Quantum imaging and metrology with incoherent light

Mark Pearce, Jasminder Sidhu and Pieter Kok

The University Of Sheffield.

EPSRC

[dstl]

QUANTUM COMMUNICATIONS HUB

What metrology improvements can we achieve with incoherent light and photodetection?

- Quantum metrology is typically presented in terms of highly coherent quantum states, such as NOON states.
- Such states are susceptible to noise, and typically very difficult to make.
- Can we get metrological (quantum) improvements with incoherent light?

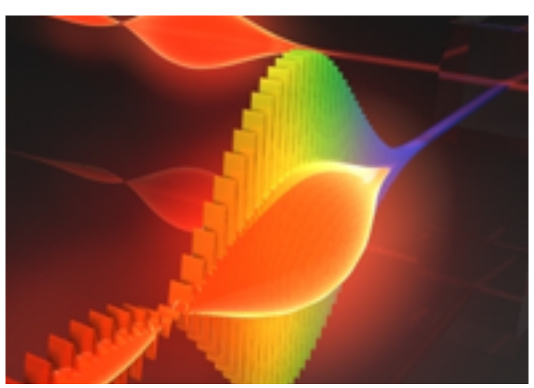
Considerations for incoherent sources

- Thermal sources and single-photon sources.
- Mode shapes and coherence properties become crucial.

We present a few examples of imaging and metrology with incoherent light:

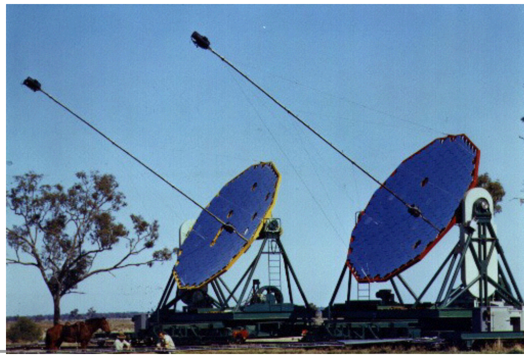
Resolution beyond the Abbe limit from intensity correlations;

optical thermometry via intensity measurements;

