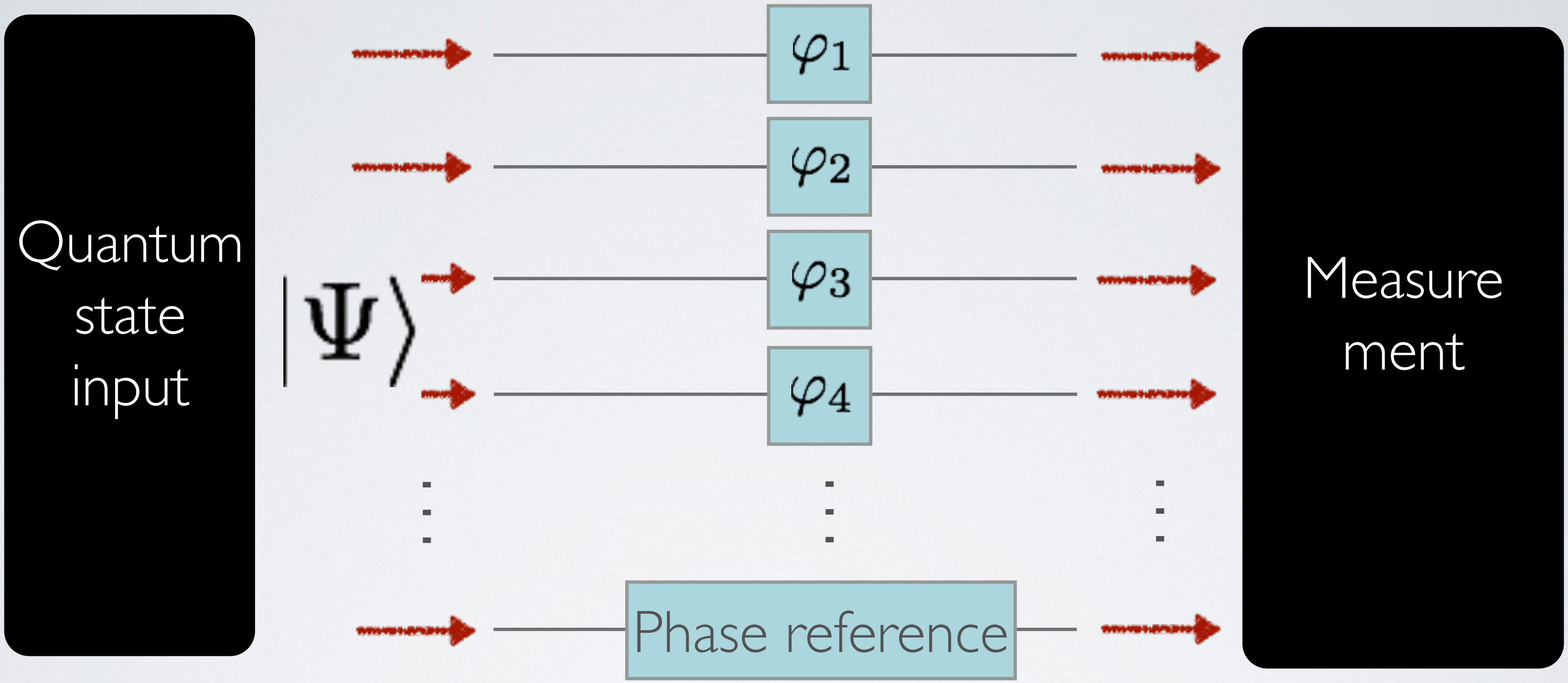


Aim: Estimate *d* parameters $(\phi_1, \phi_2, \ldots, \phi_d)$

Functions of the *M* physical φ_j parameters

T. J. Proctor, P. A. Knott, A. J. Hayes, J. F. Ralph, P. Kok, J. A. Dunningham arXiv:1601.05912

Multi-parameter optical metrology



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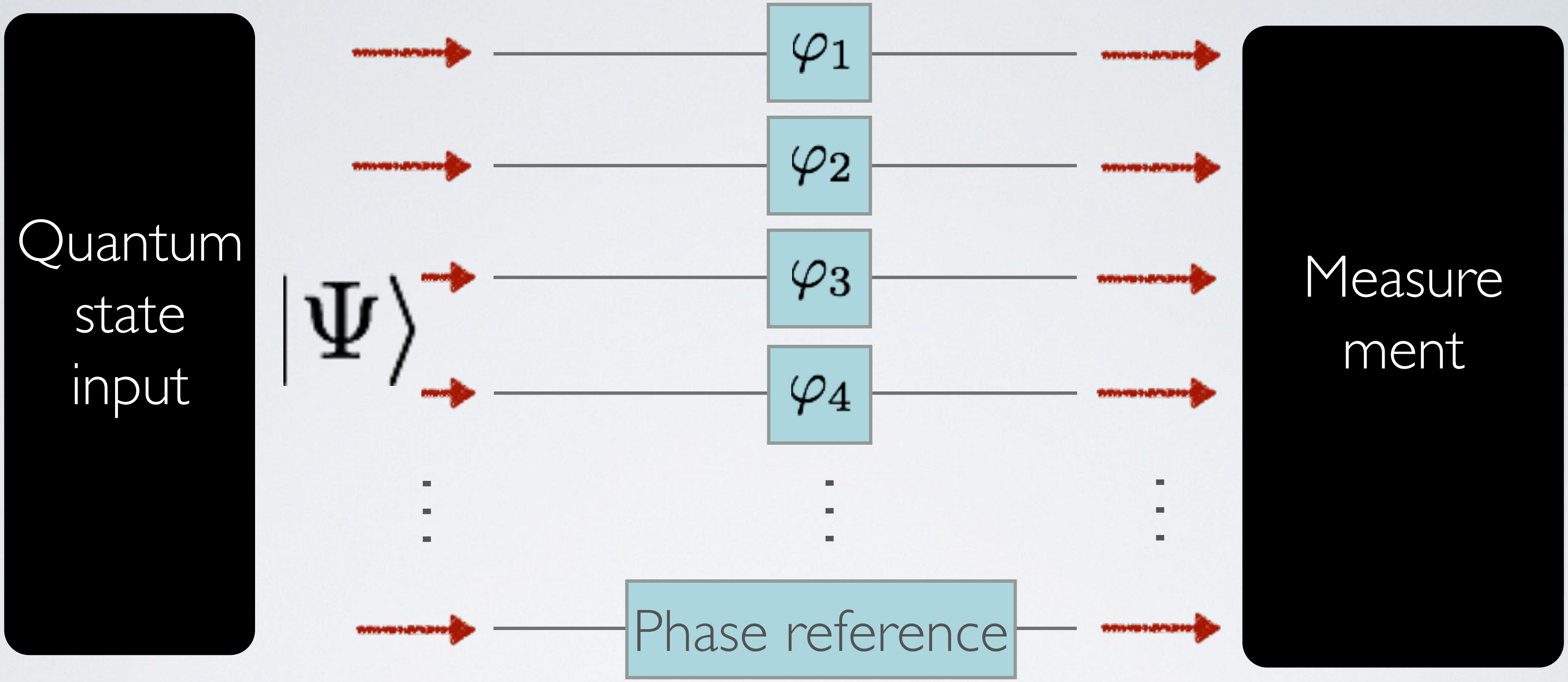
P. C. Humphreys, M. Barbieri, A. Datta, and I. A. Walmsley, Phys. Rev. Lett. 111, 070403 (2013).

