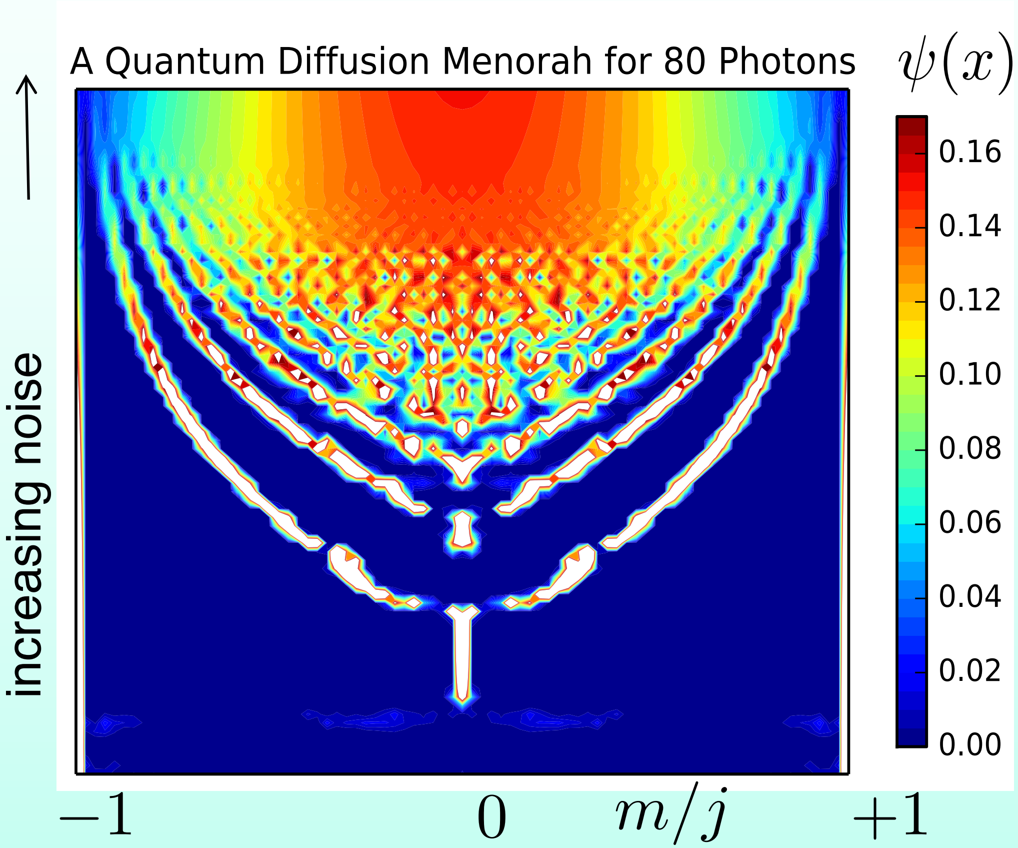
Goldilocks Probes via Quantum Annealing to Criticality

March 2-4th, RAQM 2016, Warsaw, Poland



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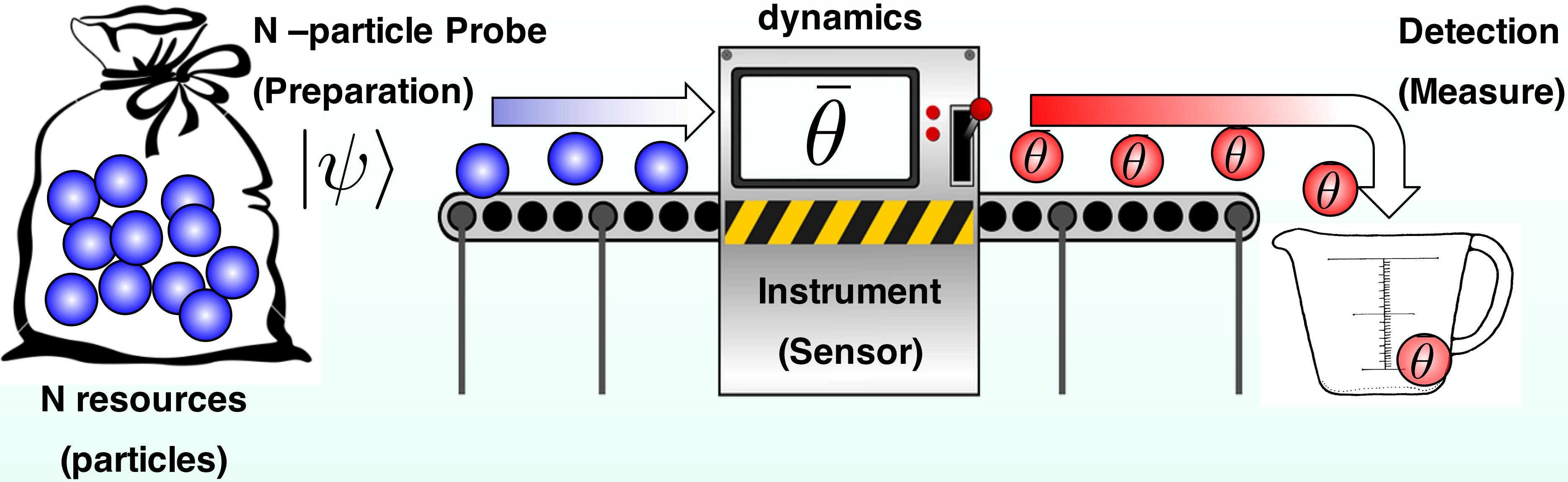
Instrument - A Quantum Metrology View



Scaling Laws:

$$\boxed{\delta\theta^2 > \frac{1}{\nu N}}$$

SHOT-NOISE LIMIT

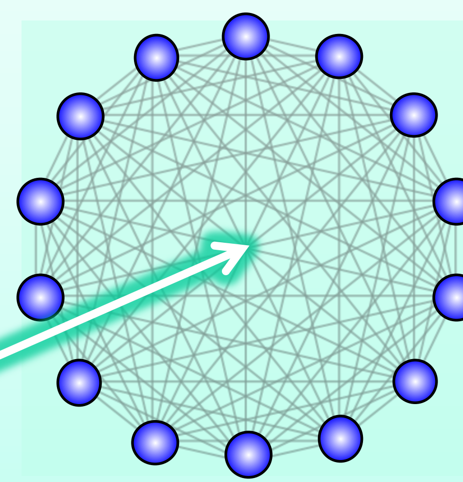
(standard limit)

$$\boxed{\delta\theta^2 > \frac{1}{\nu N^2}}$$

HEISENBERG LIMIT

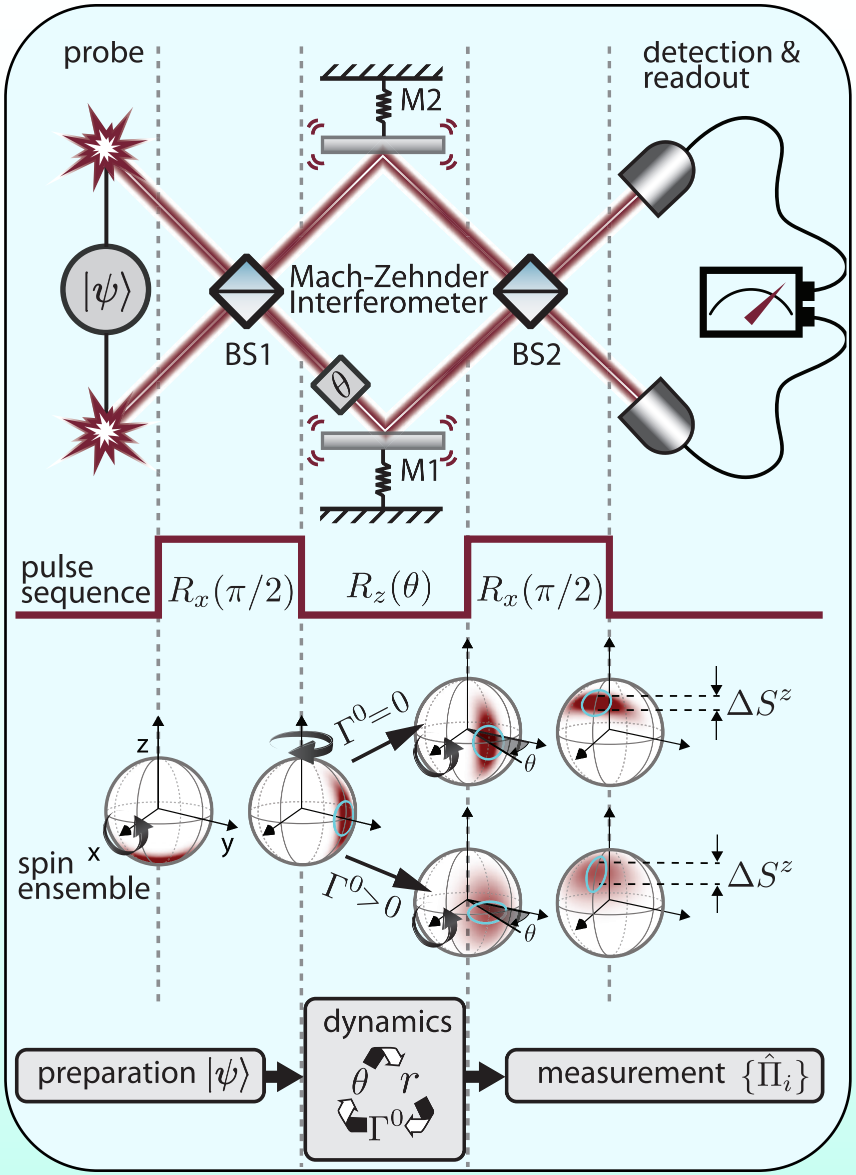
(Holy Grail)

Possible with Entanglement



N=14 Entanglement

Phase Estimation in presence of Decoherence



Start with fixed N: qubits, spins, photons

Dephasing or Phase Diffusion occurs because of thermal or acoustic motion of mirrors M1, M2 or noise intrinsic to laser source. Coherence lost but not energy.

Dissipation or Loss occurs because of imperfect detection, scattering and absorption in optical elements

Excitation/Relaxation occurs if qubits coupled to bath at finite temperature.

Dynamics of Andrea Smirne's talk

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Noisy Quantum Dynamics – Lindblad form

$$\frac{d\rho}{d\theta} = -i\kappa[S^z, \rho] - \frac{1}{2}\frac{\partial\Gamma}{\partial\theta}\hat{\hat{L}}\left(\sum_i s_i\right)\rho$$

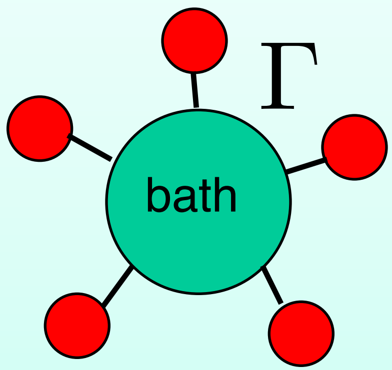
Collective noise (correlated)

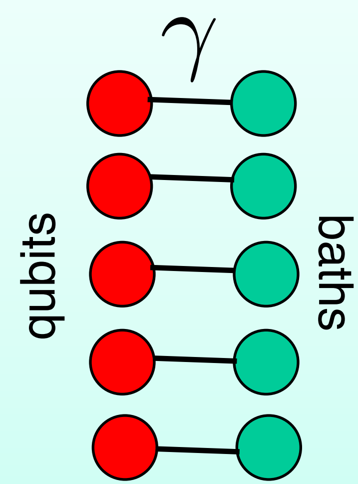
$$\boxed{\hat{\hat{L}}(s)\rho = s^\dagger s\rho + \rho s^\dagger s - 2s\rho s^\dagger}$$

$s \mapsto s^+$	excitation
$s \mapsto s^-$	relaxation
$s \mapsto s^z$	dephasing

$$\frac{d\rho}{d\theta} = -i\kappa[S^z, \rho] - \frac{1}{2}\frac{\partial\gamma}{\partial\theta}\sum_i \hat{\hat{L}}(s_i)\rho$$

Individual noise (uncorrelated)





$$S^z = \sum_i s_i^z$$

Collective spin z operator produces unitary shift

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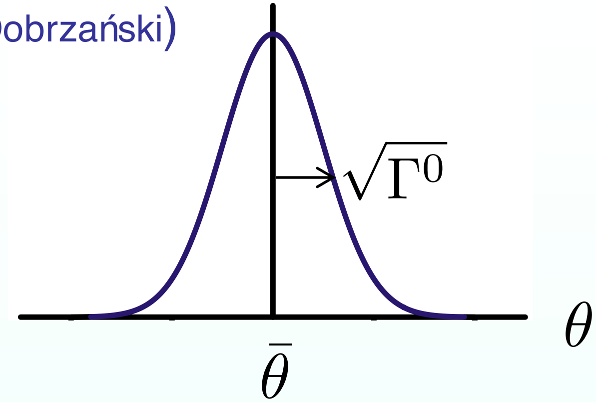
Collective Dephasing – e.g. Mirror Fluctuations

(equivalent to prior phase uncertainty in Bayesian approach:
2014 New J. Phys. 16 113002, Macieszczak, Fraas and Demkowicz-Dobrzański)



$$\frac{d\rho}{d\theta} = -i\frac{d\bar{\theta}}{d\theta}[S^z,\rho] - \frac{1}{2}\frac{d\Gamma^0}{d\theta}[S^z,[S^z,\rho]]$$

Lindblad Master equation

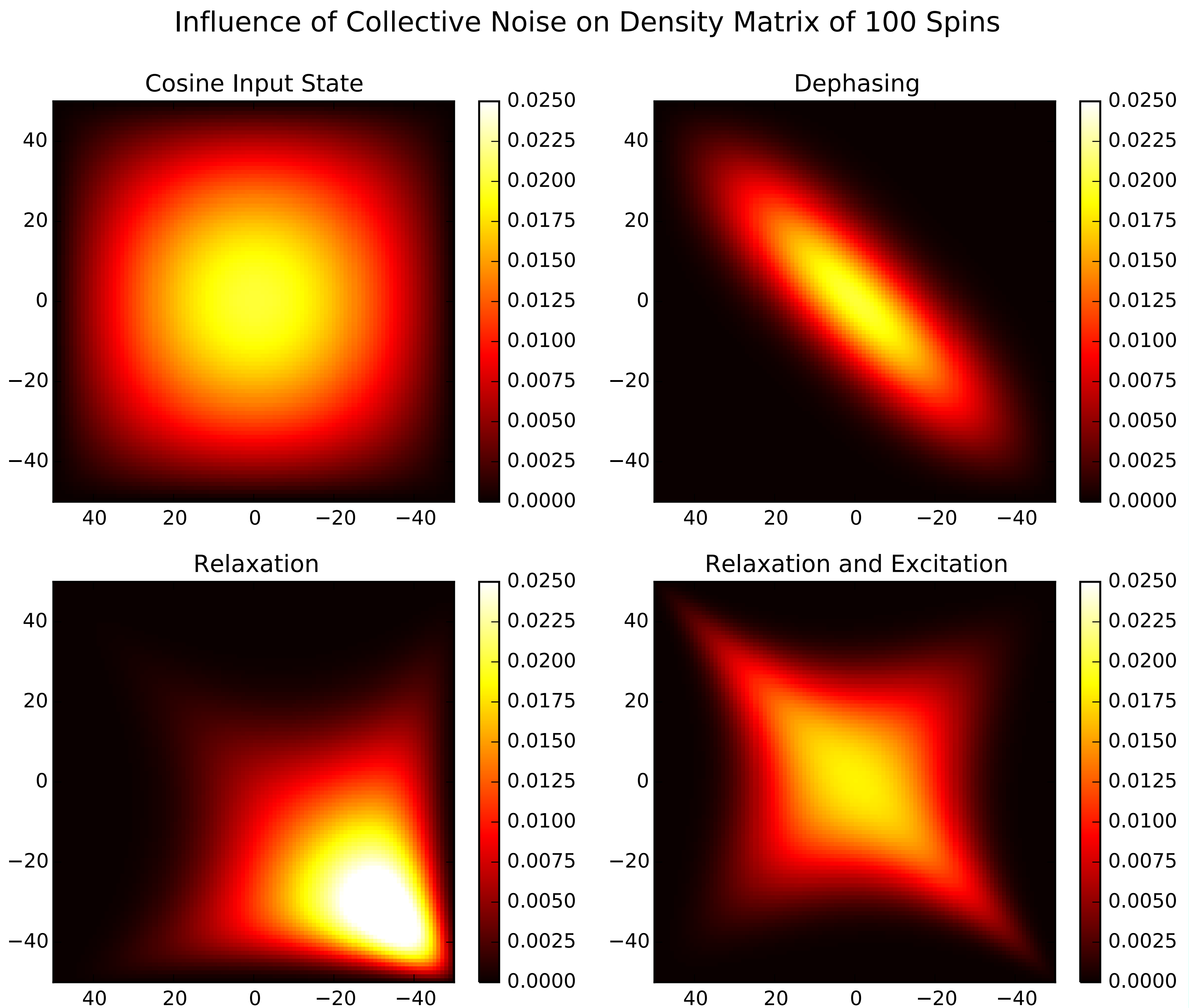
$$\langle n|\rho|m\rangle = \psi_n\psi_m \exp\{-i\bar{\theta}(n-m)\}\exp\{-\Gamma^0(n-m)^2/2\}$$