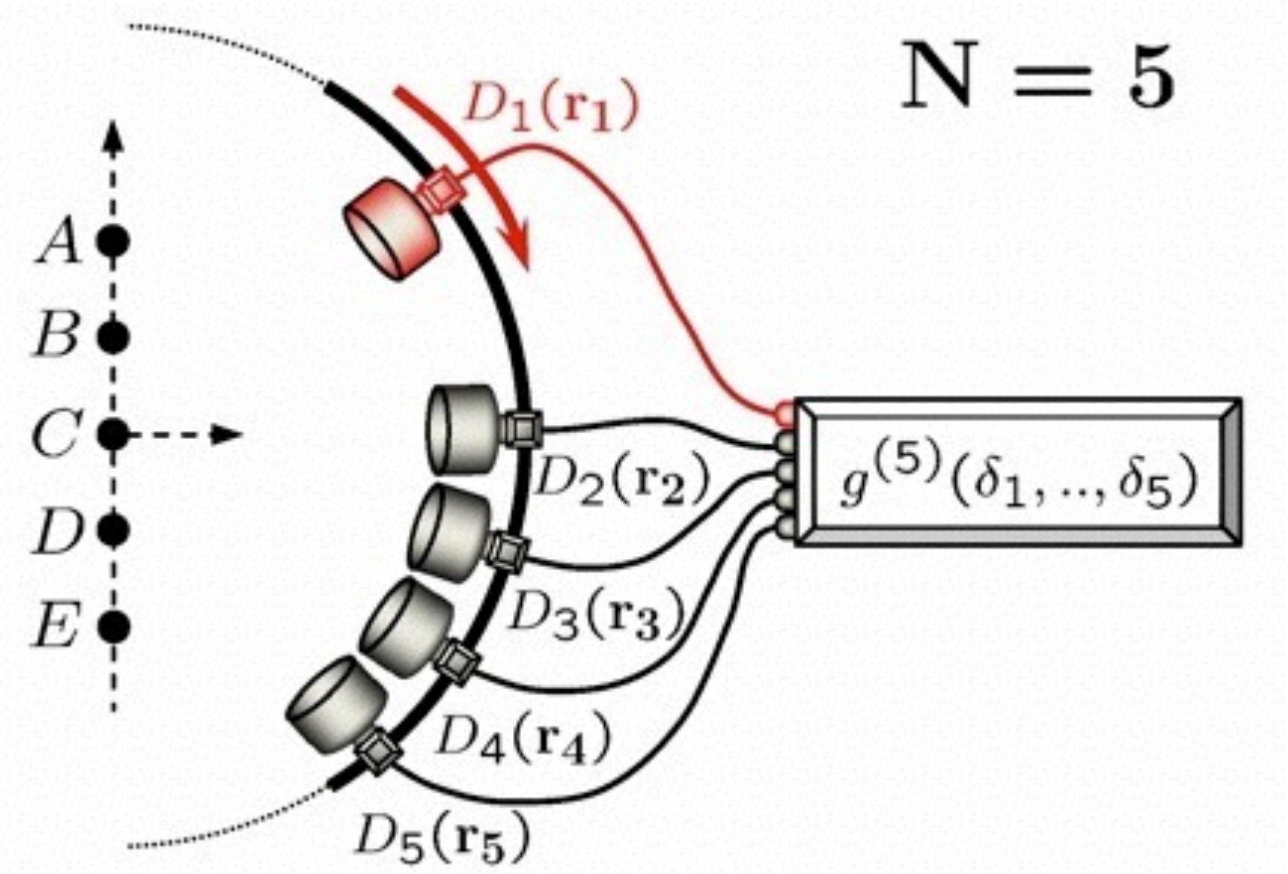


single-photon metrology probes.

Photon correlations in multiple detectors increase the resolution beyond Abbe limit.



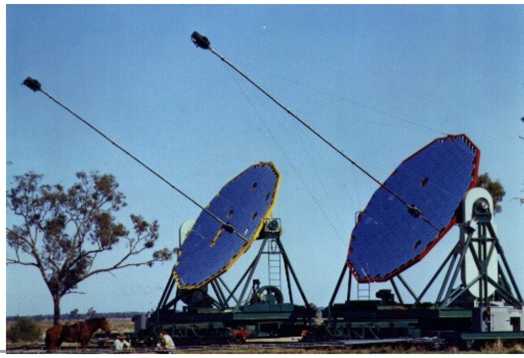
Measure light intensity in *N* detectors placed at very particular positions, the so-called *magic angles*.