Multi-parameter optical metrology



Local strategy

$$|\Psi_{\text{NOON}}\rangle = \frac{1}{\sqrt{2}}(|N',0\rangle + |0,N'\rangle)$$

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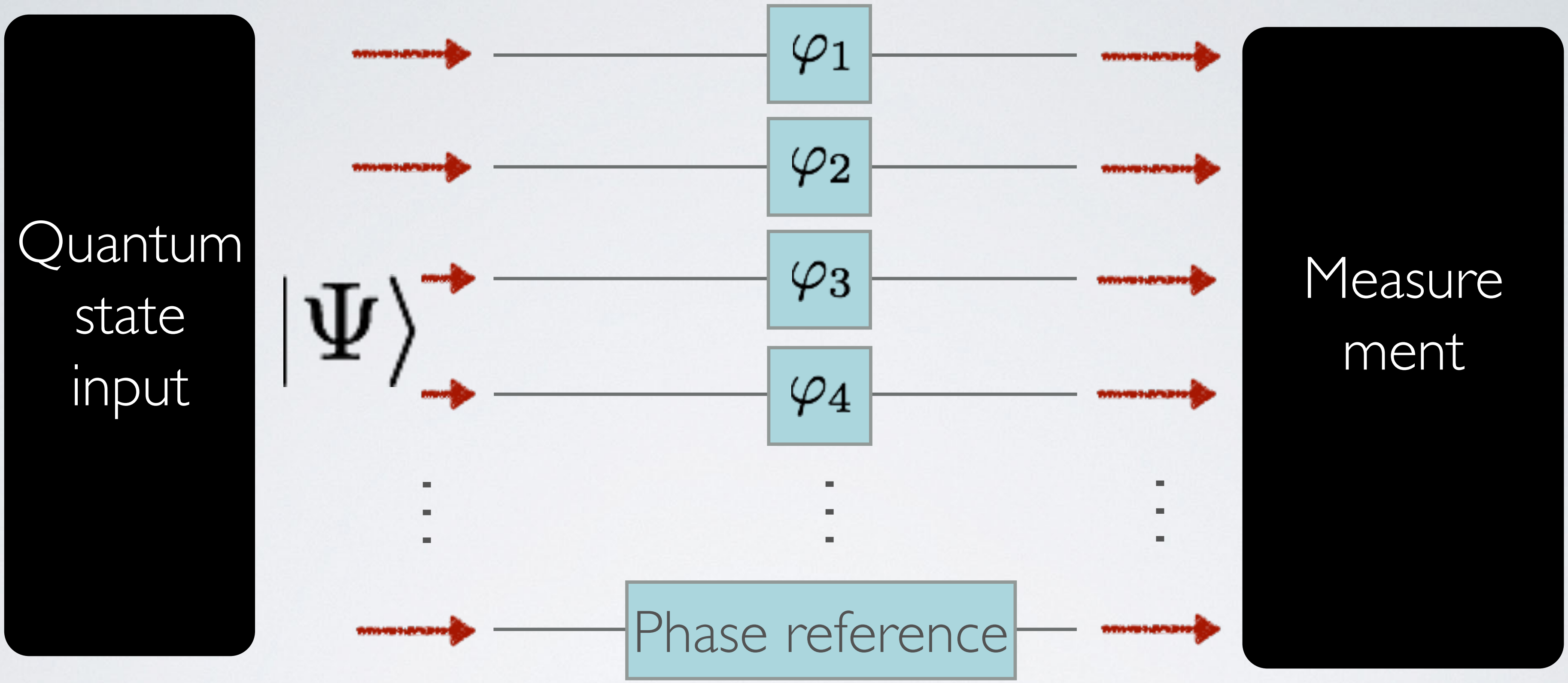

Global strategy | *Local strategy*

$$|\Psi_{\text{GNS}}\rangle = \frac{1}{\sqrt{M}}(|N,0,\ldots,0\rangle + |0,N,\ldots,0\rangle + \ldots |0,0,\ldots,N\rangle)$$

beats

$$|\Psi_{\text{NOON}}\rangle = \frac{1}{\sqrt{2}}(|N',0\rangle + |0,N'\rangle)$$

Enhancement: $O(M)$

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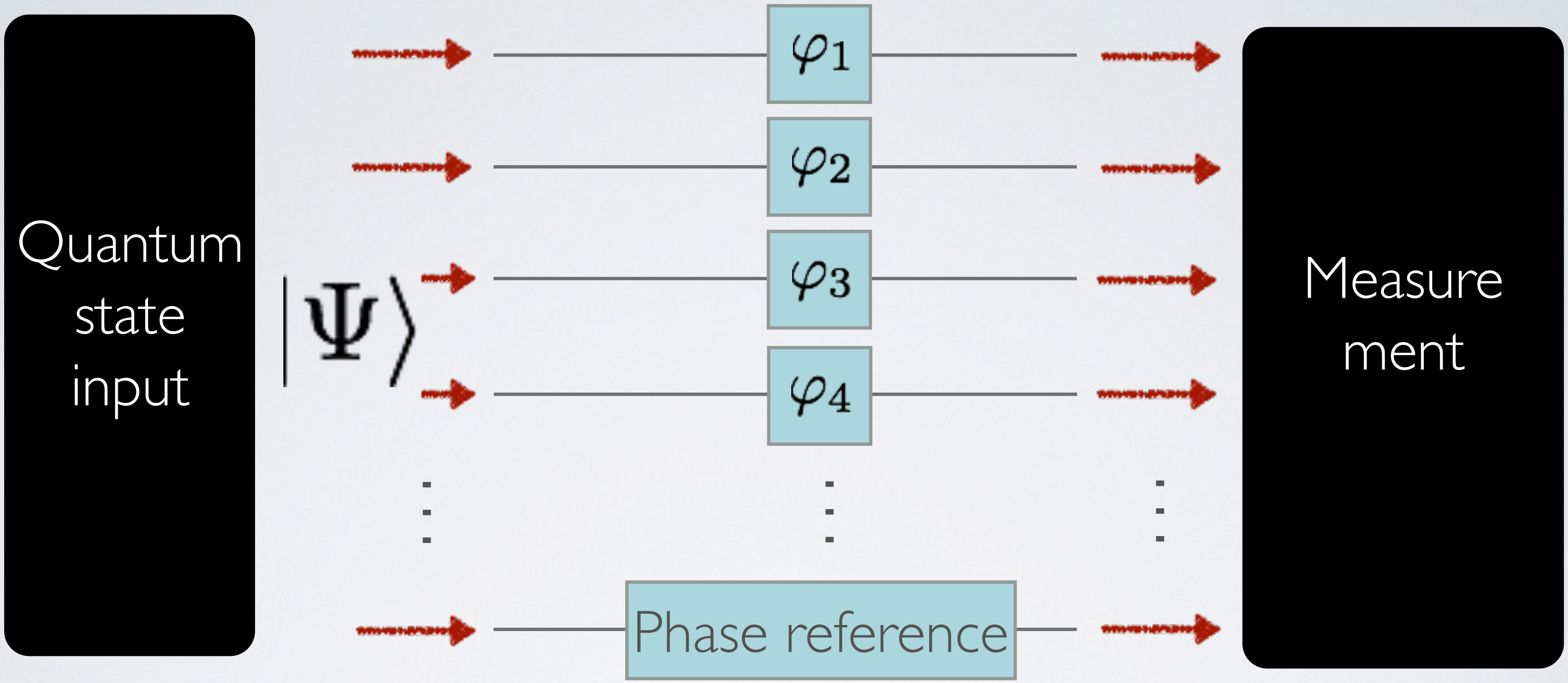

Global strategy *Local strategy*

$|\Psi_{\mathrm{GECS}}\rangle \propto |\alpha, 0, \ldots, 0\rangle + |0, \alpha, \ldots, 0\rangle + \ldots |0, 0, \ldots, \alpha\rangle$ ***beats*** $|\Psi_{\mathrm{ECS}}\rangle \propto |\alpha', 0\rangle + |0, \alpha'\rangle$

Enhancement: $O(M)$

J. Liu, X.-M. Lu, Z. Sun, and X. Wang, arXiv preprint arXiv:1409.6167 (2014).

Multi-parameter optical metrology

Global strategy | *Local strategy*

$$|\Psi_{\text{GECS}}\rangle \propto |\alpha, 0, \ldots, 0\rangle + |0, \alpha, \ldots, 0\rangle + \ldots |0, 0, \ldots, \alpha\rangle$$ ***beats*** $$|\Psi_{\text{ECS}}\rangle \propto |\alpha', 0\rangle + |0, \alpha'\rangle$$

$$|\Psi_{\text{GNS}}\rangle = \frac{1}{\sqrt{M}}(|N, 0, \ldots, 0\rangle + |0, N, \ldots, 0\rangle + \ldots |0, 0, \ldots, N\rangle)$$ ***beats*** $$|\Psi_{\text{NOON}}\rangle = \frac{1}{\sqrt{2}}(|N', 0\rangle + |0, N'\rangle)$$

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Multi-parameter optical metrology

Global strategy | *Local strategy*

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$$|\Psi_{\text{GNS}}\rangle = \frac{1}{\sqrt{M}}(|N, 0, \ldots, 0\rangle + |0, N, \ldots, 0\rangle + \ldots |0, 0, \ldots, N\rangle)$$ ***beats*** $$|\Psi_{\text{NOON}}\rangle = \frac{1}{\sqrt{2}}(|N', 0\rangle + |0, N'\rangle)$$

Is a *global strategy* really better than *local strategy*?

Enhancement: $O(M)$

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J. Liu, X.-M. Lu, Z. Sun, and X. Wang, arXiv preprint arXiv:1409.6167 (2014).

Multi-parameter estimation theory

Vector of parameters: $\boldsymbol{\phi} = (\phi_1, \phi_2, \ldots, \phi_d)$

State dependent on the parameters: $|\Psi_{\boldsymbol{\phi}}\rangle$

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Vector of parameters: $\boldsymbol{\phi} = (\phi_1, \phi_2, \ldots, \phi_d)$

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Information about parameters quantified by QFI ***matrix***:

$$\mathcal{F}_{lm} = \frac{1}{2}\langle\psi_{\boldsymbol{\phi}}|(L_l L_m + L_m L_l)|\psi_{\boldsymbol{\phi}}\rangle$$

where

$$L_l = 2\left(|\partial_{\phi_l}\psi_{\boldsymbol{\phi}}\rangle\langle\psi_{\boldsymbol{\phi}}| + |\psi_{\boldsymbol{\phi}}\rangle\langle\partial_{\phi_l}\psi_{\boldsymbol{\phi}}|\right)$$

(The symmetric logarithmic derivative)

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Cramer-Rao bound: $\Delta\phi_j^2 \geq \frac{1}{\mu}(\mathcal{F}^{-1})_{jj}$

Multi-parameter estimation theory

Vector of parameters: $\boldsymbol{\phi} = (\phi_1, \phi_2, \ldots, \phi_d)$

Special case: $|\Psi_{\boldsymbol{\phi}}\rangle = e^{i\sum_{i=1}^{d}\phi_i \hat{O}_i}|\Psi\rangle$

Input probe state

Hermitian and commuting generators of the phases.