Density Matrix element unitary shift dephasing

$$\rho = \int_{-\infty}^{\infty} |\psi(\theta)\rangle\langle\psi(\theta)| \frac{e^{-(\theta-\bar{\theta})^2/2\Gamma^0}}{\sqrt{2\pi\Gamma^0}} d\theta. \qquad |\psi(\theta)\rangle = e^{-i\theta S^z}|\psi(0)\rangle$$

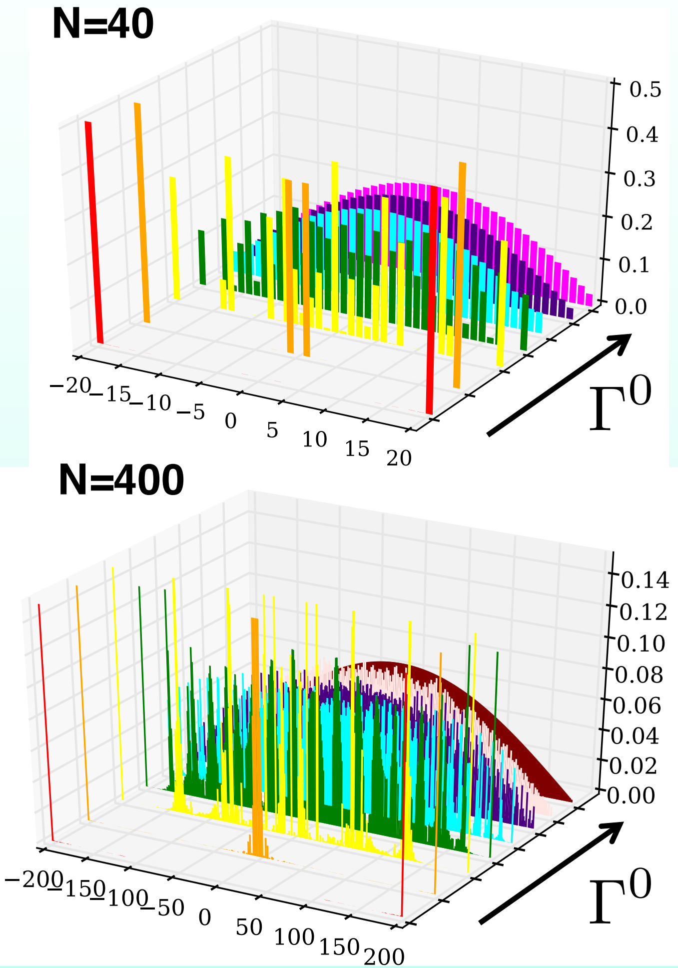
Mixture of pure states, evolved by random phases. Gaussian-distributed with mean $\bar{\theta}$ and variance Γ^0

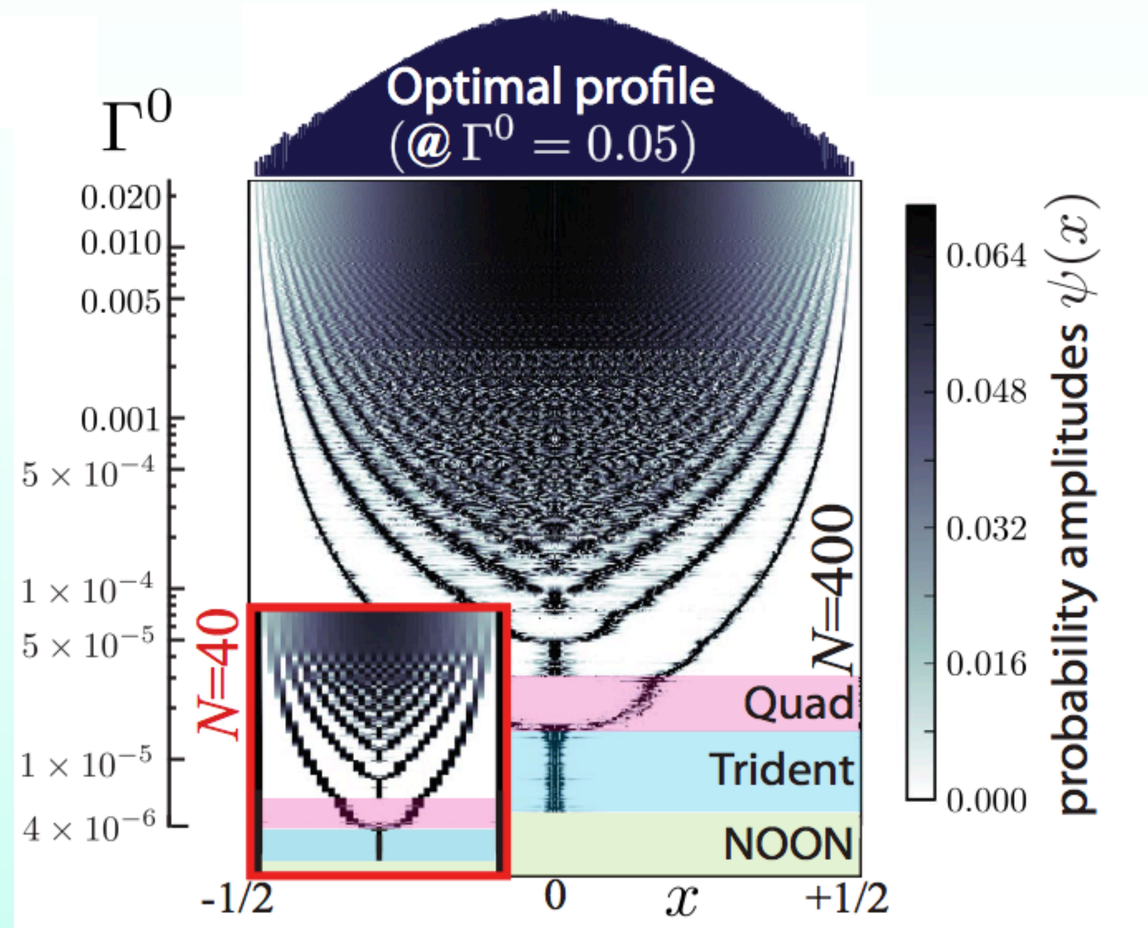
Γ^0 increases linearly in time for Markovian dynamics


Influence of Collective Noise on Density Matrix of 100 Spins
Cosine Input State
Dephasing
Relaxation
Relaxation and Excitation
40
20
0
−20
−40
0.0250
0.0225
0.0200
0.0175
0.0150
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000

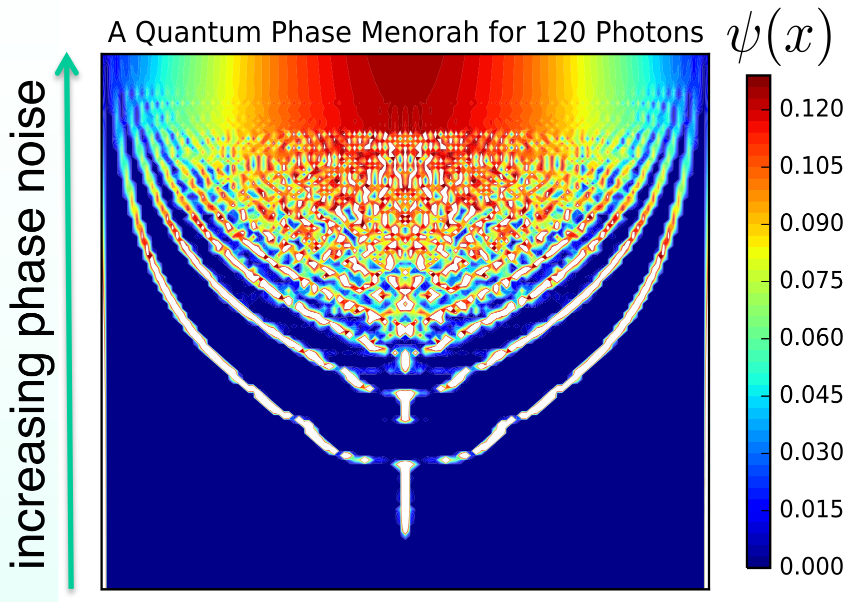
Numerics Reveal Bifurcations in Optimal State

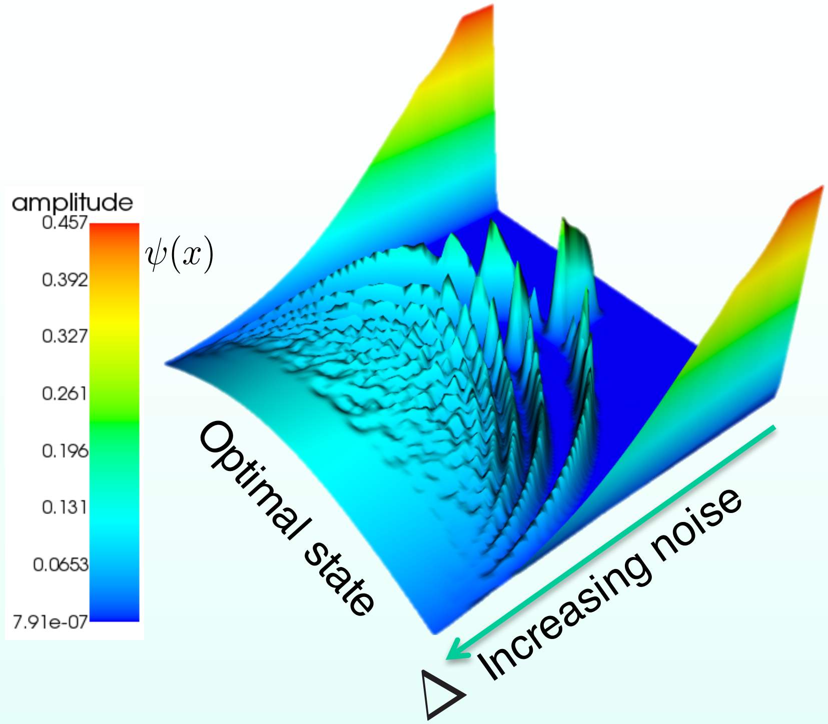
$$\psi_m \psi_n \mapsto \psi_m \psi_n \exp\{-\Gamma^0 (n-m)^2/2\}$$





$$\psi_m \mapsto \psi(x) \qquad \frac{m}{2j} = x \in [-\tfrac{1}{2}, \tfrac{1}{2}]$$





N=120 Qubits or Photons

$$\psi_m \psi_n \mapsto \psi_m \psi_n \exp\{-\Delta(n-m)^2/2\} \quad \text{(dephasing)}$$

Early numerics suggested cv approach

(for large N and/or large noise, the optimal state has smooth features)

PHYSICAL REVIEW A **89**, 032128 (2014)

Quantum Fisher information for states in exponential form

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(Received 15 February 2014; published 31 March 2014)

We derive explicit expressions for the quantum Fisher information and the symmetric logarithmic derivative (SLD) of a quantum state in the exponential form $\rho = \exp(G)$; the SLD is expressed in terms of the generator G. Applications include quantum-metrology problems with Gaussian states and general thermal states. Specifically, we give the SLD for a Gaussian state in two forms, in terms of its generator and its moments; the Fisher information is also calculated for both forms. Special cases are discussed, including pure, degenerate, and very noisy Gaussian states.

Arbitrary dynamics, for parameter Θ by Zhang Jiang

$$\boxed{L = \left(\frac{\tanh \frac{1}{2}[H,\bullet]}{\frac{1}{2}[H,\bullet]}\right)\frac{dH}{d\Theta}}$$

QFI as Asymptotic Operator Series:
Exact and Unique

Euler-Lagrange = 'Schrodinger':

$$\boxed{\frac{F}{N^2} = \int_{x=-1/2}^{+1/2} \frac{\psi^2(x)}{\mu(x)}(dx) - \int_{x=-1/2}^{+1/2} \left(\frac{\psi'(x)}{\mu(x)}\right)^2 (dx)}$$

QFI

$$\frac{F}{N^2} = \int_x \mathcal{L}(\psi, \psi', x)(dx)$$

Action : Integral of Lagrangian

$$\boxed{\frac{\partial \mathcal{L}}{\partial \psi} - \frac{d}{dx}\frac{\partial \mathcal{L}}{\partial \psi} = 0}$$

Euler-Lagrange differential equation for $\psi(x)$ that extremizes QFI: ψ_{opt}

$$-\psi''_{\text{opt}} + \mu(x)\psi_{\text{opt}} = \lambda_{\min}\psi_{\text{opt}}$$

1D particle in a potential $\mu(x)$

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Dephasing and Dissipation together – Numerics meet Analytics

30 unconstrained variables but only 1 fitting parameter in model

Coulomb sources of charge $r = N(e^{\gamma} - 1)$

$$\boxed{\mu(x) = \mu_0 + \frac{r}{4(1/2 + x)}}$$

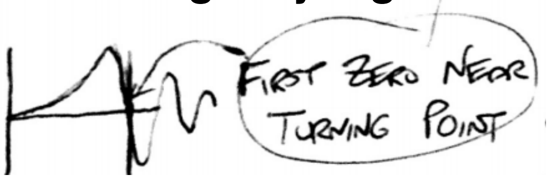
(Single mode loss, r_2=0)

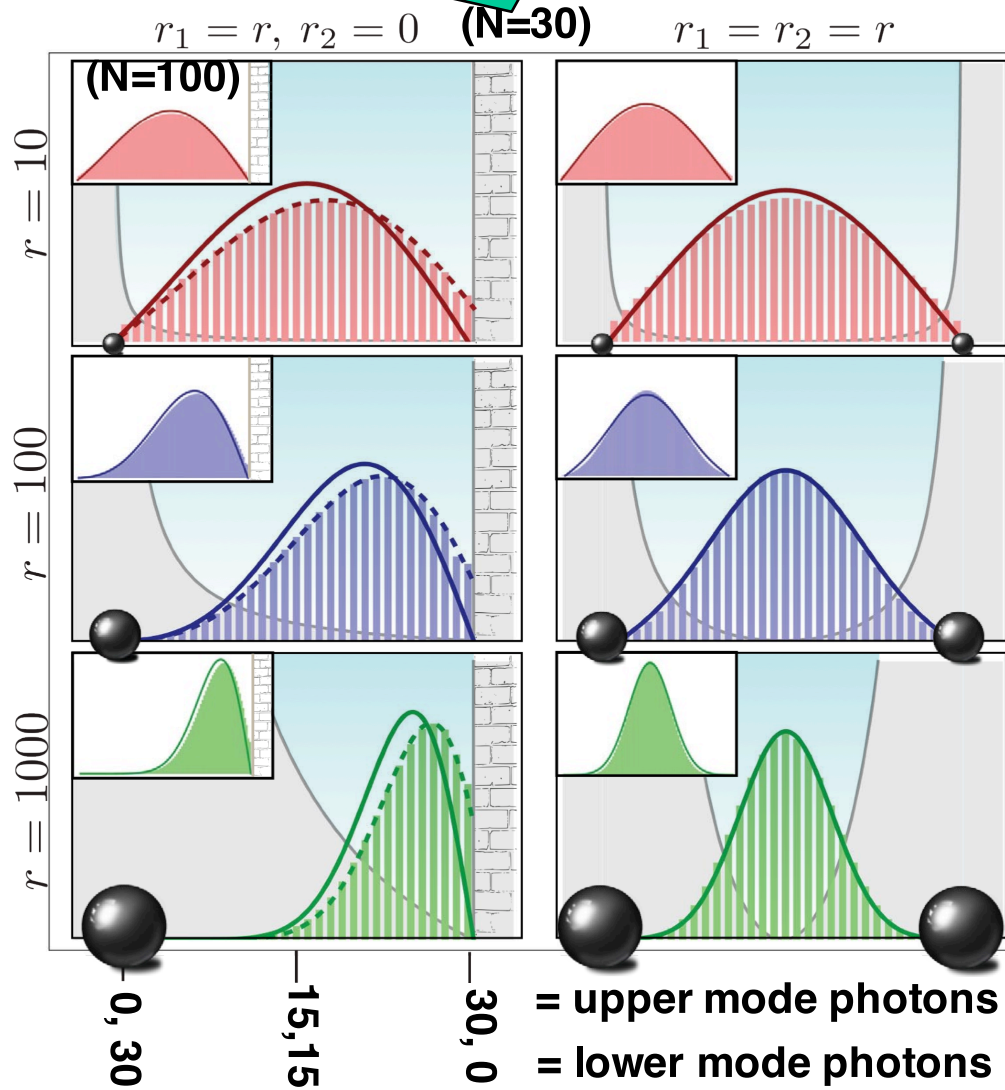
$$\boxed{\psi''(x) + \left(\lambda_{\min} - \frac{r}{4(1/2+x)}\right)\psi(x) = 0}$$