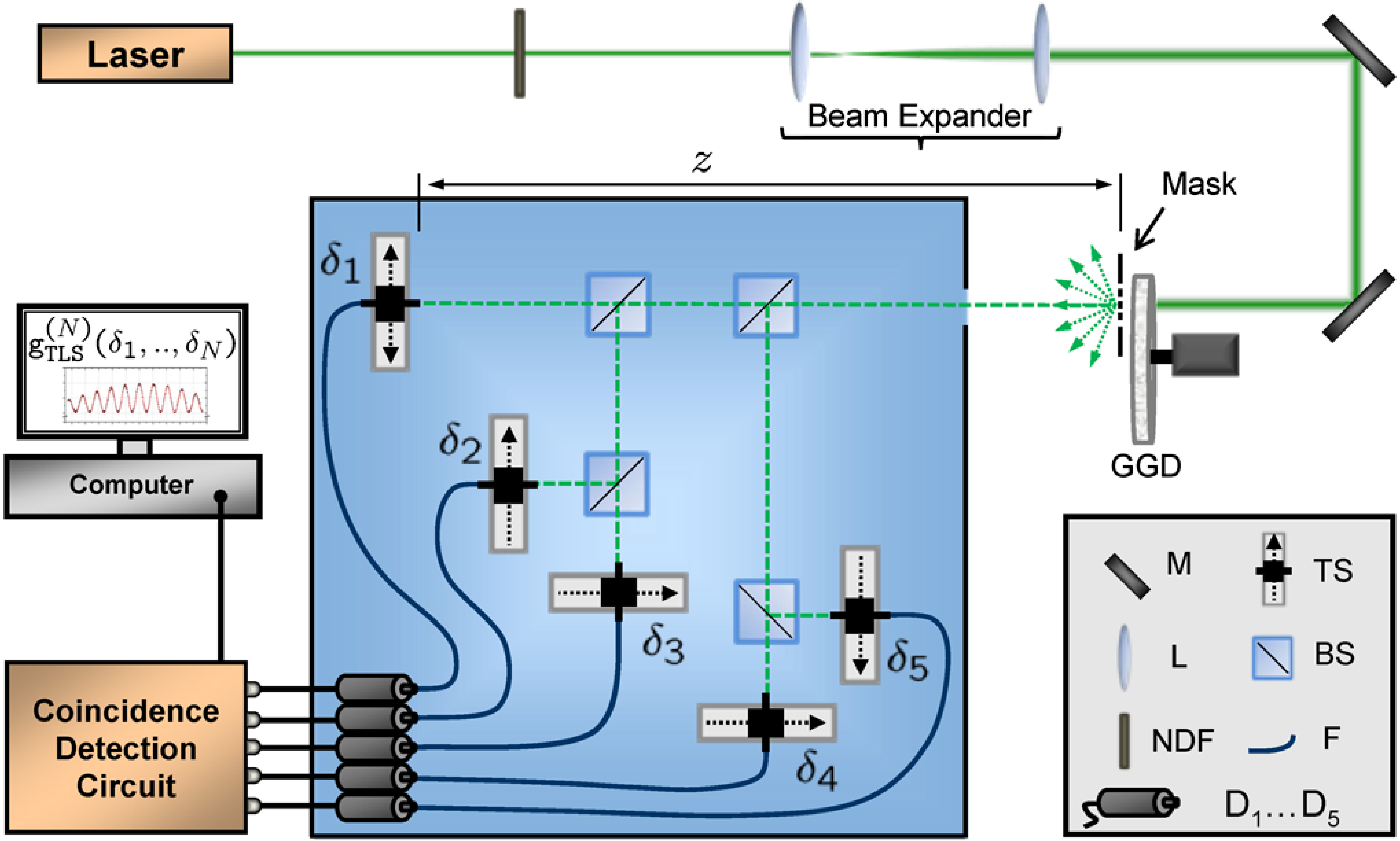


Coincidence counts change as we sweep detector 1 through a range of angles.

Oppel et al., *Phys. Rev. Lett.* **109**, 233609 (2012).

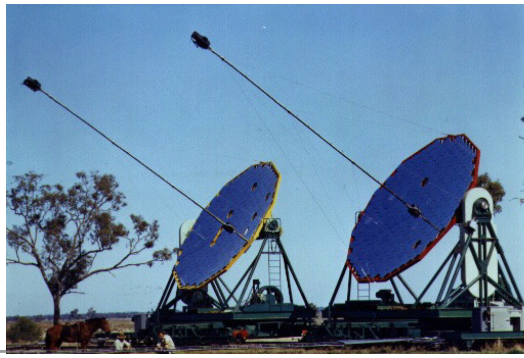
The experimental setup:

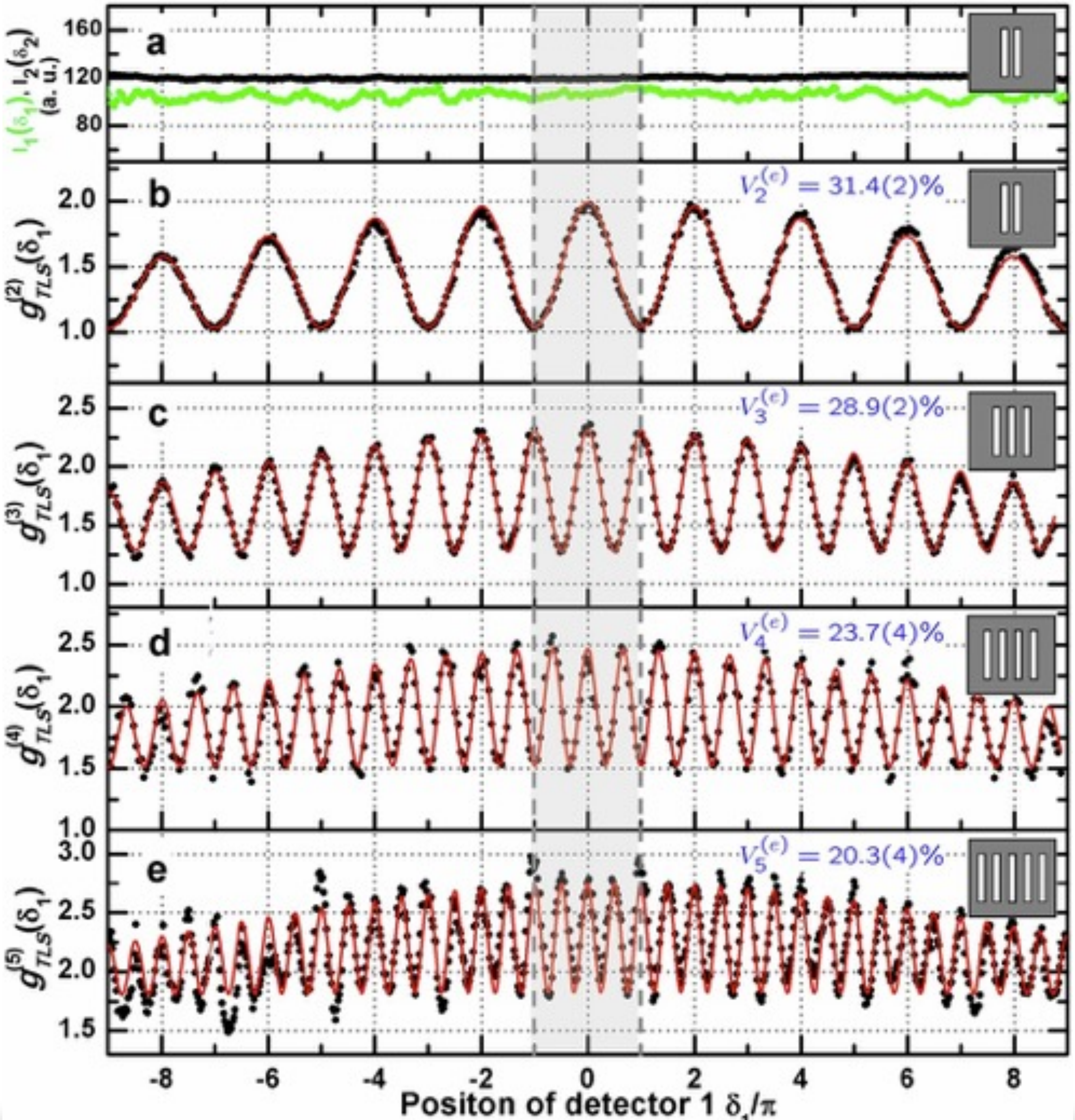


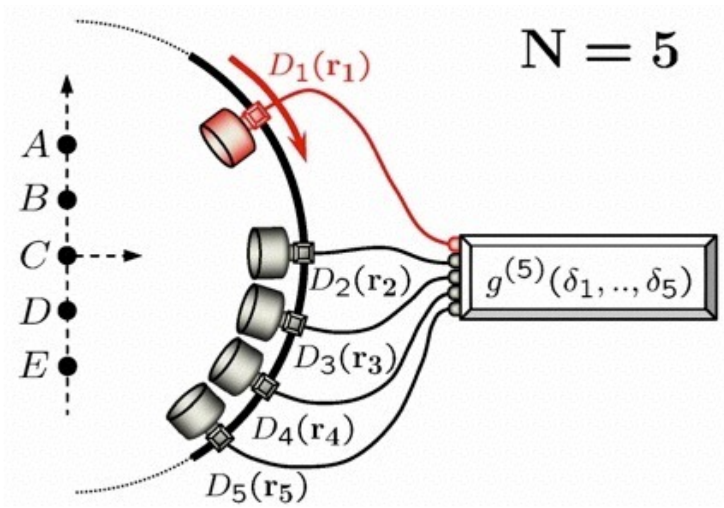


Oppel et al., *Phys. Rev. Lett.* **109**, 233609 (2012).