Cramer-Rao bound: $\Delta\phi_j^2 \geq \frac{1}{\mu}(\mathcal{F}^{-1})_{jj}$

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Input probe state

Hermitian and commuting generators of the phases.

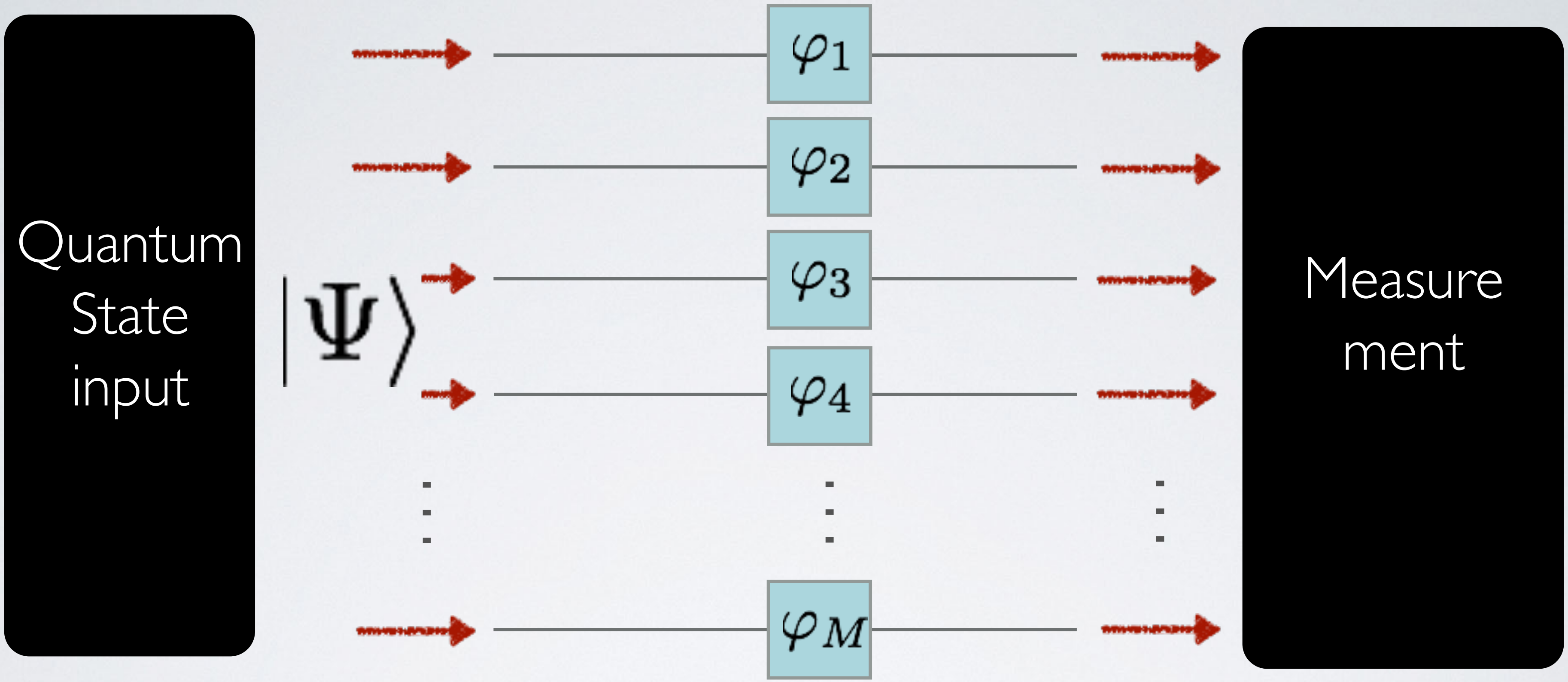
QFI ***matrix***: $\mathcal{F}_{lm} = 4\mathrm{Cov}(\hat{O}_l, \hat{O}_m)$

Cramer-Rao bound: $\Delta\phi_j^2 \geq \frac{1}{\mu}(\mathcal{F}^{-1})_{jj}$

Case 1 — Parallel Interferometers

P. A. Knott, T. J. Proctor, A. J. Hayes, J. F. Ralph, P. Kok, J. A. Dunningham arXiv:1601.05912

Case I — Parallel Interferometers



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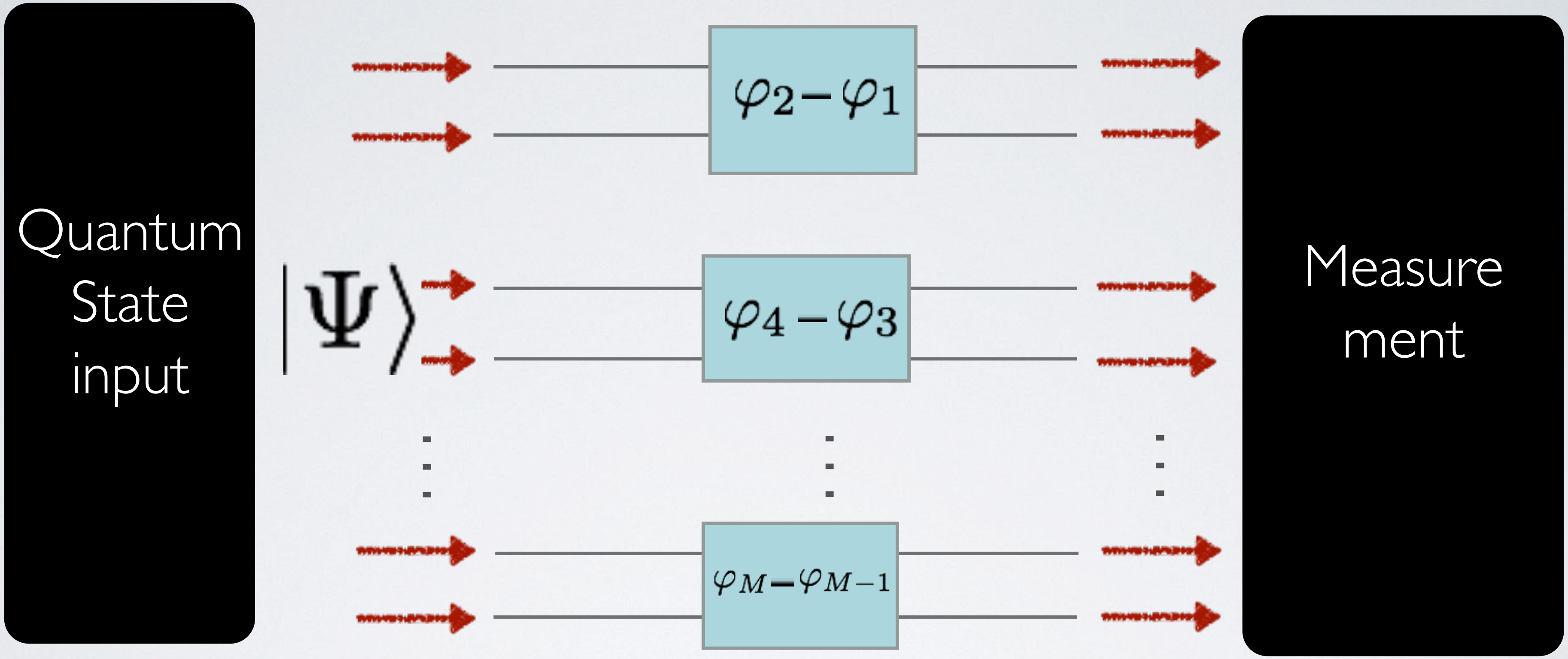