Can be re-couched as Coulomb spherical wave equation

$$\phi''(y) + \left(1 - \frac{2\eta}{y}\right)\phi(y) = 0$$

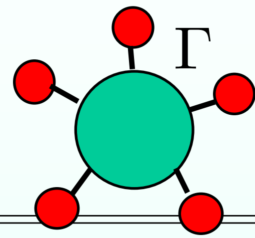
Solutions are 'Whittaker' functions with imaginary arguments

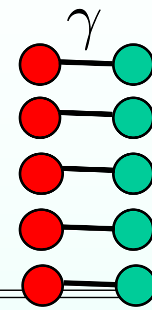


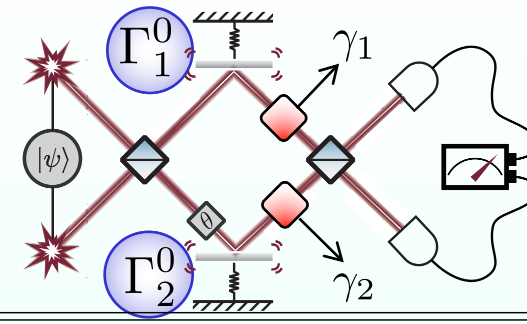


Types of noise: Optimal States and Tight Bounds

$\{0,\ -,\ +\} \mapsto \{\text{dephasing}, \text{relaxation}, \text{excitation}\}$







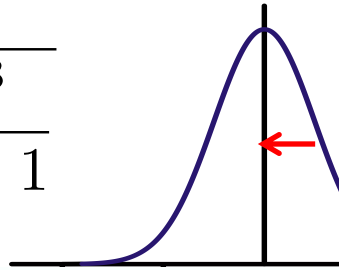
	Collective Decoherence		Individual Decoherence	Hybrid (Interferometry)
Decoherence Process	Dephasing Only (Γ^0)	General Case ($\Gamma^0, \Gamma^{\mp}$)	General Case ($\gamma^0, \gamma^{\mp}$)	Collective Dephasing, Loss ($\Gamma^0, \gamma_1, \gamma_2$)
Conserves S, S^z?	S: yes, S^z: yes	S: yes, S^z: no	S: no, S^z: no [a]	S: no [b] S^z: no
Universal Parameters	$\mu_0 = N^2\Gamma^0$	μ_0, and $\mu_1 = N^2(\Gamma^+ + \Gamma^-)$	$r = N\left(e^{\gamma^0+\gamma^-+\gamma^+} - 1\right)$	μ_0, and $r_{1,2} = N\left(e^{\gamma_{1,2}} - 1\right) \equiv 4N\epsilon^2_{1,2}$
Potential $\mu(x)$ for $N \gg 1$	$\begin{cases} \mu_0 & x \in [-\frac{1}{2}; \frac{1}{2}] \\ \infty & \lvert x\rvert > \frac{1}{2} \end{cases}$	$\mu_0 + \mu_1 \dfrac{x^2}{1/4 - x^2}$	$\dfrac{r}{1-4x^2}$	$\mu_0 + \dfrac{1}{4}\left(\dfrac{r_1}{1/2+x} + \dfrac{r_2}{1/2-x}\right)$
Optimal probe $\psi_{\text{opt}}(x)$	$\sqrt{2}\cos\pi x$	$\text{Gauss}\left[0, \frac{1}{2\mu_1^{1/4}}\right]$	$\text{Gauss}\left[0, \frac{1}{2r^{1/4}}\right]$	$\begin{cases} \text{Airy}\left[\left(\frac{4}{r_1}\right)^{\frac{1}{3}}\right] & (r_2 = 0) \\ \text{Gauss}\left[\frac{1}{2}\frac{\sqrt{r_1}-\sqrt{r_2}}{\sqrt{r_1}+\sqrt{r_2}}, \frac{(r_1 r_2)^{\frac{1}{8}}}{\sqrt{r_1}+\sqrt{r_2}}\right] & \end{cases}$
Expansion of $1/F = \nu \text{ var } \theta_{\text{est}}$	$\Gamma^0 + \frac{\pi^2}{N^2}$	$\Gamma^0 + \frac{\sqrt{\Gamma^- + \Gamma^+}}{N}$ [c]	$\frac{e^{\gamma}-1}{N} + \frac{2\sqrt{e^{\gamma}-1}}{N^{3/2}}$	$\Gamma^0 + \left(\frac{\epsilon_1}{\sqrt{N}} + \frac{\epsilon_2}{\sqrt{N}}\right)^2 + \begin{cases} \frac{\lvert a_1\rvert\epsilon_1}{N^{4/3}} & (r_2 = 0) \\ \frac{2\sqrt{\epsilon_1\epsilon_2}}{N^{3/2}} & \end{cases}$
Optimal clustering, min var θ_{est}	$\dfrac{\sqrt{2e\Gamma^0}}{\nu N}$		$\dfrac{e^{\gamma} - 1}{\nu N}$ [d]	$\dfrac{(\epsilon_1+\epsilon_2)^2}{\nu N} + \begin{cases} \frac{4}{\nu N}\left(\frac{\lvert a_1\rvert}{4}\right)^{\frac{4}{3}}\epsilon_1(\Gamma^0)^{\frac{1}{4}} & (r_2 = 0) \\ \frac{3}{2^{2/3}}\frac{(\epsilon_1+\epsilon_2)^{\frac{4}{3}}(\Gamma^0)^{\frac{1}{3}}}{\nu N(\epsilon_1\epsilon_2)^{1/3}} & \end{cases}$

[a] Individual dephasing (γ^0) conserves S^z but not S.
[b] Boson statistics ensures $S = N/2$, but N is not conserved.
[c] Only for $N \ll 1/\left[(\Gamma^- + \Gamma^+)^{1/6}(\Gamma^- - \Gamma^+)^{2/3}\right]$.
[d] Decreases monotonically ('cluster size' $N_c = \infty$)

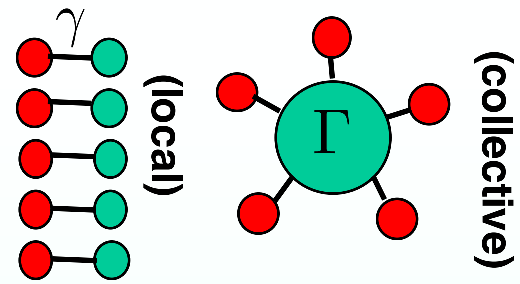
http://arxiv.org/abs/1402.0495

Optimal Probe Summary

$$\sigma \sim \sqrt[4]{\frac{N^3}{e^\gamma - 1}}$$



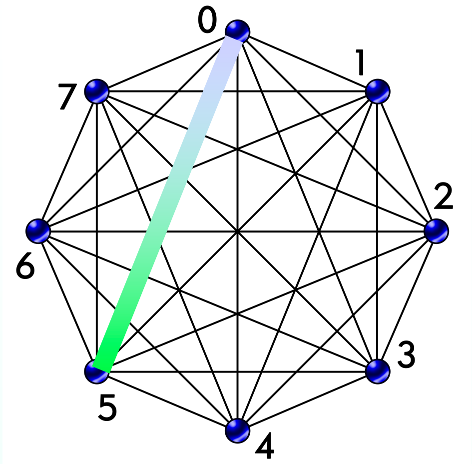
$$\gamma = \gamma^+ + \gamma^- + \gamma^0$$



1. We examined quantum precision for dephasing, excitation, relaxation, and particle loss – for both correlated and uncorrelated noise types.
2. By a direct asymptotic approach we discover tight precision bounds that are achievable but *cannot* be improved on in the N>>1 limit.
3. The approach is constructive – in tandem we learn the structure of the unique optimal probe states $|\psi\rangle$
4. Optimal probes have smooth features for large qubit ensembles N>>1 $\longrightarrow \psi'^2(x)$
5. Quite often the optimal probes have approx gaussian profile of noise-dependent and N-dependent width, e.g. for local noise $\sigma \sim N^{3/4}$ (top of this page)
6. Creating optimal probe states is challenging, some proposals do exist. Must determine a cost/benefit trade-off: engineering probes and scaling up to N>>1 , against the improved coefficient ϵ in shot-noise error scaling: $\delta\theta^2 \geq \dfrac{\epsilon}{N}$

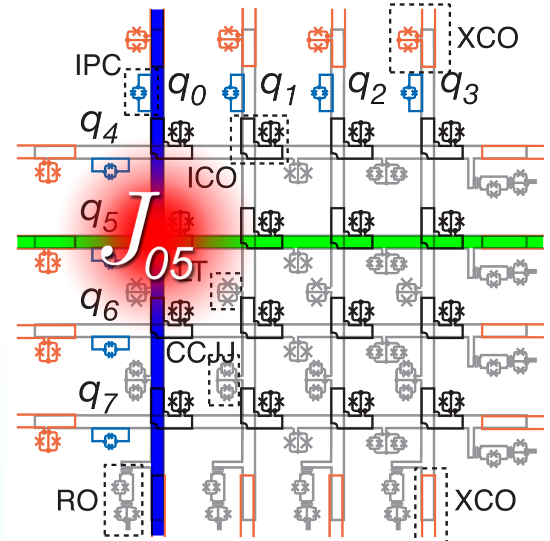
Don't squeeze too far! ☺

Quantum Annealing?

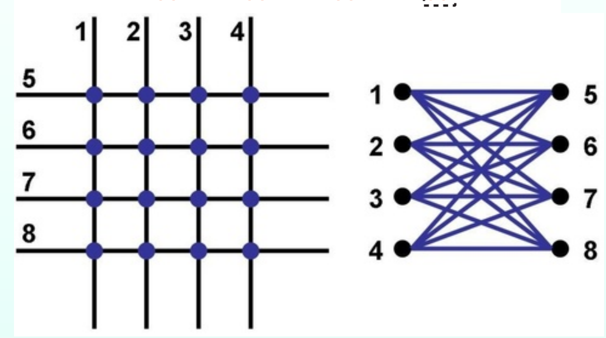


Fully-connected graph of 8 spins

DWave unit cell circuit diagram



DWave unit cell graph (8 spins)



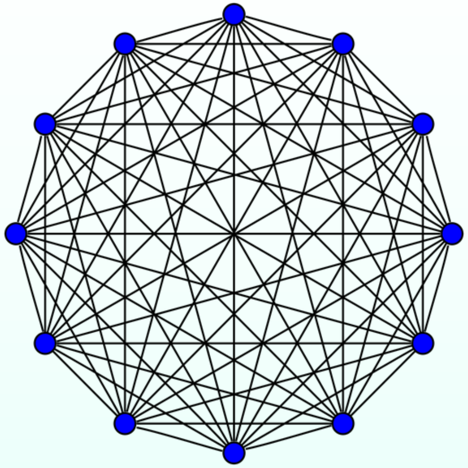
$$H(t) = -(1-t)S_x - t\sum_{i\neq j} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}$$

$$t \in [0,1]$$

Ground State at t =1 gives solution to combinatorial problem encoded in J couplings

Quadratic Spin Hamiltonian in Transverse Field

Simple Model: Infinite Range Ising Hamiltonian
Interacting Spins **without** topological features.
"**Lipkin Meshkov Glick**"



(related to Dicke model)

$$\hat{J}_z^2 = \frac{1}{4}(\hat{\sigma}_z^{(1)} + \hat{\sigma}_z^{(2)} + \hat{\sigma}_z^{(3)} + \ldots)^2$$

Complete graph of 12 spins, edges indicate spin-spin couplings $\hat{\sigma}_z^{(i)}\hat{\sigma}_z^{(j)}$

$$\boxed{\hat{H} = -\Gamma\frac{\hat{J}_x}{j} - (1-\Gamma)\frac{\hat{J}_z^2}{j^2}} \qquad ||\hat{H}|| \leq 1$$

Coupling to transverse magnetic field (Bx,0,0)

Can this Hamiltonian produce an optimal quantum probe state?

Frustration and Glassiness in Spin Models with Cavity-Mediated Interactions

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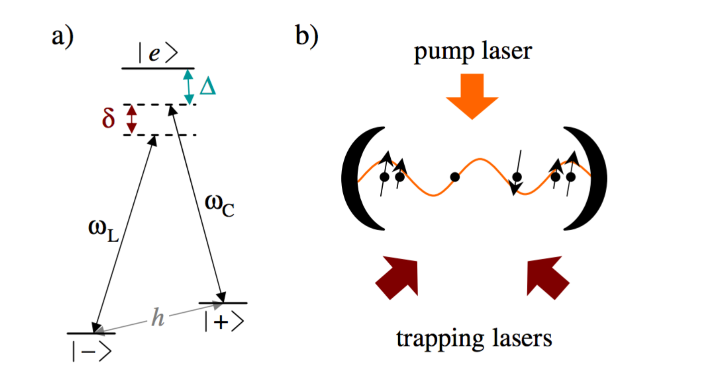
(Received 8 August 2011; published 29 December 2011)

Lev Lab
(Stanford)



We now proceed to eliminate the cavity modes perturbatively, thus arriving at an effective model for the spins, valid on time scales $\gtrsim 2\pi/\delta$:

$$H = H_{\mathrm{at}} + \sum_{\alpha;i<j} \frac{|\Omega(\mathbf{x}_i)|^2}{\Delta^2} \frac{g_\alpha(\mathbf{x}_i)g_\alpha^*(\mathbf{x}_j)}{\delta} \sigma_+^i \sigma_-^j + \mathrm{H.c.} \quad (2)$$

Goldilocks Probes for Noisy Interferometry

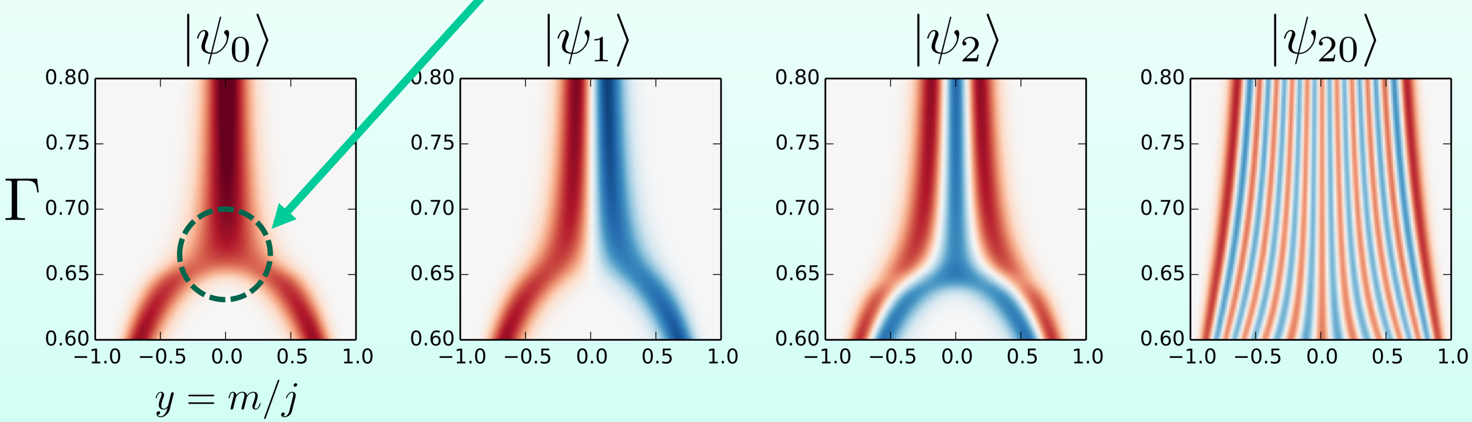
$$\hat{H} = -\Gamma \frac{\hat{J}_x}{j} - (1-\Gamma)\frac{\hat{J}_z^2}{j^2}$$

Annealing parameter is $\Gamma \in [0,1]$, decreasing monotonically as transverse field is gradually turned off

Spin-Coh. GS: $\Gamma = 1$

GHZ or NOON GS: $\Gamma = 0$

'Goldilocks' GS: $\Gamma = 2/3$



Ground State undergoes a **pitchfork** bifurcation at $\Gamma_c = \frac{2}{3}$

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Prior Work (2-mode BECs)

$$\hat{J}_x = \frac{\hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}}{2}$$

$$\hat{J}_z = \frac{\hat{b}^\dagger \hat{b} - \hat{a}^\dagger \hat{a}}{2}$$

Ground state of the double-well condensate for quantum metrology

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We discuss theoretically the ground state of a Bose-Einstein condensate with attractive atom-atom interactions in a double-well trap as a starting point of Heisenberg-limited atom interferometry. The dimensionless parameter governing the quality of the ground state for this purpose is identified. The near-degeneracy between the ground state and the first excited state severely curtails the prospects of the thermally prepared ground state in quantum metrology.