Photon correlations in multiple detectors increase the resolution beyond Abbe limit.



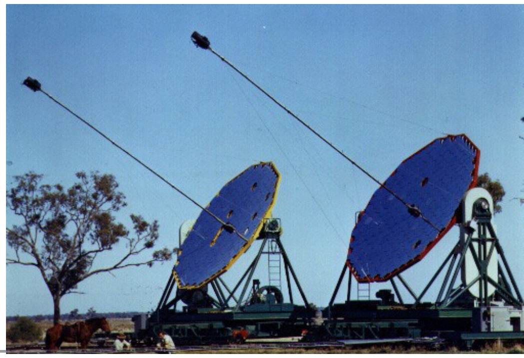




The modulation scales with *N* – 1, where *N* is the number of detectors. This allows us to measure the distance *d* between the sources with increased precision.

Oppel et al., *Phys. Rev. Lett.* **109**, 233609 (2012).

We can beat the Abbe limit using higher-order photon correlations.



- Classically, the resolution limit is given by

$$d_{\min} = \lambda/(2\mathcal{A})$$

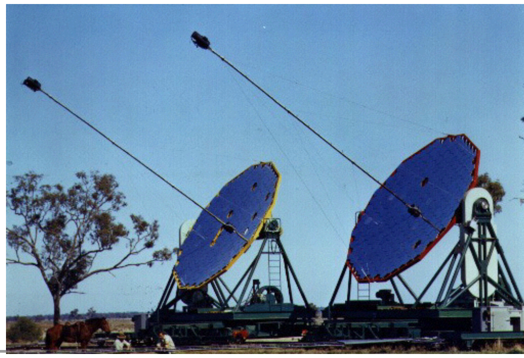
- With *M* the number of fringes, we can calculate the resolution limit as