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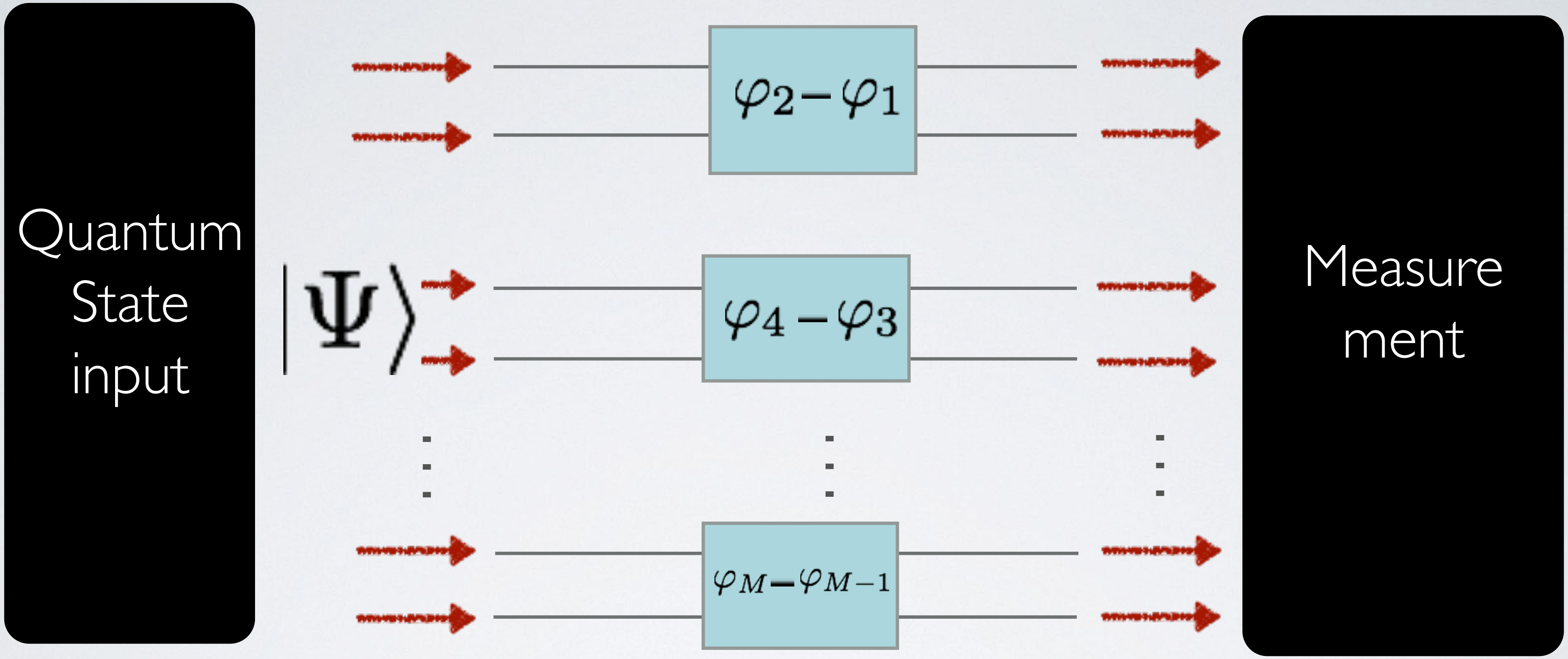

Why is this an interesting model?

— Model for a network of quantum sensors

— Some problems involve multiple optical interferometers

E.g. Gravitational wave astronomy (e.g. A. Freise et al. Class. Quantum Grav. 26, 085012 (2009))

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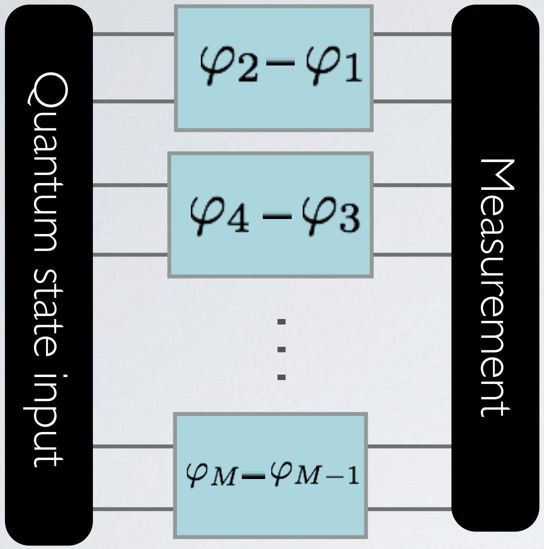

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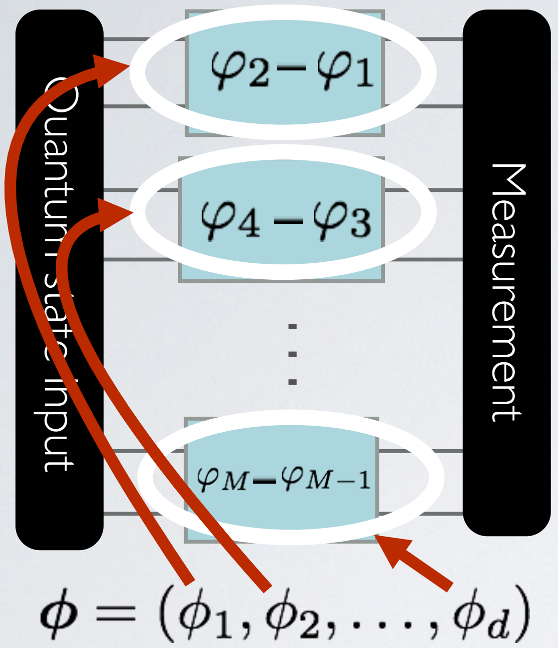

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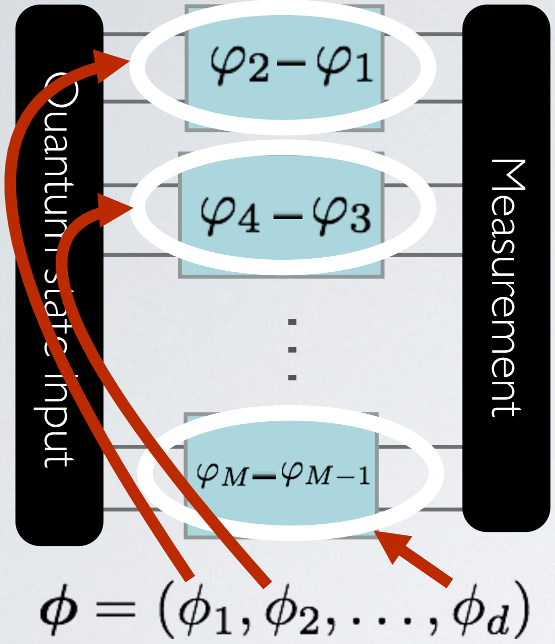

Natural assumptions:

Consider states which have…

1. Symmetry between arms of each interferometer
2. Symmetry between each pair of interferometers

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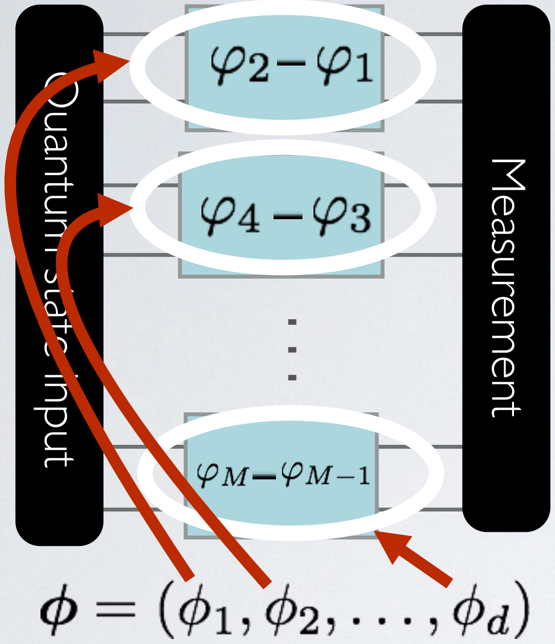

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CRB for each ϕ :

$$\Delta\phi^2 \geq \frac{1}{2(V - C_{\text{Intra}})}$$

Photon number covariance between any two modes in the same interferometer.

Photon number variance in any mode.

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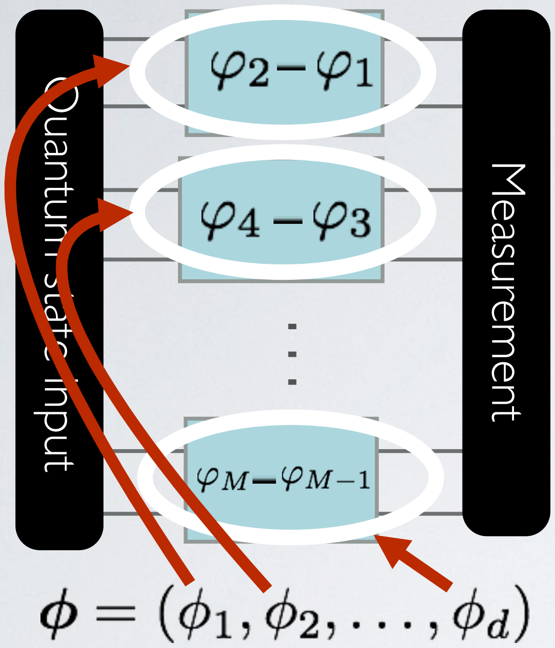

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Photon number variance in any mode.