Quantum superposition states of Bose-Einstein condensates

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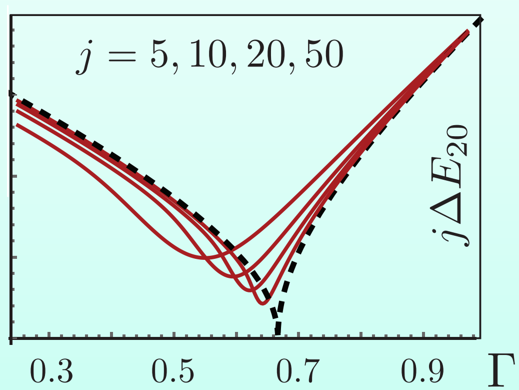
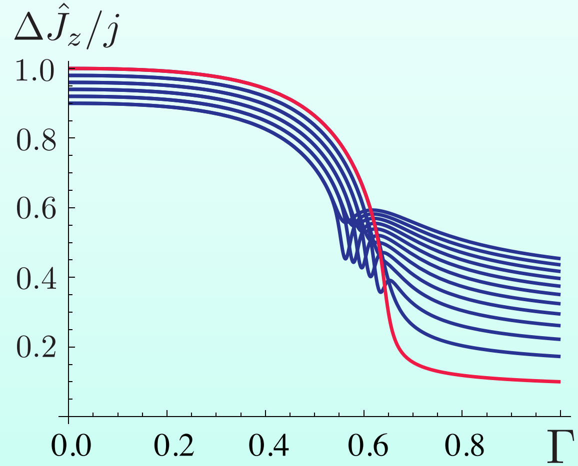
[2]*Commissariat a l'Enegie Atomique, DSM/DRECAM/SPAM, Centre d'Etudes de Saclay, 91191 Gif-sur-Yvette, France*
[3]*Institute of Physics and Astronomy, University of Aarhus, DK-8000 Århus C, Denmark*

(Received 16 June 1997)

We propose a scheme to create a macroscopic ''Schrödinger-cat'' state formed by two interacting Bose condensates. In analogy with *quantum optics*, where the control and engineering of quantum states can be maintained to a large extent, we consider the present scheme to be an example of *quantum atom optics* at work.



Minimum Gap



'Width' Ground State

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difference equation to *differential* equation.

$$\boxed{-\langle m|\hat{H}|\psi\rangle = \langle m|\left\{\Gamma\frac{\hat{J}_x}{j} + (1-\Gamma)\frac{\hat{J}_z^2}{j^2}\right\}|\psi\rangle = -E\langle m|\psi\rangle}$$

$$\hat{J}_z|m\rangle = m|m\rangle$$

$$\hat{J}_x = \frac{\hat{J}^{(+)} + \hat{J}^{(-)}}{2}$$

$$\gamma = \frac{\Gamma}{1-\Gamma}$$

(Annealing ratio)

$$|\psi\rangle = \sum_{m=-j}^{j} \psi_m |m\rangle$$

$$\hat{J}^{(\pm)}|m\rangle = \sqrt{j^2 - m^2 + j \mp m}\,|m \pm 1\rangle$$

$$\boxed{\psi_{(m-1)}\sqrt{1+\frac{1}{j-m}} + \psi_{(m+1)}\sqrt{1+\frac{1}{j+m}} = -\frac{2j}{\gamma}\frac{1}{\sqrt{j^2-m^2}}\left(\frac{E}{1-\Gamma} + \frac{m^2}{j^2}\right)\psi_m}$$

$$\frac{1}{j} \mapsto \delta, \quad \frac{m}{j} \mapsto y \in [-1, 1], \quad \psi_m \mapsto \psi(y), \quad \psi_{(m\pm1)} \mapsto \psi(y \pm \delta)$$

$$\boxed{\frac{\psi(y+\delta) + \psi(y-\delta) - 2\psi(y)}{\delta^2} \equiv \frac{d^2\psi}{dy^2}}$$

$$\boxed{\frac{\psi(y+\delta) - \psi(y-\delta)}{2\delta} \equiv \frac{d\psi}{dy}}$$

Maps to 1D variable-mass 'particle' in a potential V(y)

$$\boxed{\left[\frac{1}{2}\hat{P}\hat{M}^{-1}(y)\hat{P} + V(y)\right]\psi_n(y) = \frac{E_n}{\Gamma}\psi_n(y)}$$

Hermitian K.E.

$$\hat{M}^{-1}(y) = \left(\sqrt{1-y^2} + \frac{\delta}{2}\frac{1}{\sqrt{1-y^2}}\right) \qquad \hat{P} = -i\delta\frac{d}{dy}$$

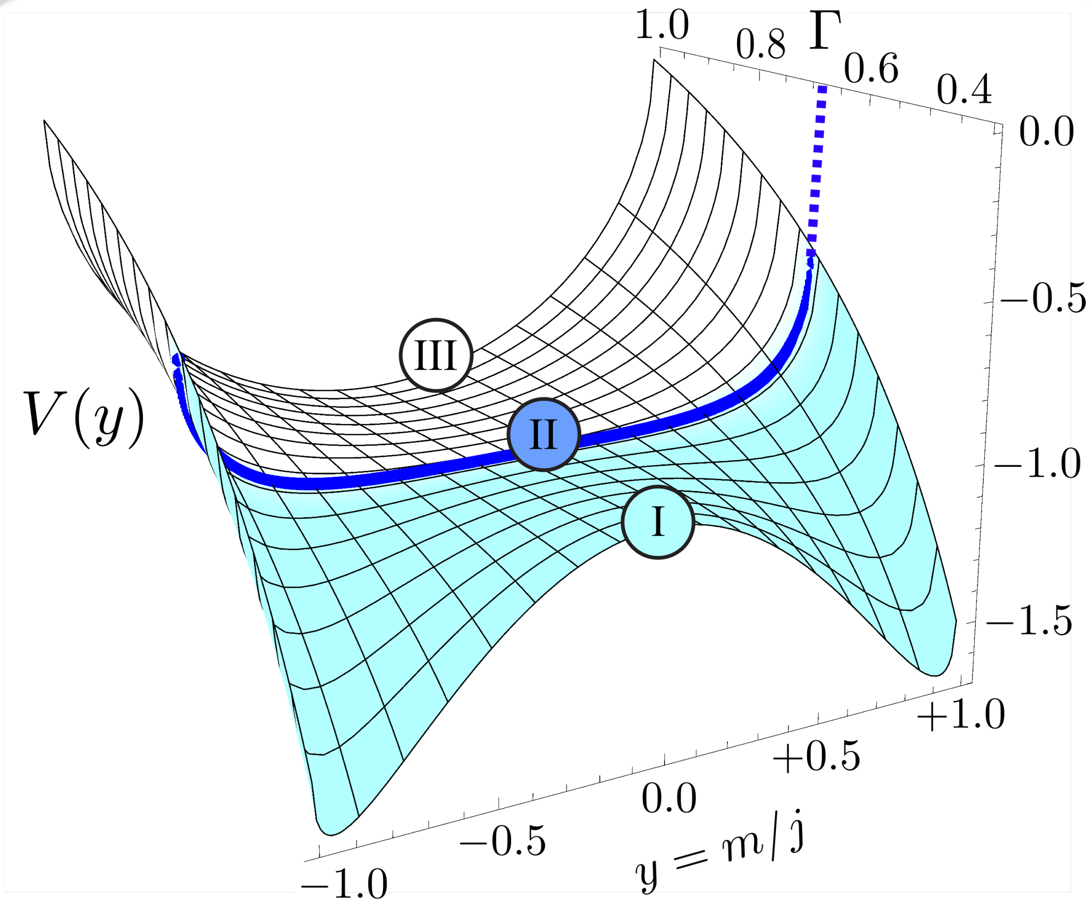
$$\boxed{V(y) = -\frac{y^2}{\gamma} - \sqrt{1-y^2} + O(\delta)}$$

potential

$$\gamma_c = 2, \quad V(y) \approx y^4/8$$

Approx. pure quartic potential

Particle in Ground State of Quartic well (at Goldilocks phase transition) is a promising Probe candidate for Quantum Metrology.



$$V(y) = -\frac{y^2}{\gamma} - \sqrt{1-y^2} + O(\delta)$$

Cramer-Rao

III $\frac{1}{F_{\mathrm{III}}} \sim \frac{1}{\eta N}$

(Coherent - loss)

II $\frac{1}{F_{\mathrm{II}}} \sim ?$

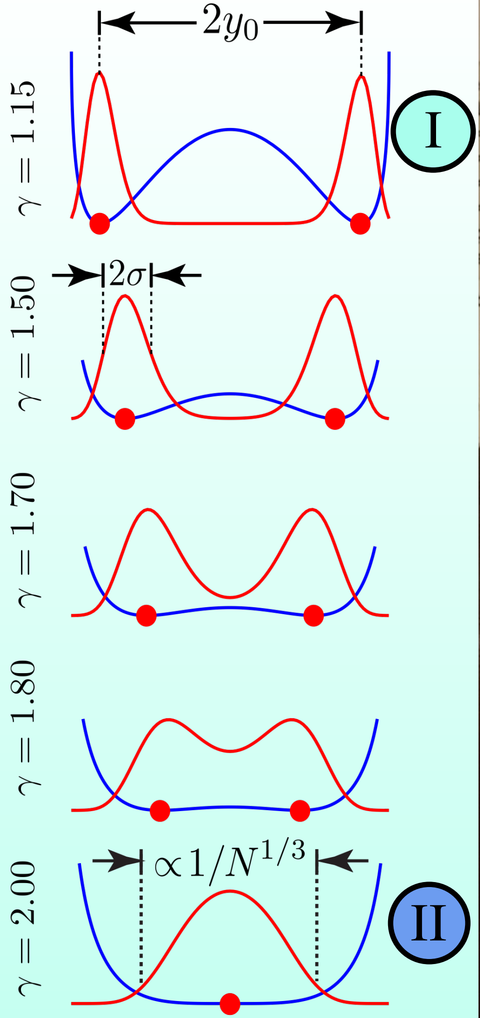
I $\frac{1}{F_{\mathrm{I}}} \sim \frac{1}{\eta^N N^2}$

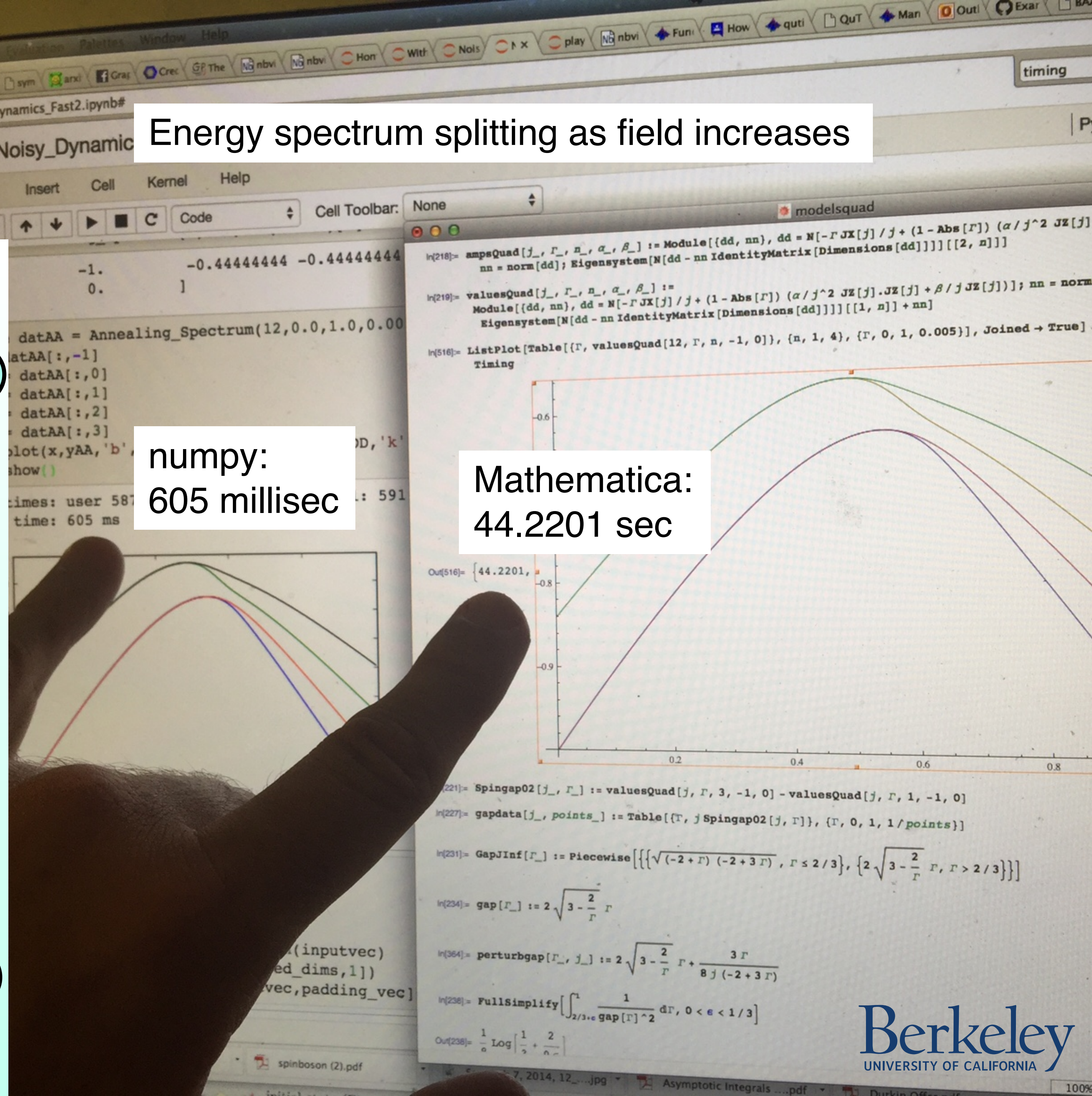
(GHZ - loss)

I $\frac{1}{F_{\mathrm{I}}} \sim \frac{e^{\kappa^0} N^2}{N^2}$

(GHZ -dephasing)

Energy spectrum splitting as field increases





numpy:
605 millisec

Mathematica:
44.2201 sec

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Prior Work on Double-Well Potentials: Tunnel-Splitting Factors

1.