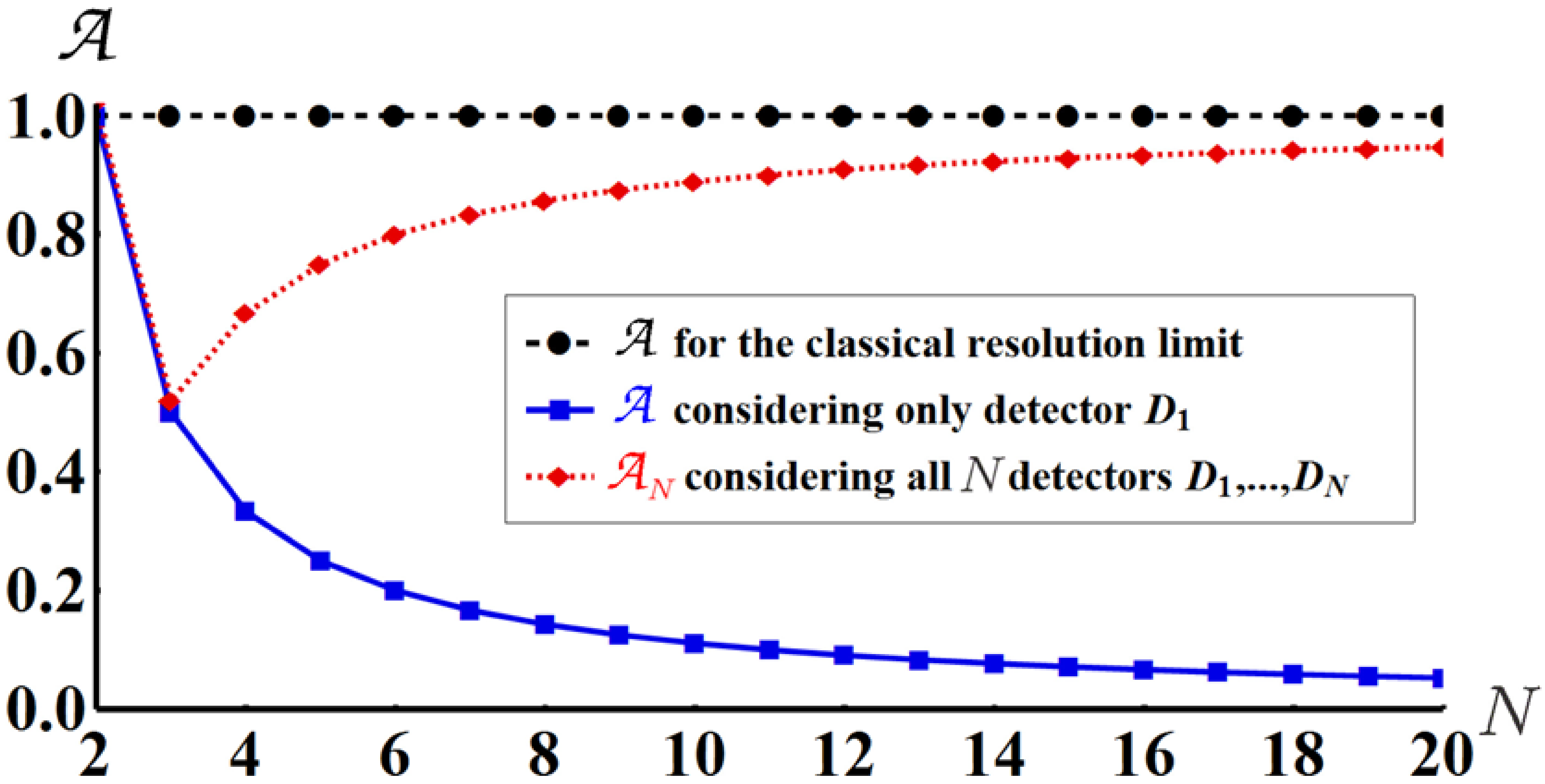
$$d = \frac{M\lambda}{2\mathcal{A}(N-1)},$$

$$\Delta d = \Delta M \left| \frac{\partial M}{\partial d} \right|^{-1} < \frac{\lambda}{4\mathcal{A}(N-1)}.$$

Oppel et al., *Phys. Rev. Lett.* **109**, 233609 (2012).

The aperture is determined by the position of the detectors.

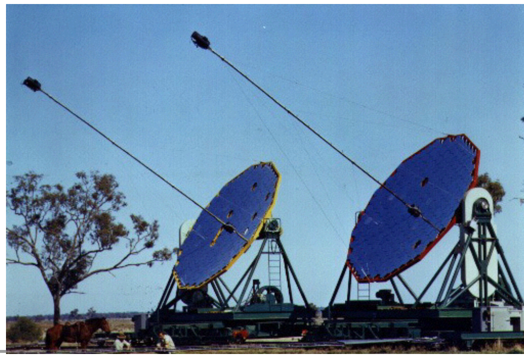




Oppel et al., *Phys. Rev. Lett.* **109**, 233609 (2012).

Can we use these higher order correlations for increased precision in imaging?