Independent of correlations between interferometers

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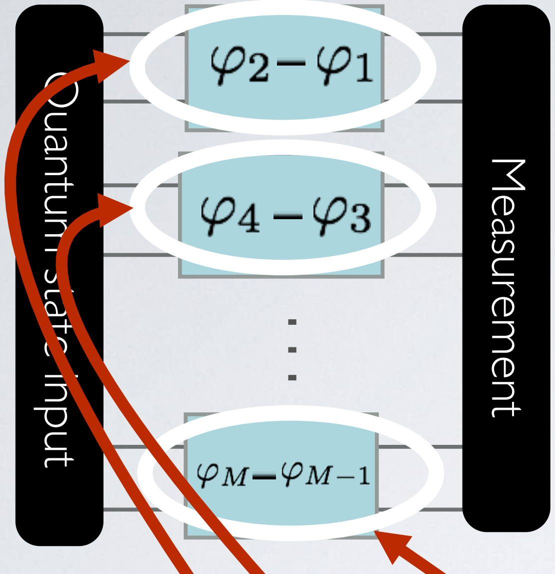

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Natural assumptions:

Consider states which have…

1. Symmetry between arms of each interferometer

2. Symmetry between each pair of interferometers

CRB for each ϕ :

$$\Delta\phi^2 \geq \frac{1}{2\bar{n}(1+\mathcal{Q})(1-\mathcal{J}_{\mathrm{Intra}})}$$

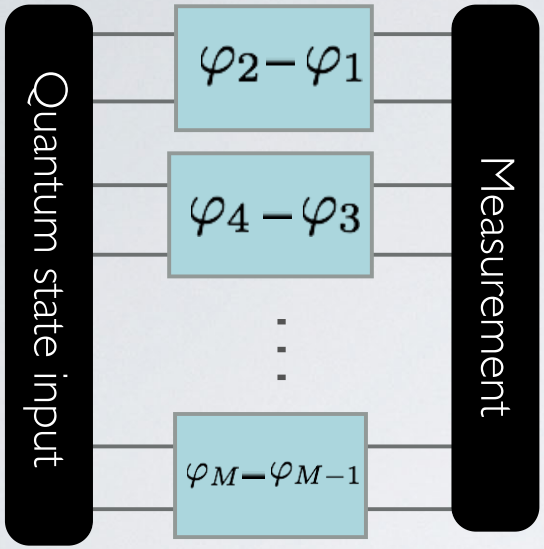
Mode correlations - at most factor of 2 enhancement

Average number of photons in a single mode

Provides quantum enhancement

Independent of correlations between interferometers

Case 1 — Parallel Interferometers



$$\Delta\phi^2 \geq \frac{1}{2\bar{n}(1+\mathcal{Q})(1-\mathcal{J}_{\mathrm{Intra}})}$$

What about this expected enhancement?

P. A. Knott, T. J. Proctor, A. J. Hayes, J. F. Ralph, P. Kok, J. A. Dunningham arXiv:1601.05912

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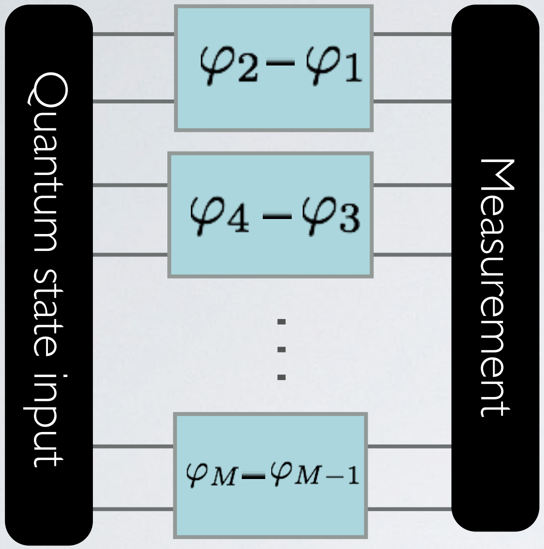

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Global strategy:

$$|\Psi_{\text{GECS}}\rangle \propto |\alpha, 0, \ldots, 0\rangle + |0, \alpha, \ldots, 0\rangle + \ldots |0, 0, \ldots, \alpha\rangle \Longrightarrow \Delta\phi^2_{\text{GECS}} \geq \frac{M}{2\left(\bar{N}^2 + \bar{N}\right)}$$

(approximately)

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average *TOTAL photon number*

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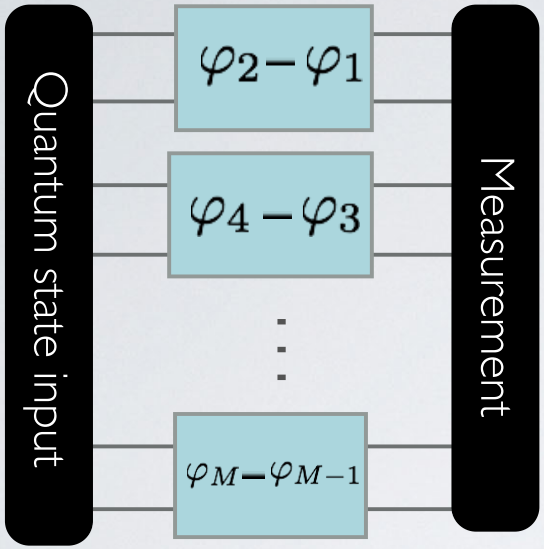

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Local strategy:

Roughly expect: $\Delta\phi^2_{\mathrm{QL}} \geq \frac{1}{\bar{n}^2} = \frac{M^2}{\bar{N}^2}$

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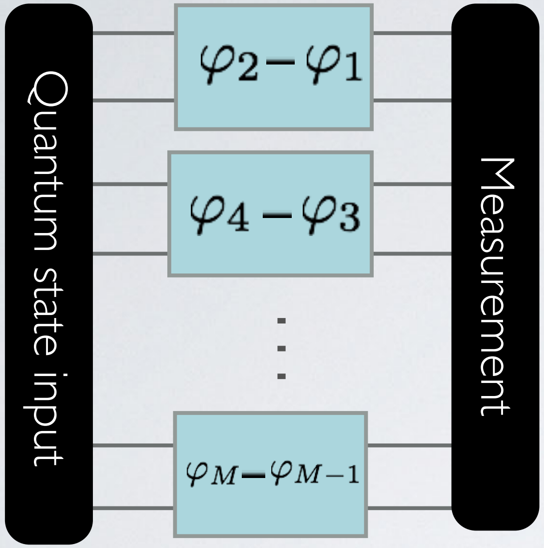

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$$|\Psi_{\mathrm{UCS}}\rangle \propto \left(|\alpha'\rangle + \nu|0\rangle\right)^{\otimes M} \Rightarrow \Delta\phi^2_{\mathrm{UCS}} \geq \frac{M}{2\left(\frac{\nu^2}{M}\bar{N}^2 + \bar{N}\right)}$$

balancing parameter

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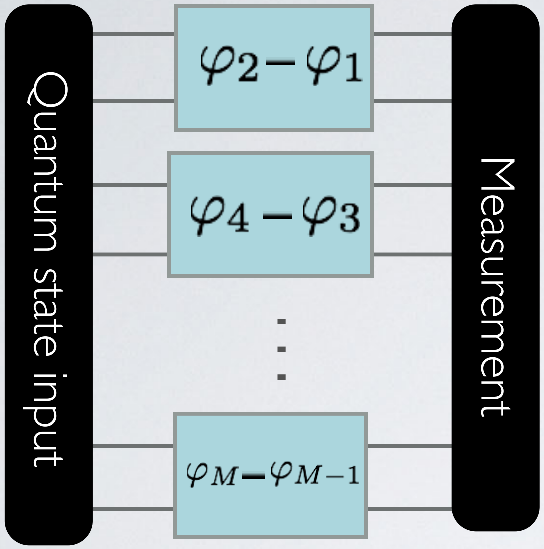