J. Phys. A: Math. Gen. **20** (1987) 4309–4319.

Tunnelling of a large spin: mapping onto a particle problem

G Scharf†, W F Wreszinski† and J L van Hemmen‡
† Institut für Theoretische Physik der Universität Zürich, Schönberggasse 9, CH-8001 Zürich, Switzerland
‡ Sonderforschungsbereich 123 der Universität Heidelberg, D-6900 Heidelberg, Federal Republic of Germany

Received 30 January 1987

Abstract. Tunnelling of a single quantum spin is studied in the limit of large spin quantum number S. The problem is mapped onto a *particle* problem on the positive half-line, with a Hamiltonian which is invariant under inversion $x \to 1/x$. Not only the ground-state energy but also all the other energy levels and corresponding level splittings (if any) are computed by using the conventional WKB methods for the particle problem and an excellent agreement with numerical data is found.

1. Introduction

In two recent papers (van Hemmen and Sütö 1986a, b) a WKB formalism was presented to describe the quantum dynamics, including tunnelling, of single spin with large spin quantum number S. A typical example is provided by the Schrödinger equation ($\hbar = 1$)

$$\mathrm{i}\frac{\mathrm{d}\psi}{\mathrm{d}t} = (-\gamma S_z^2 - \alpha S_x)\psi \qquad (1.1)$$

2. arxiv.org/abs/cond-mat/0003115

Tunnel splittings for one dimensional potential wells revisited

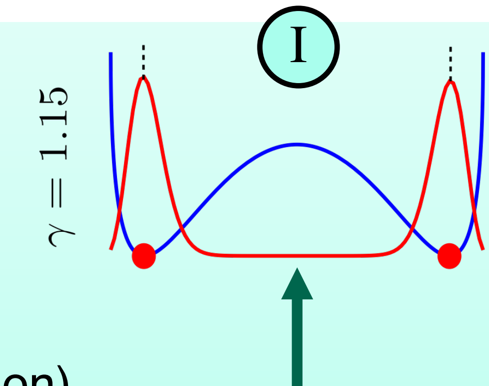
Anupam Garg
Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208
(February 1, 2008)

Abstract

The WKB and instanton answers for the tunnel splitting of the ground state in a symmetric double well potential are both reduced to an expression involving only the functionals of the potential, without the need for solving any auxilliary problems. This formula is applied to simple model problems. The prefactor for the splitting in the text book by Landau and Lifshitz is amended so as to apply to the ground and low lying excited states.

$$\Delta = \frac{2\hbar^2}{m}\psi_0(0)\psi_0'(0)$$

Herring's Formula
(exponential tails in forbidden region)



50 Qubits

Comparing original system with continuous 1D 'particle in a potential' description (numerical)

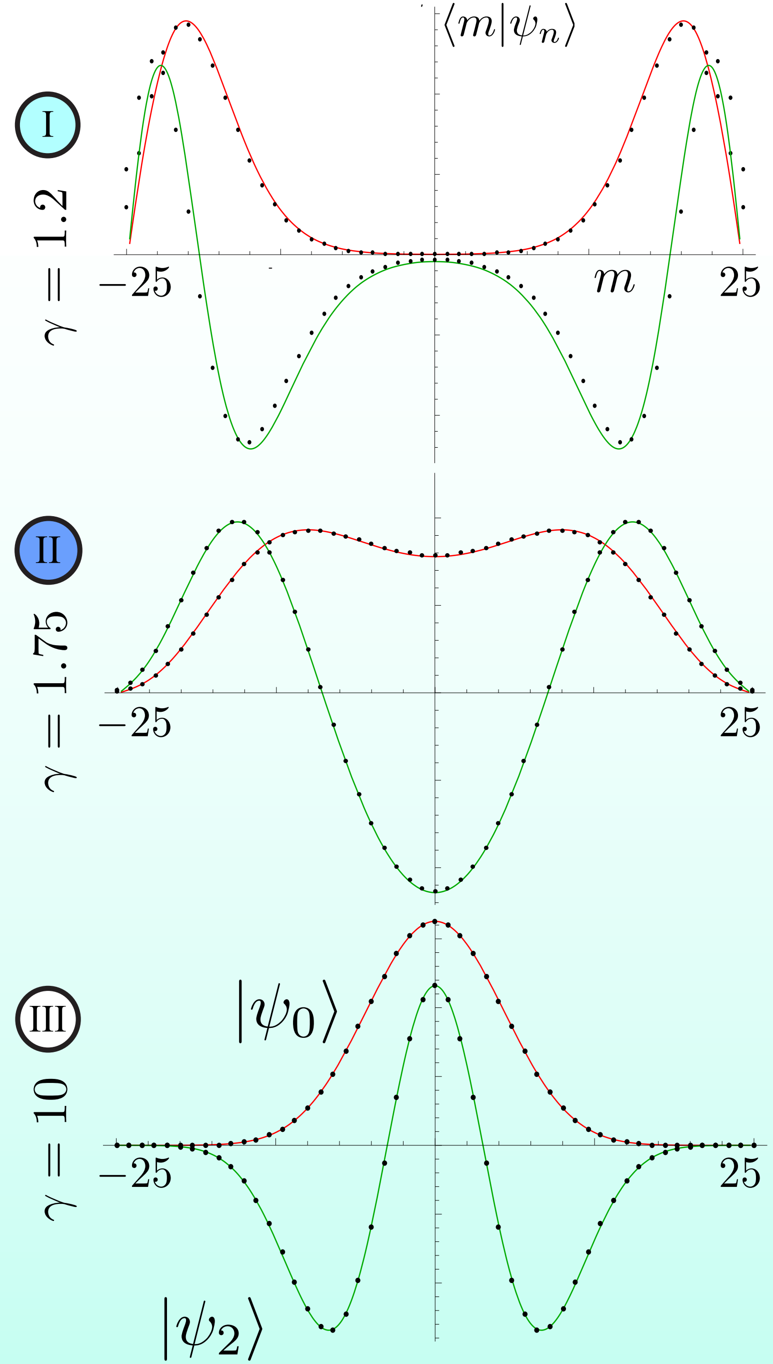
$$\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle$$

$$\boxed{\hat{H} = -\Gamma\frac{\hat{J}_x}{j} - (1-\Gamma)\frac{\hat{J}_z^2}{j^2}}$$

(dots)

$$\boxed{\left[\frac{1}{2}\hat{P}\hat{M}^{-1}(y)\hat{P} + V(y)\right]\psi_n(y) = \frac{E_n}{\Gamma}\psi_n(y)}$$

(continuous curve)



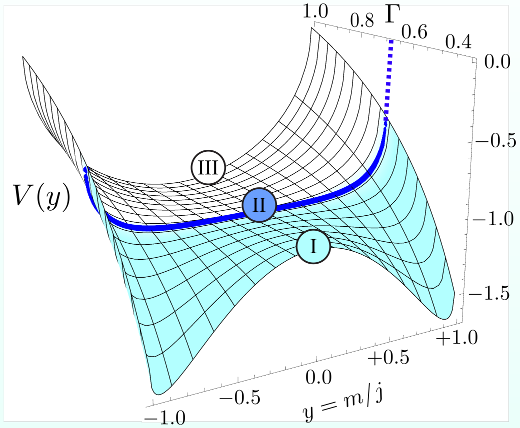
For analytics:

$$\hat{M}^{-1}(y) \mapsto 1/M(y_0) = 1/M_\gamma$$

Simplify 'variable mass' in Kinetic term (take its value at the peak amplitudes)

Scale-free Description: Region II

potential $V(y) = -\frac{y^2}{\gamma} - \sqrt{1-y^2} + O(\delta)$



$$V(y) \approx -1 + \frac{y^2}{2}\left(1 - \frac{2}{\gamma}\right) + \frac{y^4}{8}$$ II Quartic Approximation

$$\left[-\frac{d^2}{dy^2} + \nu y^2 + g y^4\right]\psi_n = \tilde{E}_n \psi_n$$

$$\boxed{\left[-\frac{d^2}{dz^2} + a z^2 + z^4\right]\phi_n = \epsilon_n(a)\phi_n}$$

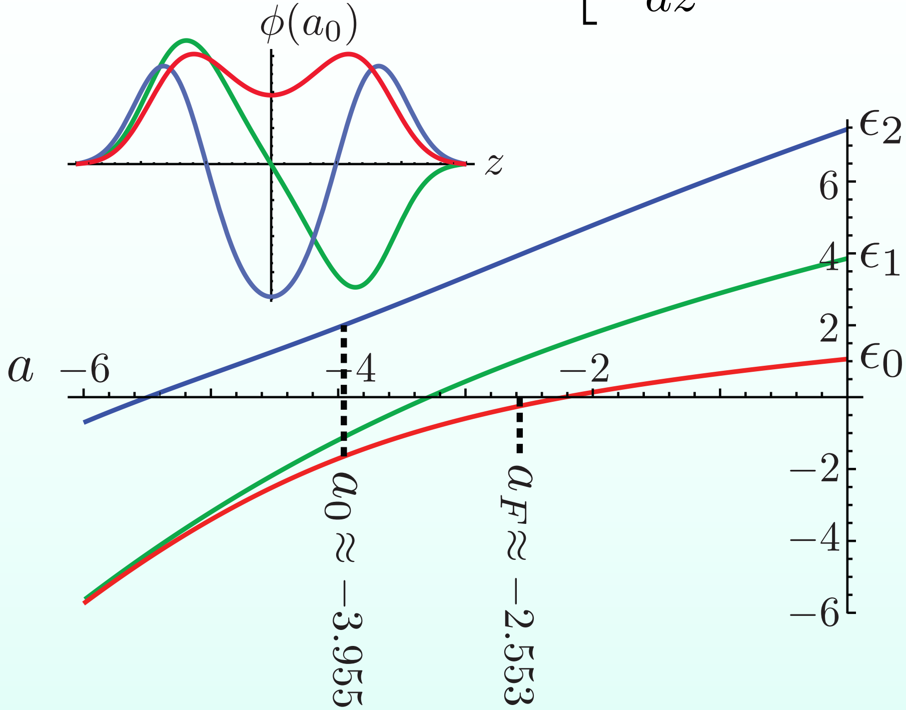
single parameter characterizing system:

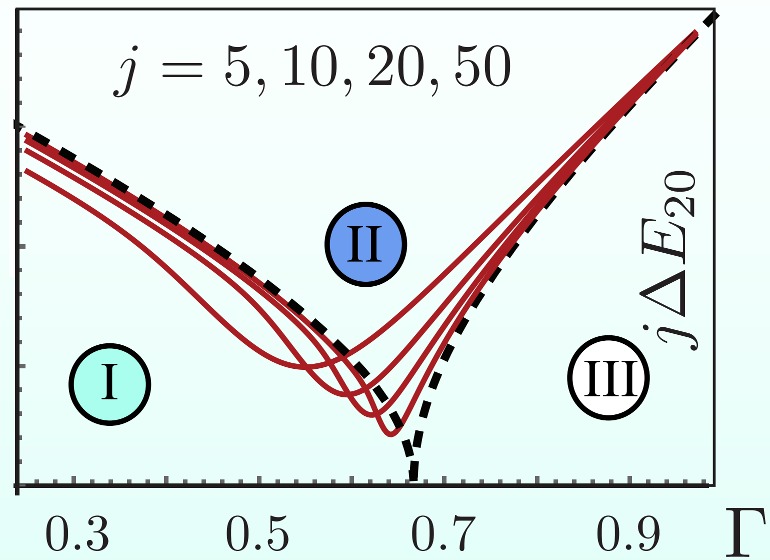
$$a(j,\Gamma) = 4(j^2/4)^{1/3}(3 - 2/\Gamma)$$

$$z = y\,(j^2/4)^{1/6} \qquad \Delta\hat{J}_z = j\,\Delta y = j \times j^{-1/3} = j^{2/3}$$

Numerically exact solution of single parameter problem:

$$\left[-\frac{d^2}{dz^2} + az^2 + z^4\right]\phi_n = \epsilon_n(a)\phi_n$$





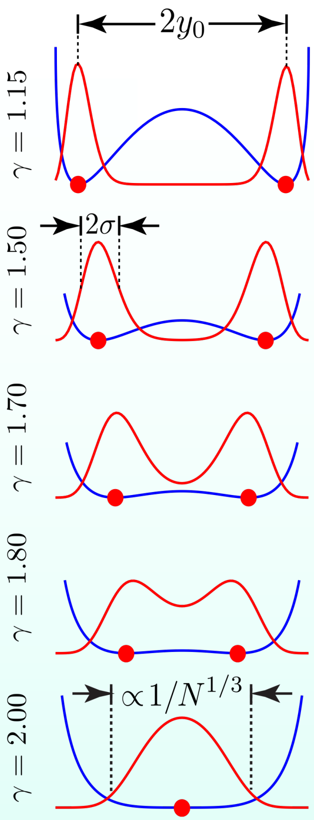
'Size' of Critical Region II : $\Delta\Gamma \sim \Gamma_0 - \Gamma_c = O(N^{-2/3})$

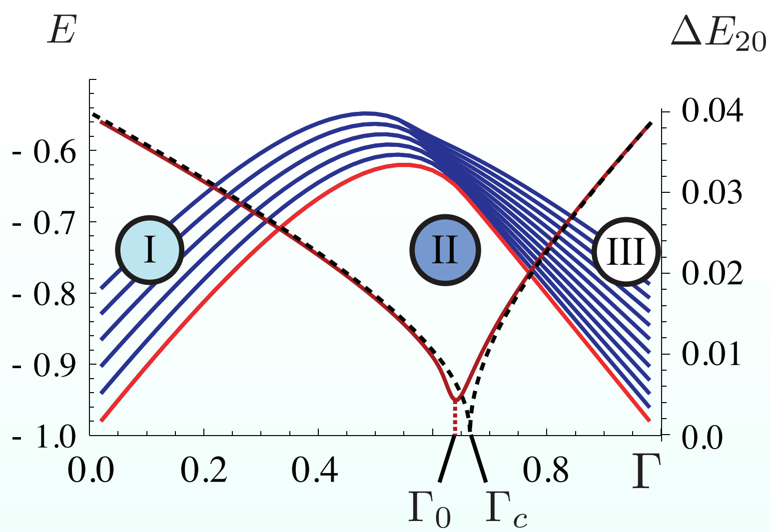
$$j\Delta E_{k\ell} = \frac{\Gamma\Delta\epsilon_{k\ell}}{2(4j)^{1/3}}$$

Why care about the location of minimum gap?

Because this is a bottleneck where annealing has to go slowest -- determines the time-scale of the process.

Approximation to states in Region I and III





Region I: weak field   Region III: strong field

Dashed line is energy gap ω in thermodynamic limit $j \sim \infty$

$$\omega_{\mathrm{I}} = \sqrt{(\Gamma - 2)(3\Gamma - 2)}\ ,\ (\Gamma < \Gamma_c)$$

$$\omega_{\mathrm{III}} = 2\Gamma\sqrt{3 - 2/\Gamma}\ ,\ (\Gamma > \Gamma_c)$$

$$\omega = \sqrt{V_0''/M_\gamma}$$

$$M_\gamma = 2/\gamma$$

Fpr N>>1 approx potential as quadratic near minima. GS is pair of SHO in Region I and a single oscillator in Region III with frequencies above.