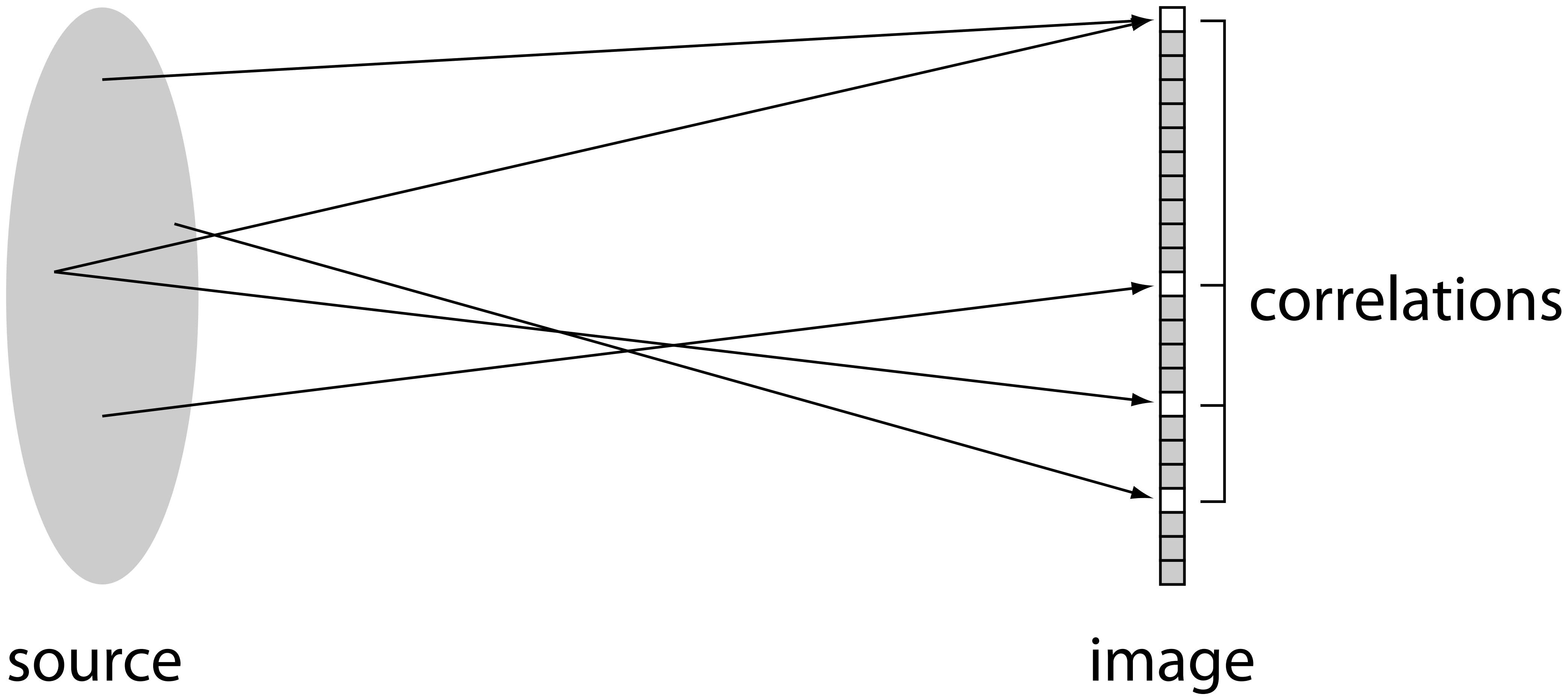
We image the size of a disk in the far field using intensity correlation measurements.



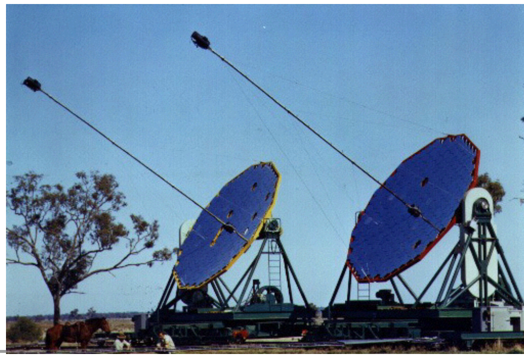
Uniform circular source with radius *a;*