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Case 2 — Quantum Imaging

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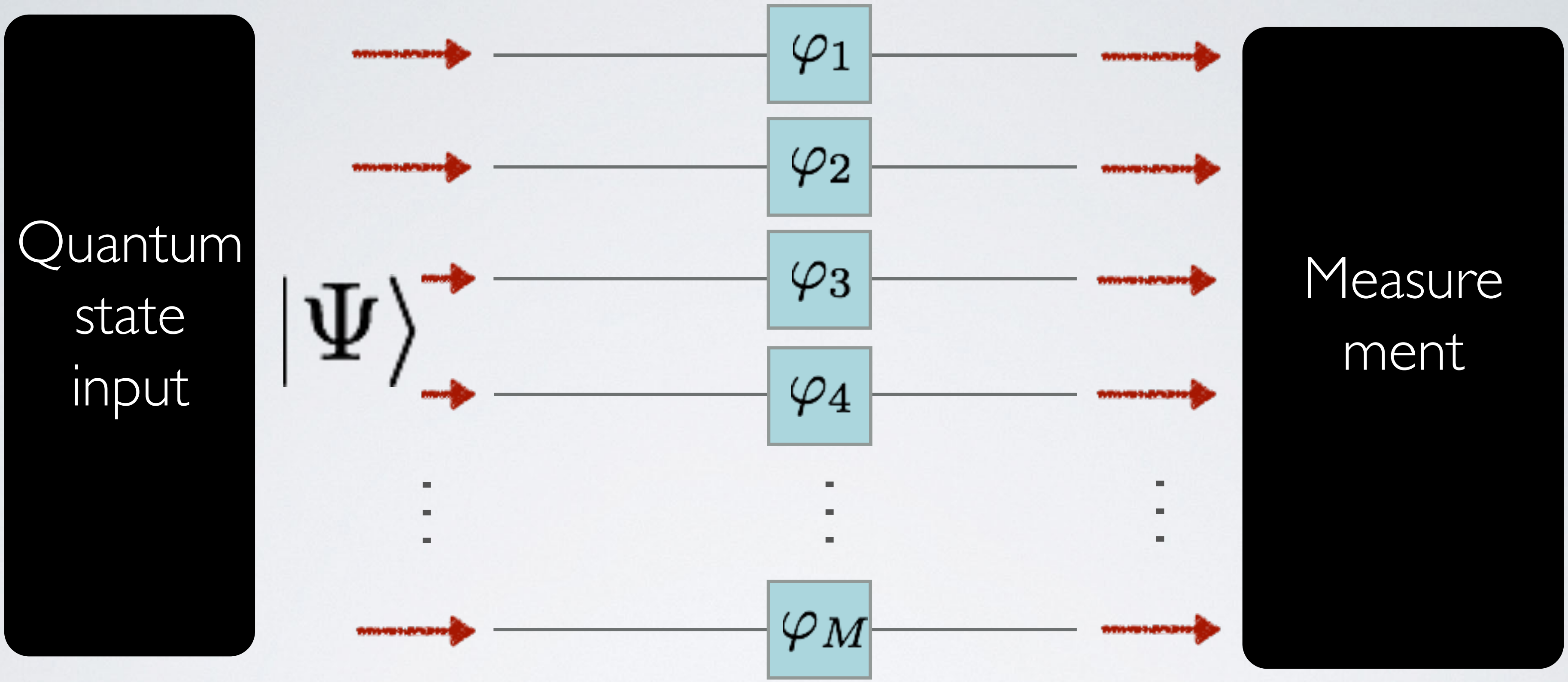
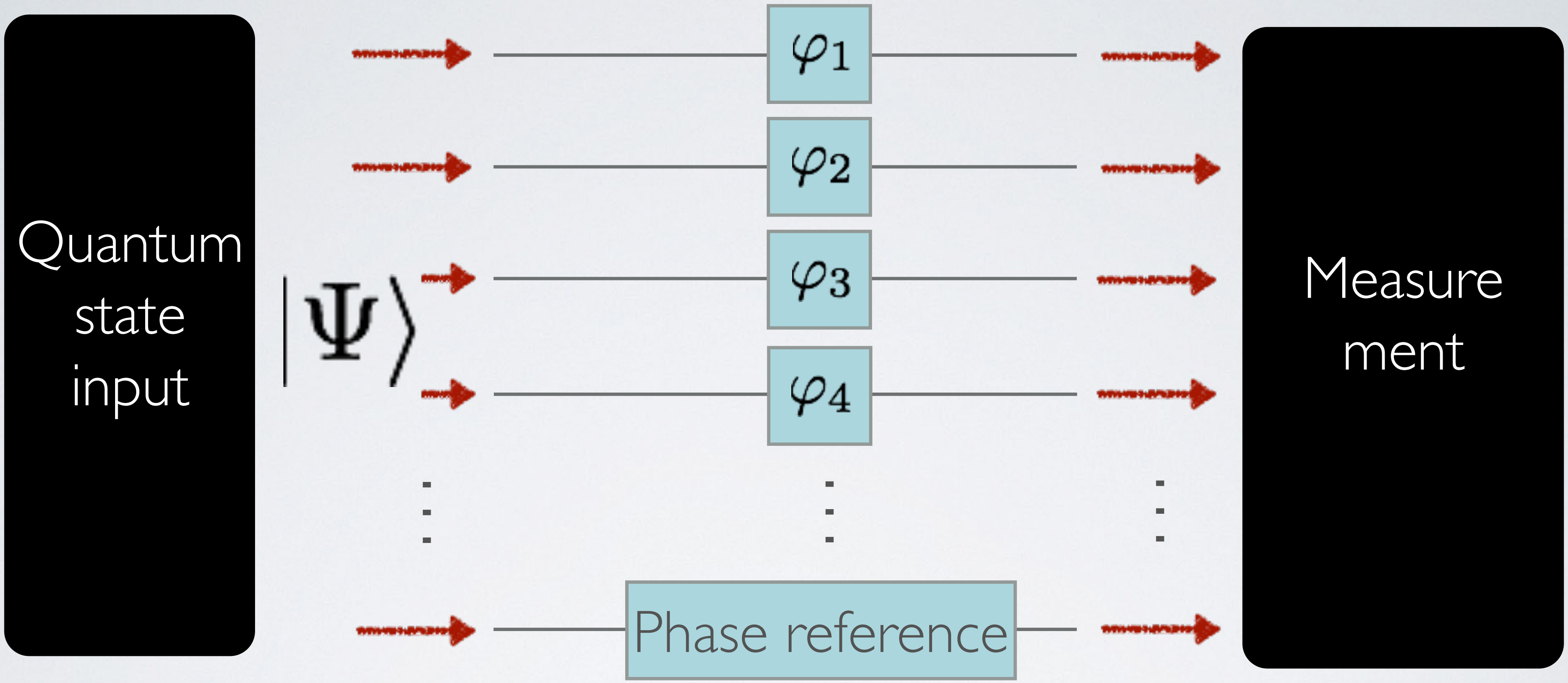


Aim: Estimate *d* parameters $(\phi_1, \phi_2, \ldots, \phi_d)$

Functions of the *M* physical φ_j parameters

Case 2 — Quantum Imaging



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Functions of the *M* physical $\phi_j = \varphi_j - \varphi_M$ parameters

P. C. Humphreys, M. Barbieri, A. Datta, and I. A. Walmsley, Phys. Rev. Lett. 111, 070403 (2013).
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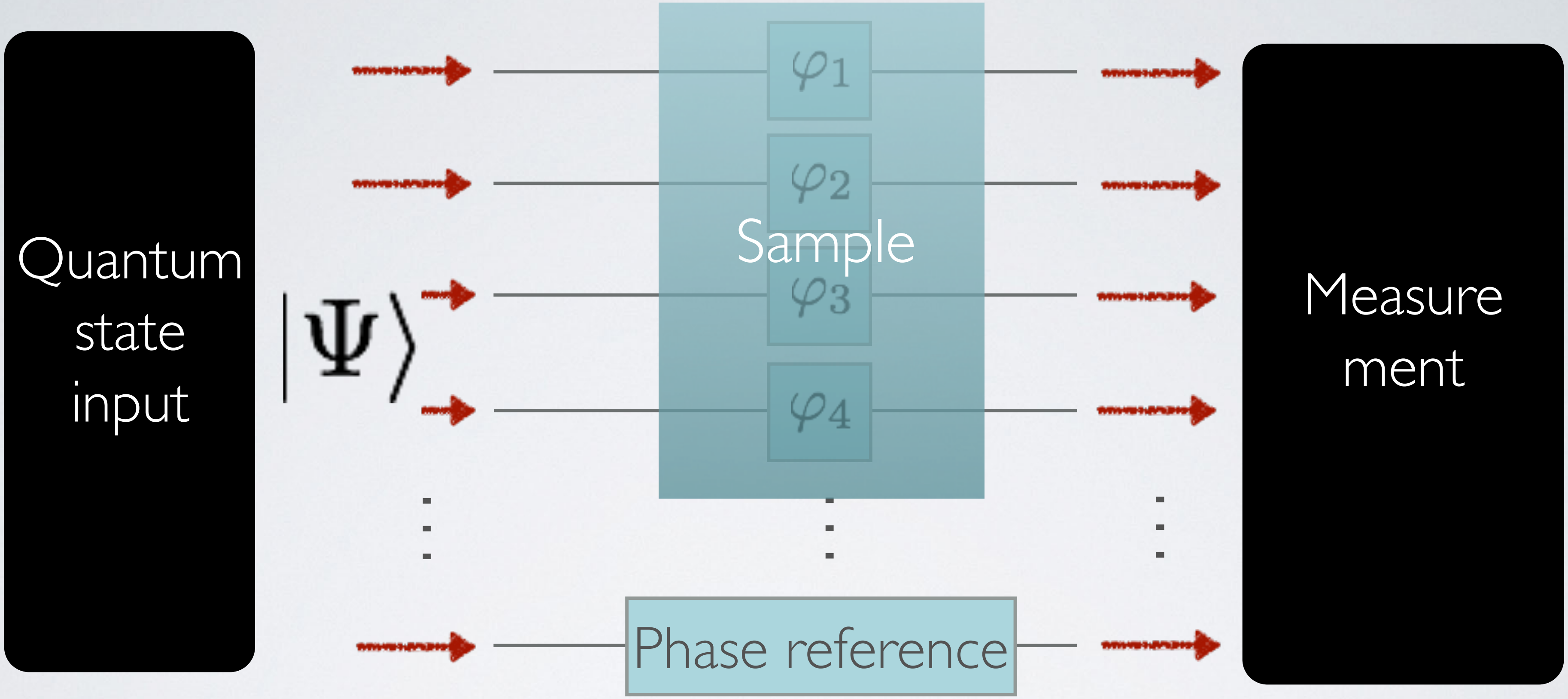

Why?