Calculation of Precision - Noisy Conditions

$N \gg 1$ QFI asymptotic expression: ‘Action’ integral

http://arxiv.org/abs/1402.0495

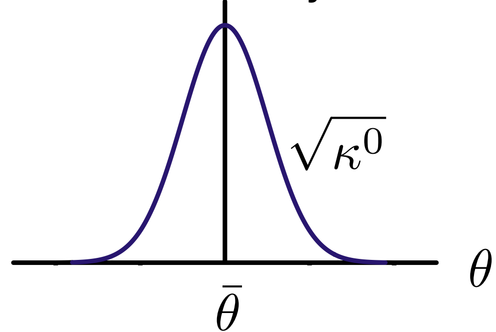
$$\frac{F}{N^2} = \int_{y=-1}^{+1} \frac{\psi^2(y)}{\mu(y)} dy \; - \; 4 \int_{y=-1}^{+1} \left(\frac{\psi'(y)}{\mu(y)} \right)^2 dy \; + O\left(\frac{1}{\mu(y)^3} \right)$$

Penalty term

Noise potential function $\mu(y) = N^2 \kappa^0 + N \kappa^{(L)} / (1 - y^2)$

Collective dephasing, or prior Bayesian uncertainty

Local Noise (dephasing, excitation and/or relaxation. Sometimes called ‘private baths assumption’)



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Dominant Penalty Term in QFI

II

Asymptotic smoothness condition (penalizes discontinuities)

$$\int_y \psi'(y)^2 (dy) = \delta^{-2} \langle \hat{P}^2 \rangle$$

1. Schrodinger Eqn. $$\langle \hat{P}^2 \rangle / \delta^2 = g^{1/3} [\epsilon_n(a) - a\langle z^2 \rangle - \langle z^4 \rangle]$$

2. Eigenstate result: $$\langle [\hat{H}, \hat{P}y] \rangle = 0$$

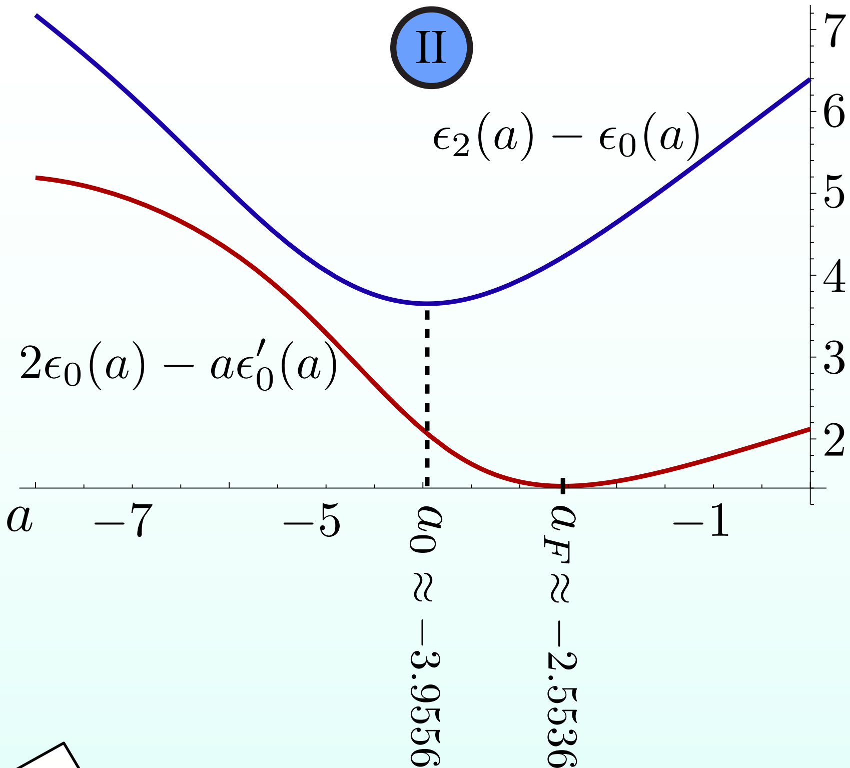
$$\boxed{\epsilon_0' = \frac{d\epsilon_0}{da} \equiv \langle z^2 \rangle}$$

(Hellman-Feynman)

1 + 2 gives result: $$\frac{\langle \hat{P}^2 \rangle}{\delta^2} = \frac{g^{1/3}}{3} \Big(2\epsilon_0(a) - a\epsilon_0'(a) \Big)$$

Precision is (asymptotically) a function **only** of the ground state energy and its rate of change with parameter – not dependent on ensemble size, noise strength or noise type.

Locations of Minimum Gap and Maximum Precision



$$\left[-\frac{d^2}{dz^2} + az^2 + z^4\right]\phi_n = \epsilon_n(a)\phi_n$$

$a = 0 \quad \Gamma_c$ Critical Annealing in thermodynamic limit

$a_F \mapsto \quad \Gamma_F$ Annealing Value at Max Precision

$a_0 \mapsto \quad \Gamma_0$ Annealing value at Min Gap

$$\boxed{\frac{1}{F} = \kappa^0 + \frac{\kappa^{(L)}}{N}} + \frac{2}{3N^{4/3}}\left(\frac{M_2}{2}\right)^{\frac{1}{3}}\left(2\epsilon_0(a_F) - a_F\epsilon_0'(a_F)\right)$$

Ultimate bound

penalty term depends only on probe

$$\Gamma_0 = \frac{\Gamma_c}{1 - \frac{a_0}{12g^{1/3}}} \qquad \Delta\Gamma \sim \Gamma_0 - \Gamma_c = O(N^{-2/3})$$

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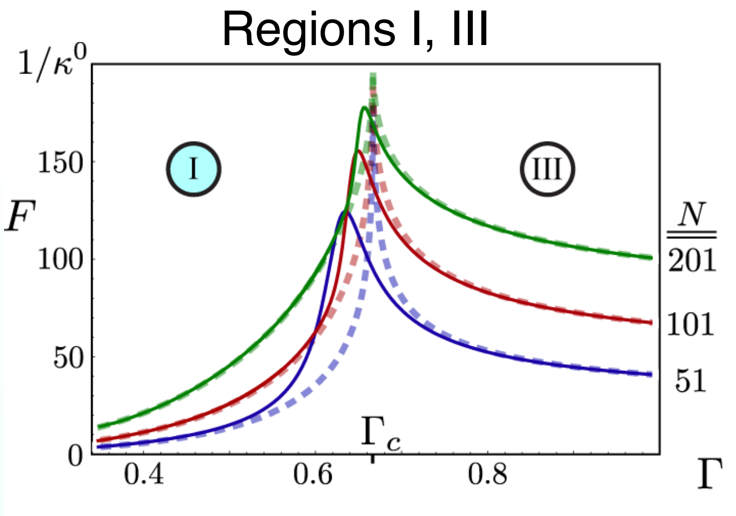
Precision in Presence of Noise



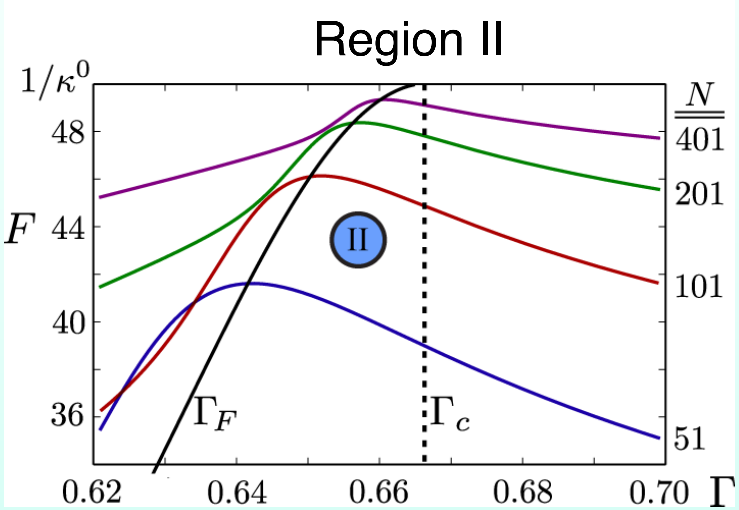
$$\frac{1}{F_\infty(\omega)} = \kappa^0 + \frac{\kappa^{(L)}}{N} + \frac{M\omega}{N}$$

ω is 'quantum' (Energy gap)

I $\quad \omega_{\mathrm{I}} = \sqrt{(\Gamma-2)(3\Gamma-2)} \;,\; (\Gamma < \Gamma_c)$

III $\quad \omega_{\mathrm{III}} = 2\Gamma\sqrt{3-2/\Gamma} \;,\; (\Gamma > \Gamma_c)$

$$\left[-\frac{d^2}{dz^2} + az^2 + z^4\right]\phi_n = \epsilon_n(a)\phi_n$$

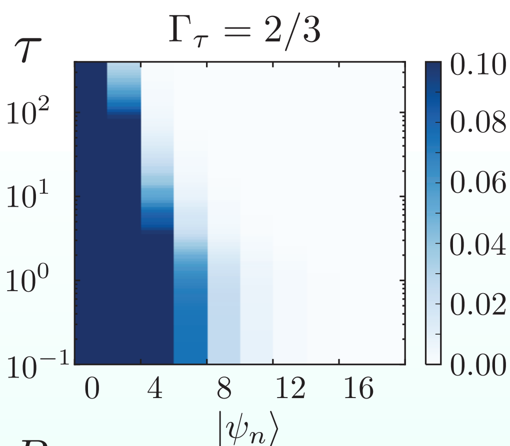


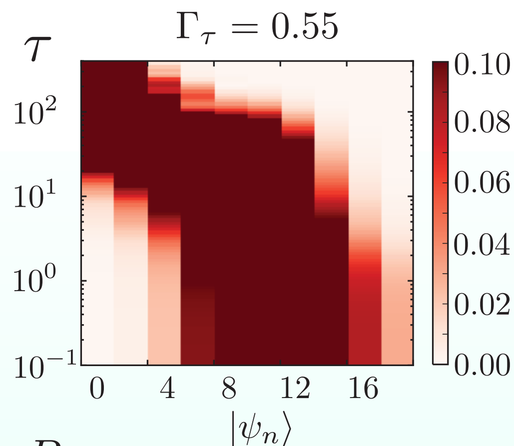
$$[\Delta^2\theta]_{\mathrm{II}} \geq \frac{1}{F} = \kappa^0 + \frac{\kappa^{(L)}}{N} + \frac{2}{3N^{4/3}}\left(\frac{M_2}{2}\right)^{\frac{1}{3}}\left(2\epsilon_0(a_F) - a_F\epsilon_0'(a_F)\right)$$

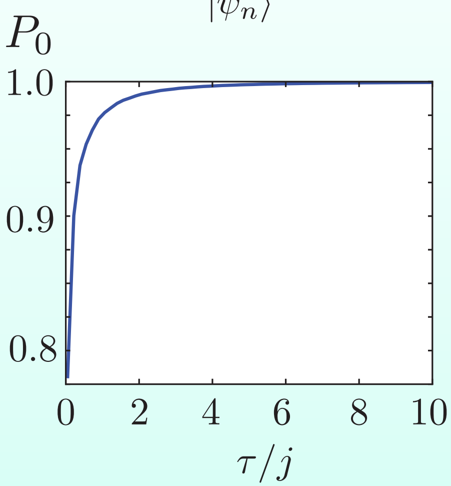
Precision is maximized when the latter term is minimized at $a \mapsto a_F$

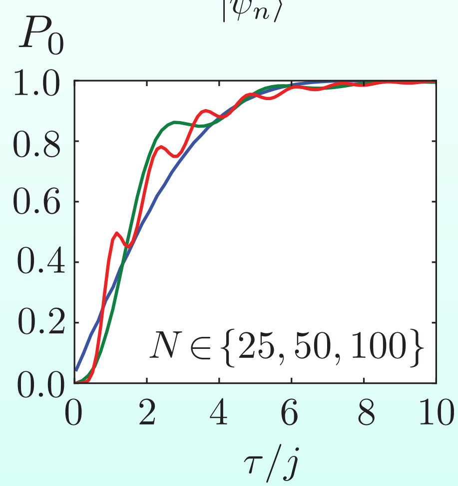
Dynamical Performance

How likely are we to stay in the ground state for a linear annealing of total time τ ? (Noiseless Case)









Terminated in Region II

Terminated in Region I

Time Complexity?

$$T \approx \int_{\Gamma_\tau}^{1} \left\| \frac{d(j\hat{H})}{d\Gamma} \right\|_2 \frac{d\Gamma}{\omega^2(\Gamma)}$$

(Van Dam, Mosca and Vazirani)

$$T \sim O(N)$$

Annealing time scales **linearly** with number of qubits, irrespective of where the annealing halts.

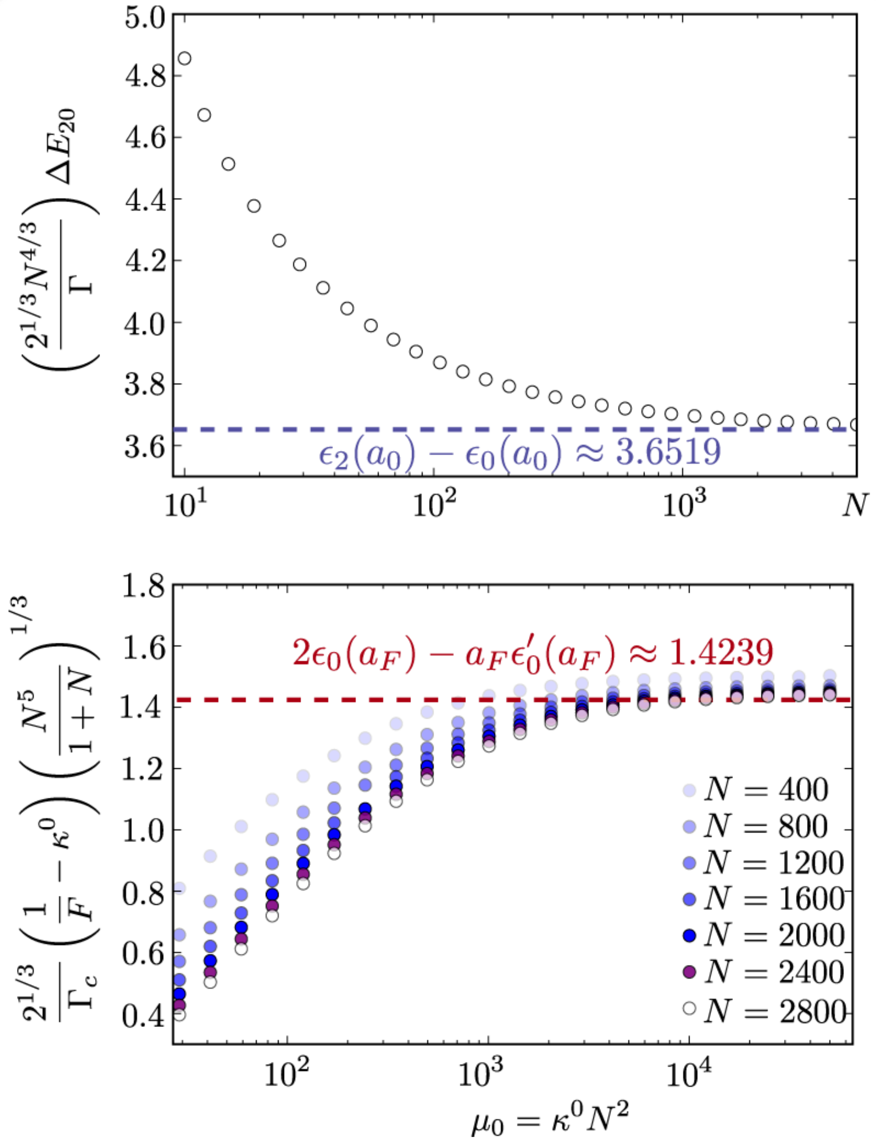
Evolution will be approximately adiabatic for longer annealing times, i.e. state will remain mostly in instantaneous ground state.

Convergence of Numerics to Asymptotic Results

Precise convergence is fairly slow, requiring spin ensembles of N>1000. For QFI this is due to the first neglected term in the asymptotic expansion being only

$$O\left(\frac{1}{N^{1/3}}\right)$$

smaller than the last included term. N must be large for the terms to separate out.



$$\rho(\theta) = \sum_i \lambda_i |\psi_i\rangle\langle\psi_i|$$

$$F(\theta) = 2\sum_{i,j} \frac{|\langle|\psi_i|\rho'(\theta)|\psi_j\rangle|^2}{\lambda_i + \lambda_j} \mapsto 2\sum_{i,j} \frac{(\lambda_i - \lambda_j)^2}{\lambda_i + \lambda_j} \left|\langle\psi_i|\hat{J}_z|\psi_j\rangle\right|^2$$

Numerics Validate Approximations:

I II III

1. Large interacting spin model = single particle in a potential: OK for metrology beyond thermodynamic limit, gives leading finite size corrections.

2. Asymptotic Formula for Quantum Fisher Info is validated.

3. Exact numerical solution of particle in quartic well gives all significant physics at Goldilocks phase transition, including **minimum gap**, **maximum precision** and **entanglement**.

4. Relevant Properties of system are universal and independent of ensemble size N or noise strength/type, as long as their product is >>1, everything determined in Goldilocks region

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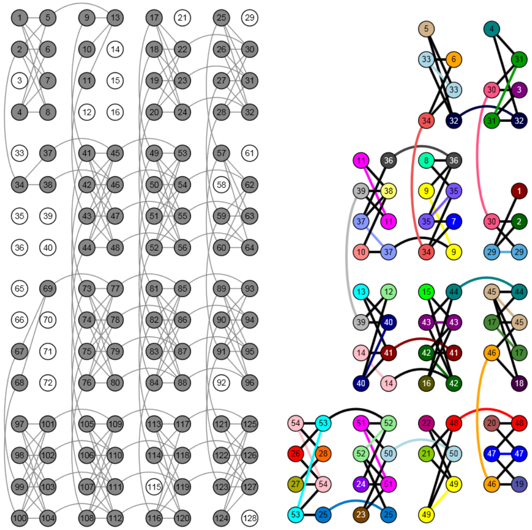
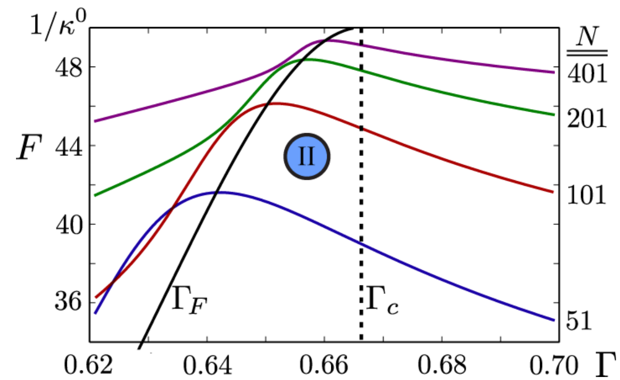
Importance of finite size corrections $f(N)$

(locating maximum correlation via control parameter)



Dwave I (N= 128 Qubits)

We have seen significance for quantum annealers of mesoscopic N



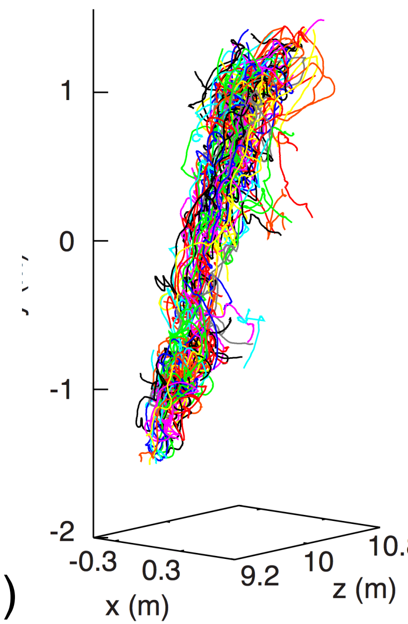
Also important for **biological** systems, e.g. N ~500 swarming midges

Selected for a Viewpoint in *Physics*
PRL **113,** 238102 (2014) PHYSICAL REVIEW LETTERS week ending 5 DECEMBER 2014

Finite-Size Scaling as a Way to Probe Near-Criticality in Natural Swarms

Alessandro Attanasi,[1,2] Andrea Cavagna,[1,2,3,†] Lorenzo Del Castello,[1,2] Irene Giardina,[1,2,3] Stefania Melillo,[1,2,*] Leonardo Parisi,[1,4] Oliver Pohl,[1,2] Bruno Rossaro,[5] Edward Shen,[1,2] Edmondo Silvestri,[1,6] and Massimiliano Viale[1,2]

midges exhibit swarming close to phase transition: **swarming** (disordered) to **flocking** (ordered)



(Viscek model)

Conclusions and Outlook

- Model: Infinite Range Ising Hamlitonian Model of Interacting Spins - no topological features. Spin-Squeezing, BECs etc.

- Anneal to ground state of width $N^{2/3}$near criticality. Requires annealing precision: $\Delta\Gamma \sim N^{-2/3}$

- GS robust against noise, provides optimal metrological precision for N >> 1

- Adiabatic quantum annealing to critical Region II faster than to GHZ state in Region I - linear in N for both

- Quantum Annealers (Dwave)? Individually addressable qubit couplings.

- Uniform couplings produce useful interesting quantum states – what if individually addressable couplings? **Annealing ~ Quantum State Preparation**!

- Molmer RISQ paper – Not just metrology, Quantum Simulation

New inverted computational task – Find optimal Hamiltonian couplings --- ground state to best approximate *known* quantum target arXiv.org > quant-ph > arXiv:1602.08752

Quantum Physics

Goldilocks Probes for Noisy Interferometry via Quantum Annealing to Criticality

Gabriel A. Durkin

(Submitted on 28 Feb 2016)

Quantum annealing is explored as a resource for quantum information beyond solution of classical combinatorial problems. Envisaged as a generator of robust interferometric probes, we examine a Hamiltonian of $N >> 1$ uniformly-coupled spins subject to a transverse magnetic field. The discrete many-body problem is mapped onto dynamics of a single one-dimensional particle in a continuous potential. This reveals all the qualitative features of the ground state beyond typical mean-field or large classical spin models. It illustrates explicitly a graceful warping from an entangled unimodal to bi-modal ground state in the phase transition region. The transitional `Goldilocks' probe has a component distribution of width $N^{2/3}$ and exhibits characteristics for enhanced phase estimation in a decoherent environment. In the presence of realistic local noise and collective dephasing, we find this probe state asymptotically saturates ultimate precision bounds calculated previously. By reducing the transverse field adiabatically, the Goldilocks probe is prepared in advance of the minimum gap bottleneck, allowing the annealing schedule to be terminated `early'. Adiabatic time complexity of probe preparation is shown to be linear in N

Goldilocks principle is all-pervasive on arxiv.org

arXiv.org > quant-ph > arXiv:1111.4982

Quantum Physics

The quantum Goldilocks effect: on the convergence of timescales in quantum transport

Seth Lloyd, Masoud Mohseni, Alireza Shabani, Herschel Rabitz

(Submitted on 21 Nov 2011)

arXiv.org > astro-ph > arXiv:0709.2309

Astrophysics

Evolutionary Catastrophes and the Goldilocks Problem

Milan M. Cirkovic

(Submitted on 14 Sep 2007)

arXiv.org > hep-ph > arXiv:0709.0297

High Energy Physics - Phenomenology

Goldilocks Supersymmetry: Simultaneous Solution to the Dark Matter and Flavor Problems of Supersymmetry

Jonathan L. Feng, Bryan T. Smith, Fumihiro Takayama

(Submitted on 4 Sep 2007 (v1), last revised 15 Jan 2008 (this version, v3))

arXiv.org > astro-ph > arXiv:1102.3926

Astrophysics > Earth and Planetary Astrophysics

Habitability of the Goldilocks Planet Gliese 581g: Results from Geodynamic Models

W. von Bloh, M. Cuntz, S. Franck, C. Bounama

(Submitted on 18 Feb 2011 (v1), last revised 22 Feb 2011 (this version, v2))

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Timeline | Saved

Mar 03, 00:33

Envisaged as a generator of robust interferometric probes, we examine a Hamiltonian of N >> 1 uniformly- couple…

Mar 02, 14:35

Scaling laws for precision in quantum interferometry and bifurcation landscape of optimal state

Text input

Envisaged as a generator of robust interferometric probes, we examine a Hamiltonian of N >> 1 uniformly- coupled spins subject to a transverse magnetic field. We map the discrete many-body problem onto dynamics of a single one-dimensional particle in a continuous potential. This reveals all the qualitative features of the ground state beyond mean-field or large classical spin models. It illustrates explicitly a graceful warping from an entangled unimodal to bi-modal ground state in the phase transition region. The transitional 'Goldilocks' probe has a component distribution of width N2/3 and exhibits characteristics for enhanced phase estimation in a decoherent environment. In the presence of noise, precision is found to be supra-classical for ground states prepared in the strong transverse field regime, peaking at criticality, and inferior to classical unentangled probes in the weak field limit. By reducing the transverse field adiabatically, the optimal probe is prepared in advance of the minimum gap, allowing the annealing schedule to be terminated 'early'. The adiabatic time complexity of probe preparation is shown to be linear in N .

Similar to input text:

Durkin, Gabriel A.

Goldilocks Probes for Noisy Interferometry via Quantum Annealing to Criticality (2016)

Quantum annealing is explored as a resource for quantum information beyond solution of classical combinatorial problems. Envisaged as a generator of robust interferometric probes, we examine a Hamiltonian of $N >> 1$ uniformly-coupled spins subject to a transverse magnetic field. The discrete many-body problem is mapped onto dynamics of a single one-…

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Allahverdyan, Armen E. / Balian, Roger / Nieuwenhuizen, Theo M.

Phase transitions and quantum measurements (2005)

In a quantum measurement, a coupling g between the system S and the apparatus A triggers the establishment of correlations, which provide statistical information about S. Robust registration requires A to be macroscopic, and a dynamical symmetry breaking of A governed by S allows the absence of any bias. Phase transitions are thus a paradigm for…

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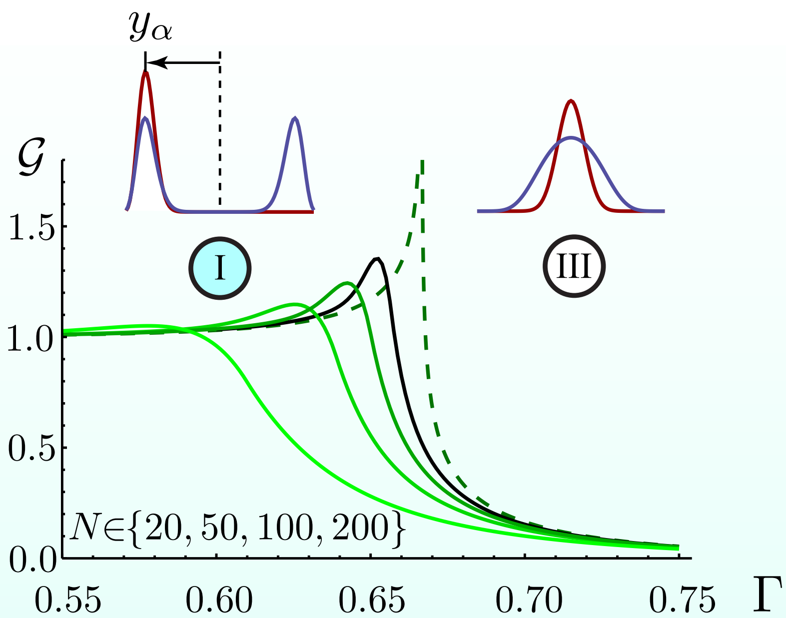
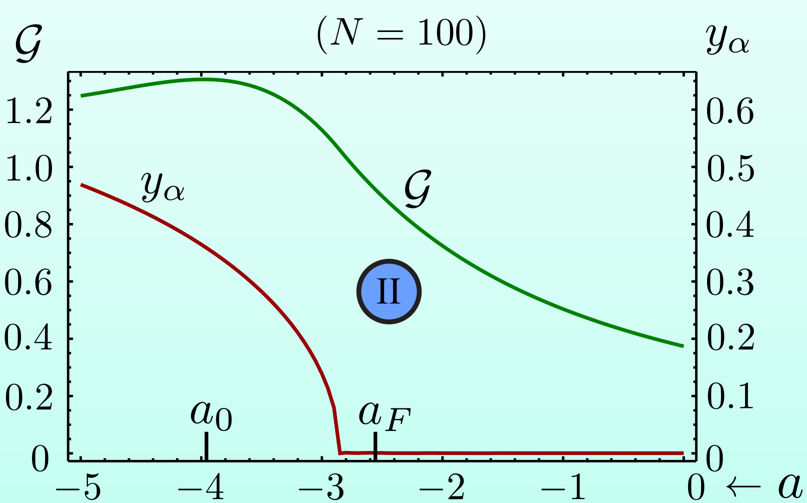
Following slides are back-up

Global Entanglement

$$\mathcal{G}[|\psi\rangle] = \min\left\{-\log_2 |\langle\chi|\psi\rangle|^2\right\} \ , \quad \forall\, |\chi\rangle \in S$$





Logarithm of squared overlap with nearest separable state (red curve)

Can be argued that in the fully symmetric j=N/2 subspace the nearest separable state is a spin coherent state. It has width:

$$\Delta y \propto \frac{1}{\sqrt{j}}$$

$$\Delta z \propto \Delta y\, j^{1/3} \propto j^{-1/6}$$

(width of spin coh. state in scale-free coords)

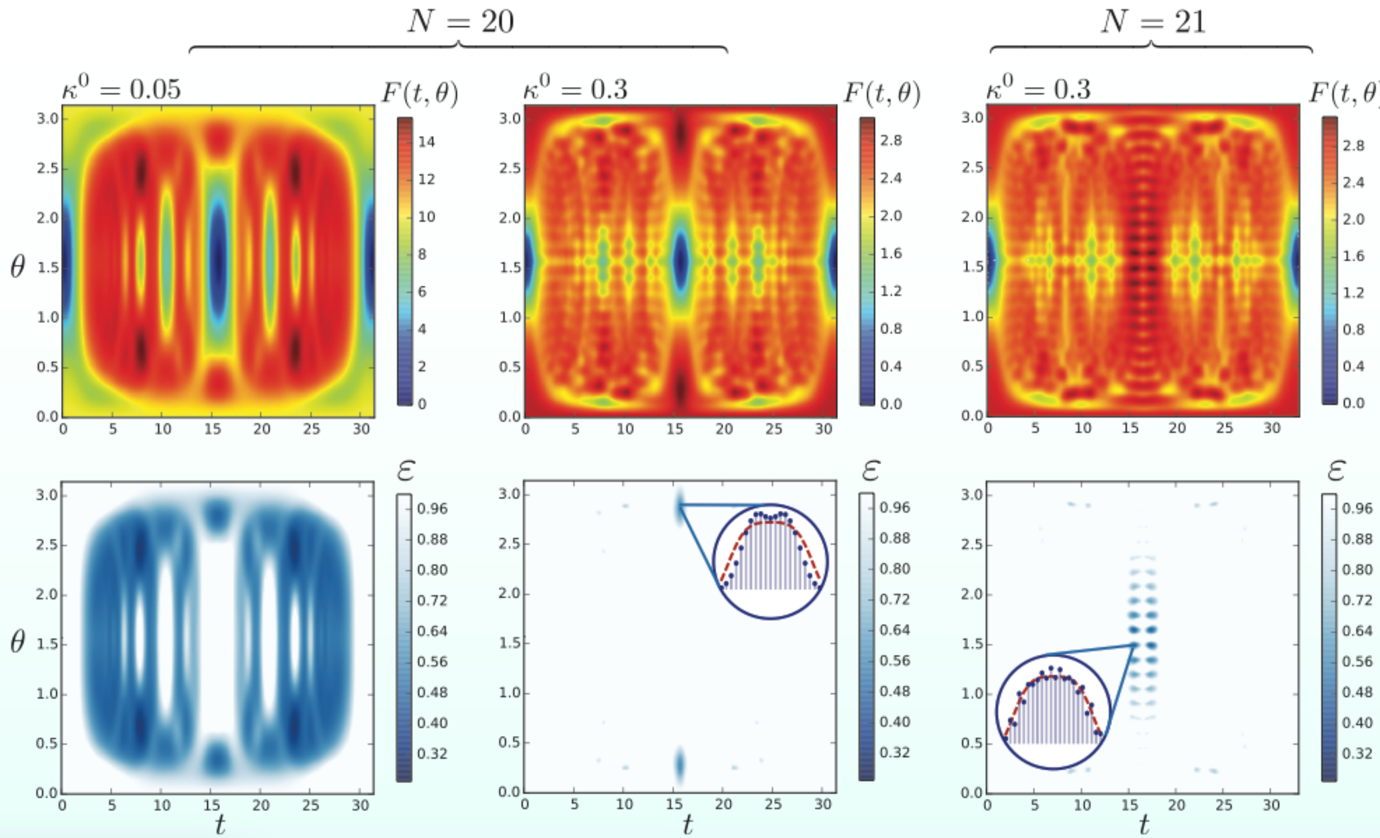
$$\mathcal{G}_\infty \sim \frac{1}{6}\log_2 N$$

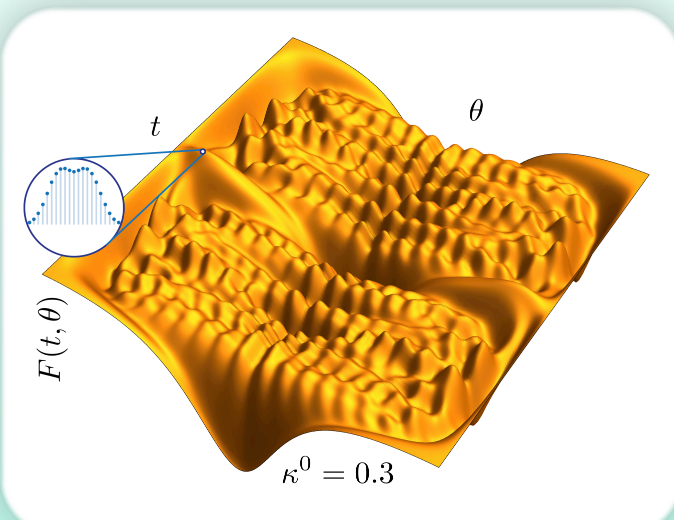
Maximum entanglement is in Critical Region II. Confirms central result of PRL 101, 025701 (2008).