Model for (quantum-enhanced) imaging

C. A. Pérez-Delgado, M. E. Pearce, and P. Kok Phys. Rev. Lett. 109, 123601 (2012)

P. C. Humphreys, M. Barbieri, A. Datta, and I. A. Walmsley, Phys. Rev. Lett. 111, 070403 (2013).

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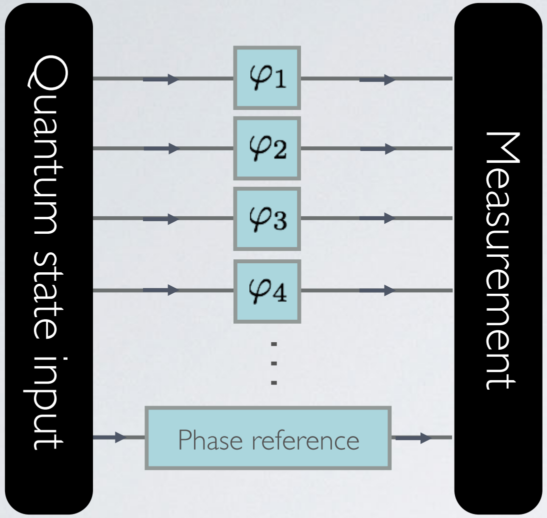

Natural assumptions:

Consider states which have…

1. Symmetry between probe modes

Using this we have:

$$f(M,\mathcal{J}) = 1 - \frac{\mathcal{J}}{M\mathcal{J}+1}$$

$$\Delta\phi^2 \geq \frac{f(M,\mathcal{J})}{4\bar{n}(1+\mathcal{Q})(1-\mathcal{J})}$$

Mode correlations - at most factor of 2 enhancement

Average number of photons in a single mode

Provides quantum enhancement

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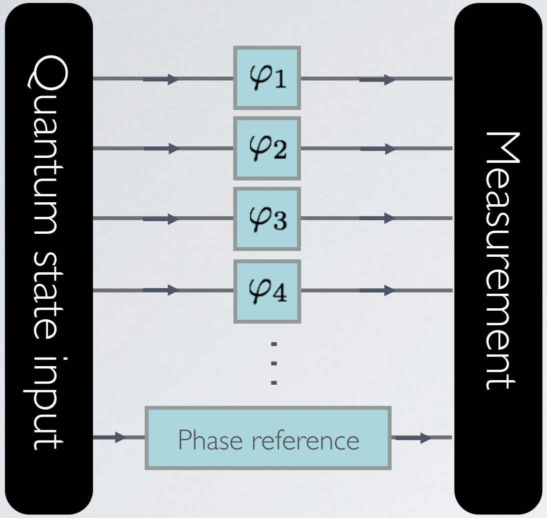

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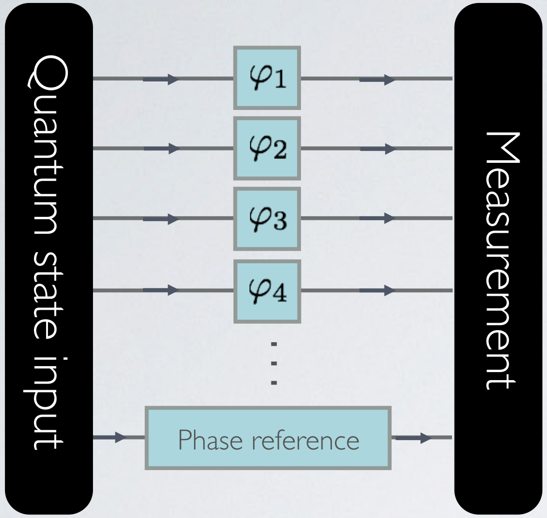

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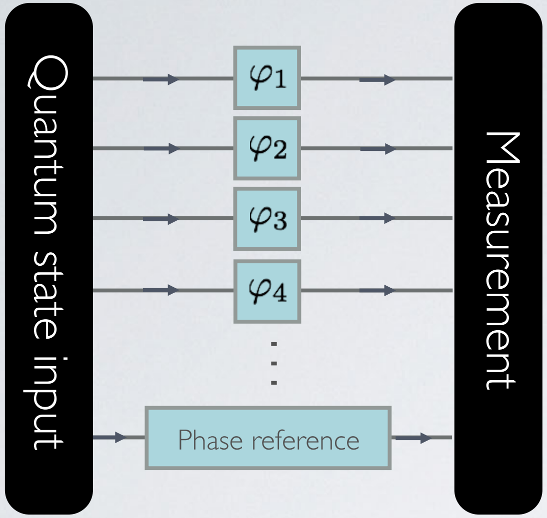

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Local strategy:

$$|\Psi_{\text{UNO}}\rangle \propto (|N\rangle + \nu|0\rangle)^{\otimes M}$$

balancing parameter

$\Rightarrow$

$\nu = 1$	Standard quantum scaling
$\nu \approx \sqrt{M}$	Same enhancement
$\nu > \sqrt{M}$	Better enhancement

What does this mean?

Local strategy can do as well as global strategy

Both strategies work because $\mathcal{Q} = \mathcal{O}(\bar{N})$ (rather than $\mathcal{Q} = \mathcal{O}(\bar{n})$)

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Entanglement

Global strategy — $|\Psi_{\mathrm{GNS}}\rangle = \frac{1}{\sqrt{M}}(|N, 0, \ldots, 0\rangle + |0, N, \ldots, 0\rangle + \ldots |0, 0, \ldots, N\rangle)$

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⇒ Amounts to the same thing.

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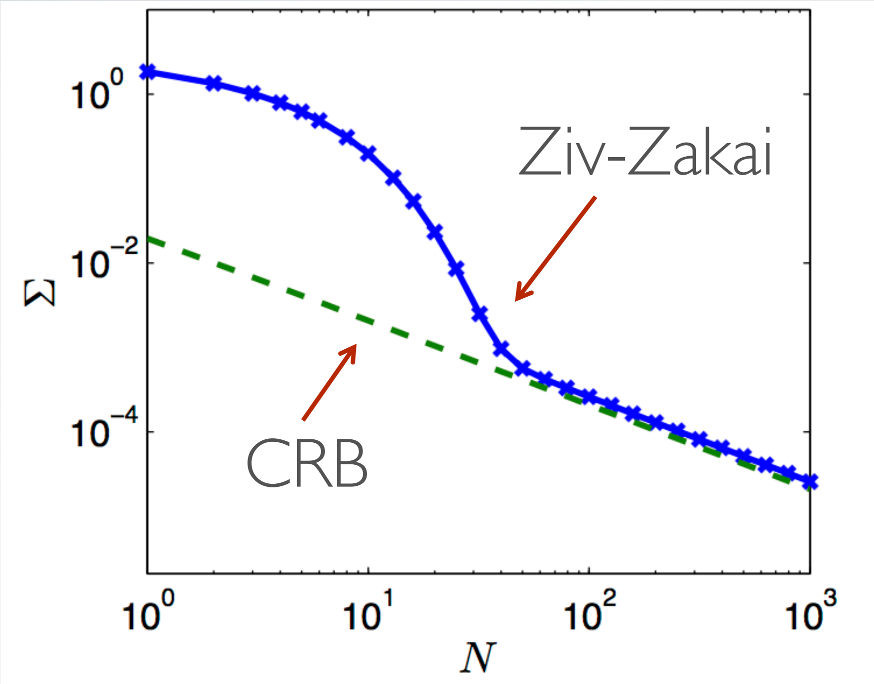
Adding more vacuum

⇒ Amounts to the same thing.

However… this may be down to an over-reliance on Fisher information

M. J. W. Hall, D. W. Berry, M. Zwierz, and H. M. Wiseman, Phys. Rev. A 85, 041802 (2012).
V. Giovannetti and L. Maccone, Phys. Rev. Lett. 108, 210404 (2012).

Caveats



M. Tsang, PRL 108, 230401 (2012)

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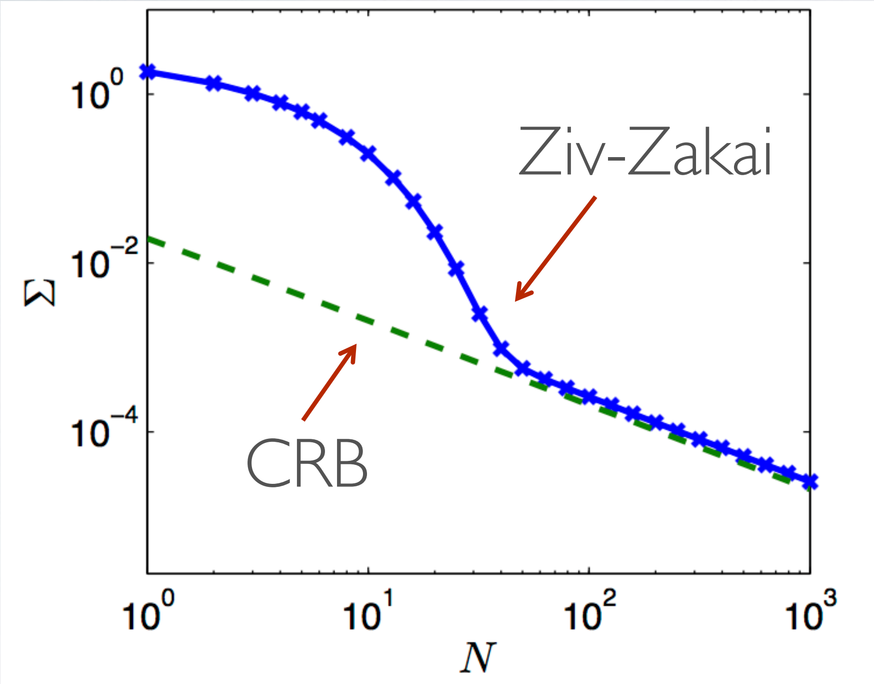