Also note $\mathcal{G}_\infty \leq N - 1$ in general.

Sudden Quench Dynamics

Collective Dephasing (also look at local noise processes)





$$\exp\{-i\hat{J}_y\theta\}\,\exp\{-i\hat{J}_z^2 t/j\}\Big(|\rightarrow\rangle^{\otimes N}\Big)$$

$$1/F = \kappa^0 + \varepsilon/N$$

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RAPID COMMUNICATIONS

PHYSICAL REVIEW A **87**, 051801(R) (2013)

Efficient spin squeezing with optimized pulse sequences

C. Shen* and L.-M. Duan

Department of Physics, University of Michigan, Ann Arbor, Michigan 48103, USA and
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Spin squeezed states are a class of entangled states of spins that have practical applications to precision measurements. In recent years spin squeezing with one-axis twisting (OAT) has been demonstrated experimentally with spinor Bose-Einstein condensates (BECs) with more than 10^3 atoms. Although the noise is below the standard quantum limit, the OAT scheme cannot reduce the noise down to the ultimate Heisenberg limit. Here we propose an experimentally feasible scheme based on optimized quantum control to greatly enhance the performance of OAT to approach the Heisenberg limit, requiring only an OAT Hamiltonian and the use of several coherent driving pulses. The scheme is robust against technical noise and can be readily implemented for spinor BECs or trapped ions with current technology.

$$\exp\{-i\hat{J}_y\theta\}\exp\{-i\hat{J}_z^2 t/j\}\left(|\rightarrow\rangle^{\otimes N}\right)$$

$$\hat{J}_z \mapsto \sqrt{\frac{N}{2}}\hat{x}$$

$$\hat{J}_x \mapsto -\sqrt{\frac{N}{2}}\hat{p}$$

$$\hat{J}_y \mapsto -\frac{N+1}{2} + \frac{1}{2}\left(\hat{p}^2 + \hat{x}^2\right)$$

closed oscillator algebra

$$\{\hat{x}^2,\ \hat{p}^2,\ (\hat{x}\hat{p} + \hat{p}\hat{x})\}$$

Most general propagator within this algebra

$$\exp\{i\alpha\hat{J}_z^2 + i\beta\hat{J}_y + i\gamma(\hat{J}_z\hat{J}_x + \hat{J}_x\hat{J}_z)\}$$

Now, a 3D optimization, no ordering problem

Estimation of Phase and Diffusion: Combining Quantum Statistics and Classical Noise

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(Dated: July 22, 2013)

Coherent ensembles of N qubits present an advantage in quantum phase estimation over separable mixtures, but coherence decay due to classical phase diffusion reduces overall precision. In some contexts, the strength of diffusion may be the parameter of interest. We examine estimation of both phase and diffusion in large spin systems using a novel mathematical formulation. For the first time, we show a closed form expression for the quantum Fisher information for estimation of a unitary parameter in a noisy environment. The optimal probe state has a non-Gaussian profile and differs also from the canonical phase state; it saturates a new tight precision bound. For noise below a critical threshold, entanglement always leads to enhanced precision, but the shot-noise limit is beaten only by a constant factor, independent of N. We provide upper and lower bounds to this factor, valid in low and high noise regimes. Unlike other noise types, it is shown for $N \gg 1$ that phase and diffusion can be measured simultaneously and optimally by canonical phase measurements.

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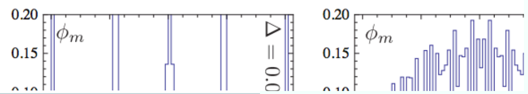
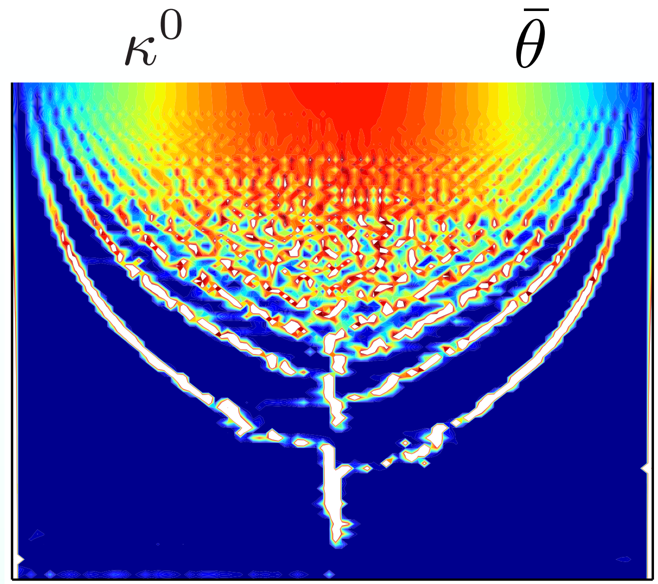
Dephasing, or a random uncontrollable phase accumulation, is one of the most important types of noise in quantum systems, responsible for a transition from quantum to classi-





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Joint estimation of phase and phase diffusion for quantum metrology

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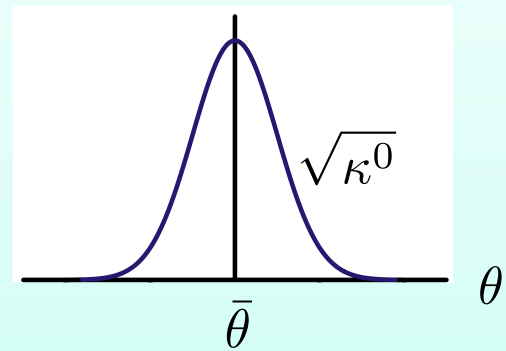
Phase estimation, at the heart of many quantum metrology and communication schemes, can be strongly affected by noise, whose amplitude may not be known, or might be subject to drift. Here we investigate the joint estimation of a phase shift and the amplitude of phase diffusion at the quantum limit. For several relevant instances, this multiparameter estimation problem can be effectively reshaped as a two-dimensional Hilbert space model, encompassing the description of an interferometer probed with relevant quantum states—split single-photons, coherent states or NOON states. For these cases, we obtain a trade-off bound on the statistical variances for the joint estimation of phase and phase diffusion, as well as optimum measurement schemes. We use this bound to quantify the effectiveness of an actual experimental set-up for joint parameter estimation for polarimetry. We conclude by discussing the form of the trade-off relations for more general states and measurements.

No trade-off generally in joint optimal measurement of phase and strength of phase noise for N>>1

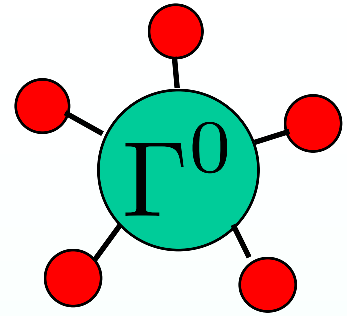


Optimal probe is Pegg-Summy-Berry-Wiseman (Sine), optimal measurement is canonical phase measurement.

Dephasing: Operator Approach

$$\frac{m}{2j} = x \in [-\tfrac{1}{2}, \tfrac{1}{2}]$$

$$\psi_m \mapsto \psi(x)$$



Probe state $$|\psi\rangle = \sum_{m=-j}^{+j} \psi_m |m\rangle$$

before dephasing $$|\psi\rangle\langle\psi| \mapsto \int_{x,y} \psi(x)\psi(y)|x\rangle\langle y|\, dx dy$$

after dephasing $$\rho(\Gamma^0, N) = \int_{x,y} \psi(x) \exp\{-N^2\Gamma^0(x-y)^2/2\} \psi(y)|x\rangle\langle y|\, dx dy$$

Free particle Feynman propagator in imaginary time!

$$\boxed{\int_{x,y} dx dy \exp\{-\mu_0(x-y)^2/2\}|x\rangle\langle y| \mapsto e^{-\hat{P}^2/(2\mu_0)}}$$

$$\rho = e^{-\hat{V}/2} e^{-\hat{T}} e^{-\hat{V}/2}, \qquad \hat{V} = \log \frac{1}{\psi^2(\hat{X})}, \quad \hat{T} = \hat{P}^2/2\mu_0$$

$$= \psi(\hat{X}) e^{-\frac{\hat{P}^2}{2\mu_0}} \psi(\hat{X})$$

Dephased density matrix is in terms of canonical X and P operators of the Heisenberg group

Mass parameter $$\boxed{\mu_0 = N^2\Gamma^0}$$

Operator Orderings

http://arxiv.org/pdf/math-ph/0506007v1.pdf (Suzuki)

$$\boxed{\rho = e^{-\hat{V}/2} e^{-\hat{T}} e^{-\hat{V}/2} = e^{-H}}$$

Symmetric 'Strang' splitting of Density Matrix

No even-order contributions - first non-trivial contribution at 3rd order

$$e^{xA} e^{xB} = e^{x(A+B)+O(x^2)}$$ Typical BCH

$$e^{\frac{x}{2}A} e^{xB} e^{\frac{x}{2}A} = e^{x(A+B)+O(x^3)}$$ Strang

$$\rho = e^{-H_0 - H_1 - \cdots}$$

Dephased Density Matrix: thermal state of Abstract Hamiltonian

$$H_0 = T + V,$$ Kinetic + Potential Energies

$$H_1 = \frac{1}{12}[T,[T,V]] + \frac{1}{24}[V,[T,V]] = -\frac{\{P,\{P,V''\}\}}{48\mu_0^2} + \frac{V'^2}{24\mu_0},$$

First non-trivial contribution

Asymptotic series in mass parameter

$$\mu_0 = N^2 \Gamma^0$$

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New Operator Form for QFI

$$F = \mathrm{Tr}\left(L^2\rho\right)$$

$$\rho = e^{-H}$$ **Write mixed state in exponential form, and evolve via parameter** θ

$$\frac{1}{2}\{e^{-H}, L\} = \frac{d\rho}{d\theta} = -i\left[S^z, e^{-H}\right]$$ **Sylvester and Schrodinger equations**

$$\frac{1}{2}\left(e^{-\frac{H}{2}}L\,e^{+\frac{H}{2}} + e^{\frac{H}{2}}L\,e^{-\frac{H}{2}}\right) = -i\left(e^{-\frac{H}{2}}S^z e^{+\frac{H}{2}} - e^{\frac{H}{2}}S^z e^{-\frac{H}{2}}\right)$$

Using identity $$\boxed{e^A B e^{-A} = \exp([A,\bullet])B \equiv B + [A,B] + \frac{1}{2!}[A,[A,B]] + \cdots}$$

$$\cosh\left(\tfrac{1}{2}[H,\bullet]\right)L = -2i\sinh\left(\tfrac{1}{2}[H,\bullet]\right)S^z$$

$$\longrightarrow \quad \boxed{L = -2i\tanh\left(\tfrac{1}{2}[H,\bullet]\right)S^z}$$

using algebraic properties of 'inner derivation' $[H,\bullet]$ **, *L* is a series of nested commutators**

Operator Method for Phase Diffusion

$$\rho = e^{-V/2} e^{-T} e^{-V/2} = e^{-H}$$

Combining Two Asymptotic Operator Series to 3rd Order: $\tanh x \approx x - x^3/3$

1. $F = \left\langle \left[S^z, 2\tanh\left(\tfrac{1}{2}[H, \bullet]\right) S^z \right] \right\rangle$ Fisher Information for Unitary Shift by S^z

2. $H = (T+V) + \frac{1}{12}[T,[T,V]] + \frac{1}{24}[V,[T,V]] + \ldots$ Suzuki-Trotter-Strang

After some very careful book-keeping…

$$V = \log 1/\psi^2(X) \quad , \quad T = P^2/2\mu_0 \quad , \quad [X,P] = i \quad , \quad [P, f(X)] = -if'(X) \,.$$

$$\boxed{\frac{F}{N^2} = \frac{1}{\mu_0} - \frac{1}{\mu_0^2} \int \left(\frac{d\psi}{dx} \right)^2 dx + O\left(\frac{1}{\mu_0^3} \right)}$$

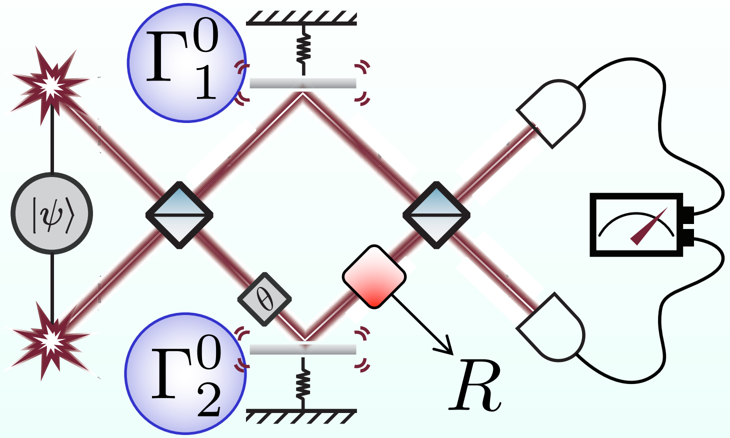
$\mu_0 = N^2 \Gamma^0$

Mass parameter

….Semi-Classical asymptotic expansion in mass parameter, e.g. $\frac{1}{\mu_0} \sim \hbar$

Idea to combine phase noise Γ^0+ loss R

$$\Gamma^0 = \Gamma^0_1 + \Gamma^0_2$$



$$\boxed{\frac{1}{F} \approx \Gamma^0 + \frac{1}{N^2}\int \psi'^2(x)\,dx,}$$

Dephasing result respects total particle number, e.g. n-k, if k photons lost.

$$\psi_n \mapsto \psi_n \sqrt{\frac{p(n,k)}{w_k}} = \tilde{\psi}_{n,k}\ ,\ w_k = \sum_{n=k}^{N} p(n,k)\psi_n^2$$

Probability to lose k photons from n in lower mode

$$p(n,k) = \binom{n}{k} R^k (1-R)^{n-k} \sim \sqrt{\frac{1}{2\pi k(1-R)}} \exp\left\{-\frac{R^2}{2k(1-R)}\left(n - \frac{k}{R}\right)^2\right\}$$

Gaussian Approximation

$$\left(\tilde{\psi}'_{n,k}\right)^2 = \frac{p(n,k)}{w_k}\left(\psi'_n + \frac{\psi_n}{2}\frac{d}{dn}[\log p(n,k)]\right)^2$$

Replace in dephasing expression for 1/F and integrate over k lost photons

$$\boxed{\frac{1}{F} \approx \Gamma^0 + \frac{1}{N^2}\int_x (dx)\left(\psi'^2(x) + \frac{r}{4(1/2+x)}\psi^2(x)\right)}$$

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Quantum Fisher Information as Classical Action

$$\boxed{\frac{F}{N^2} = \frac{1}{\mu_0} - \frac{1}{\mu_0^2}\int\left(\frac{d\psi}{dx}\right)^2 dx + O\left(\frac{1}{\mu_0^3}\right)}$$

Dephasing case just worked out

$$\mu_0 \mapsto \mu(x)$$

$$\boxed{\frac{F}{N^2} \approx \int\left[\frac{\psi^2}{\mu(x)} - \frac{\psi'^2}{\mu^2(x)}\right]dx,}$$

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