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$$|\psi\rangle \propto \nu|0\rangle + |N\rangle$$

To reach CRB regime, we need:

$$\mu/\nu^2 > 1$$

i.e. no. of repeats depends on the state

Take fixed total number, N_t

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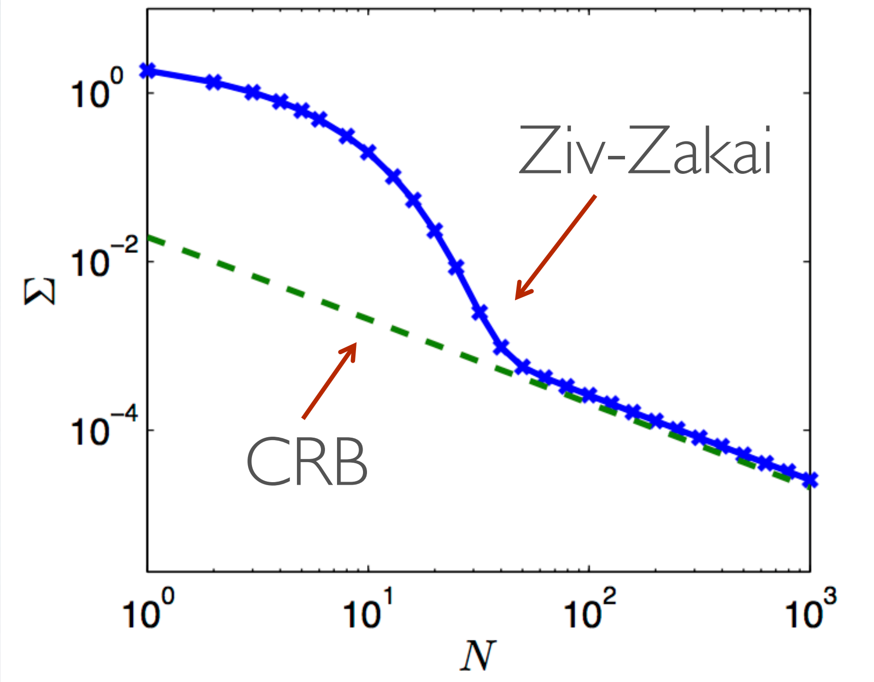

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NOON: $N_t = \mu_1 n M$

μ_1 is a (constant) number of repeats

$$(\Delta\phi)^2 \geq \frac{1}{\mu_1 n^2} = \frac{1}{\mu_1 \left(\frac{N_t}{\mu_1 M}\right)^2} \sim \frac{M^2}{N_t^2}$$

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Unbalanced state: $|\psi\rangle \propto \nu|0\rangle + |N\rangle$

Get factor M improvement for : $\nu \gtrsim \sqrt{M}$

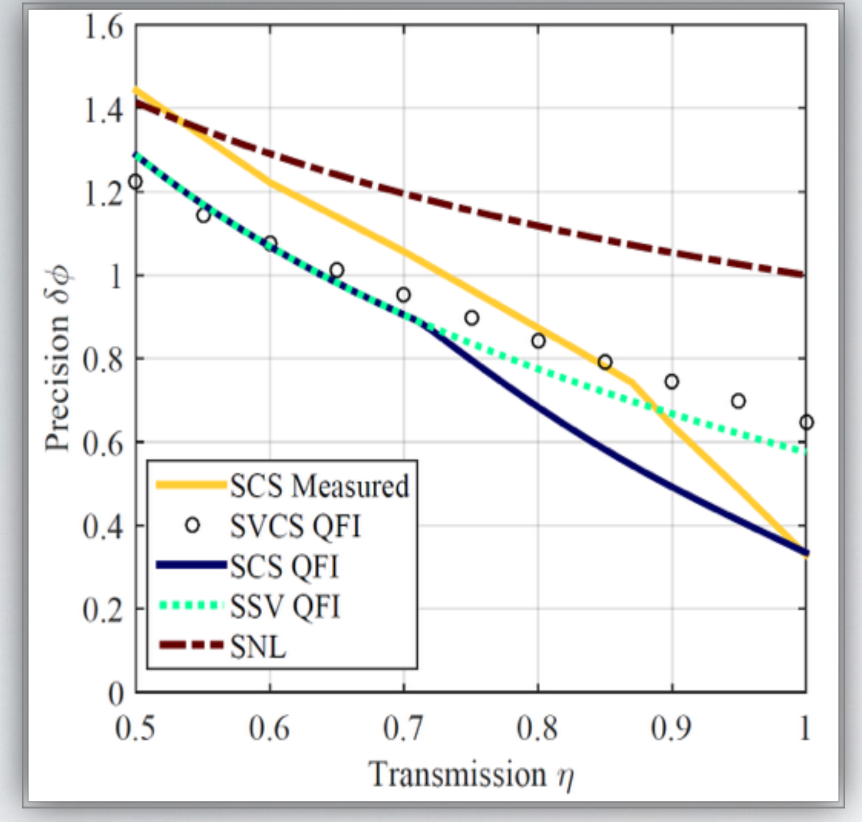
$$N_t = \mu_2 n M$$

$$(\Delta\phi)^2 \geq \frac{1}{M}\frac{1}{\mu_2 n^2} = \frac{1}{M\mu_2 \left(\frac{N_t}{\mu_2 M}\right)^2} = \frac{\mu_2 M}{N_t^2}$$

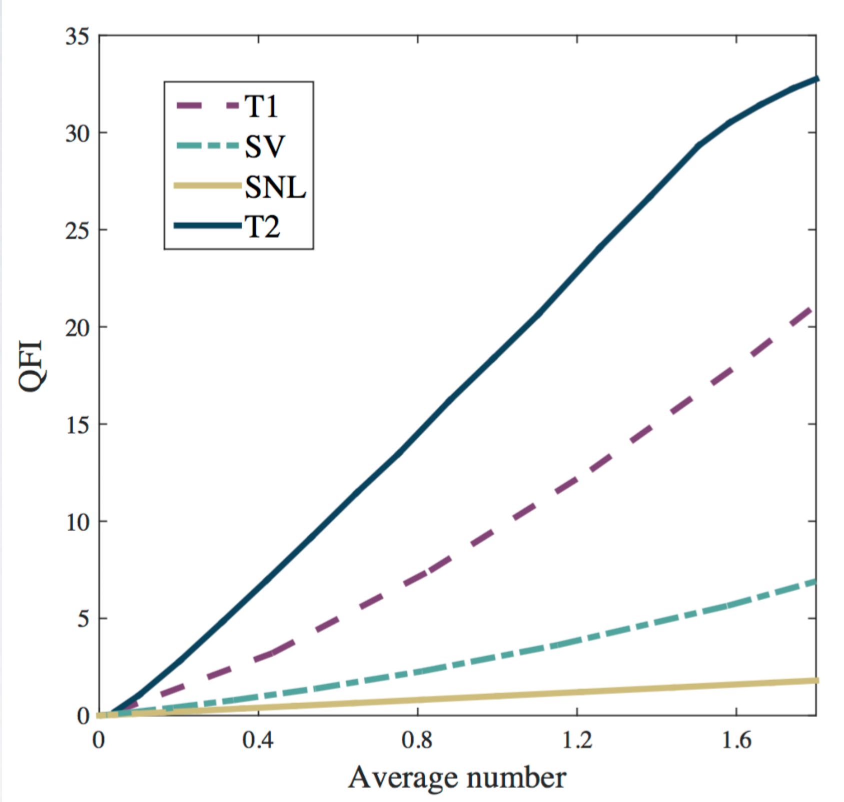
However this assumes CRB regime: $\mu_2/\nu^2 > 1 \implies \mu_2 > M$

So: $$(\Delta\phi)^2 \sim \frac{M^2}{N_t^2}$$

Same as for NOON state, may get factor advantage but not scaling



Squeezed cat state



Evolutionary algorithm

Factor advantage for multi-parameter estimation could be achieved in a local strategy with multiple copies of the states seen earlier

Conclusions

— Squeezed non-Gaussian states good for quantum metrology

— Apparent M-fold enhancement when M phases are estimated can be achieved with local strategies as well as global ones

—This scaling advantage reduces to a factor when we account for the repeats needed to reach the CRB.

Acknowledgements

Collaborators

P. A. Knott
T.P. Proctor
A. J. Hayes
J. F. Ralph
P. Kok
J. P. Cooling
J. Rubio Jiménez

Funding

University of Leeds Research Scholarship
University of Sussex PhD Scholarship
NQIT
DSTL