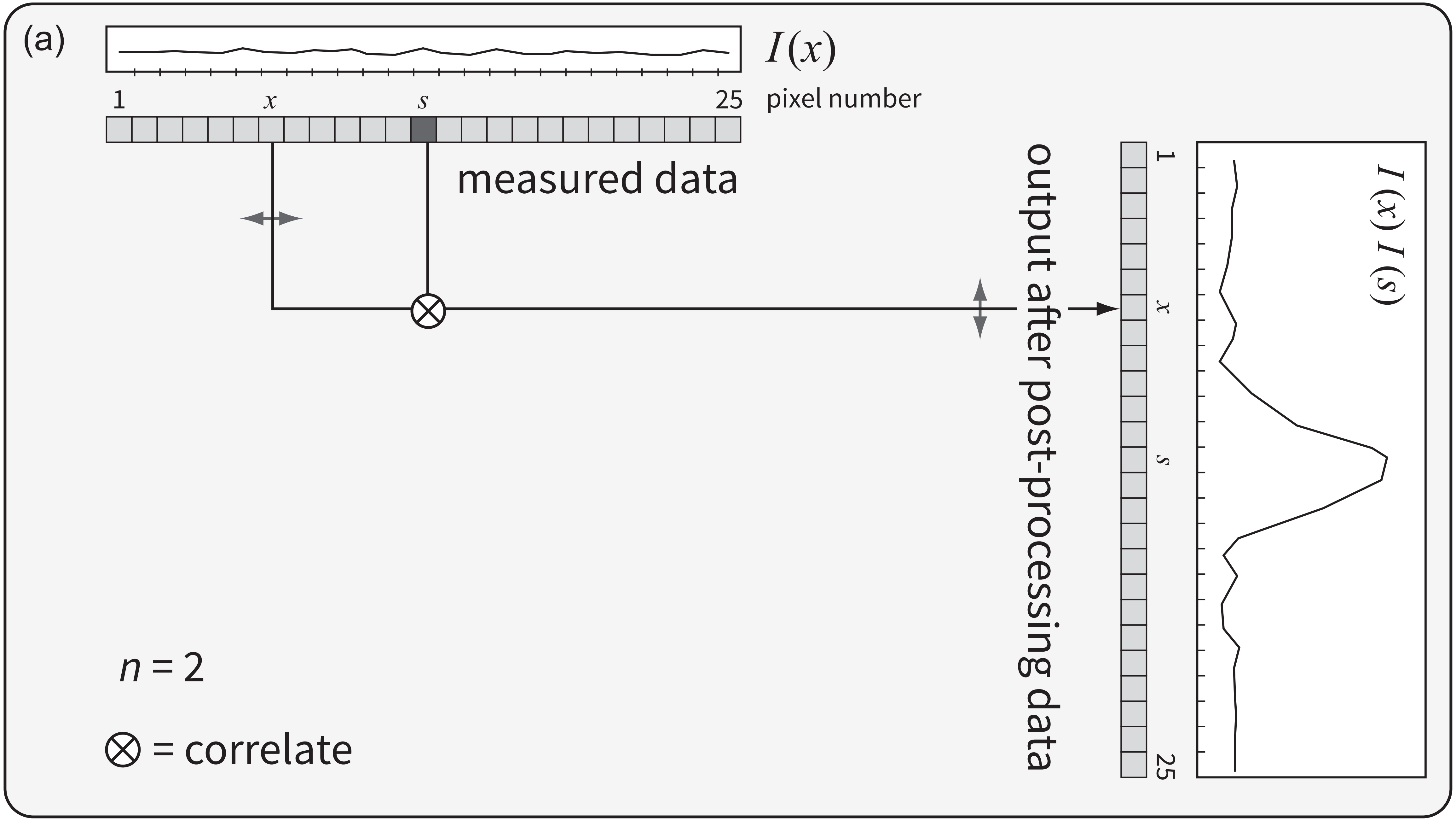
pseudo-thermal, monochromatic light with wave number *k*;

a distance *d* from the imaging plane.



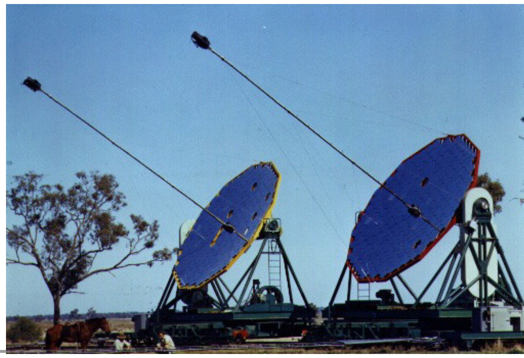
There are a number of methods for combining data of multiple pixels.

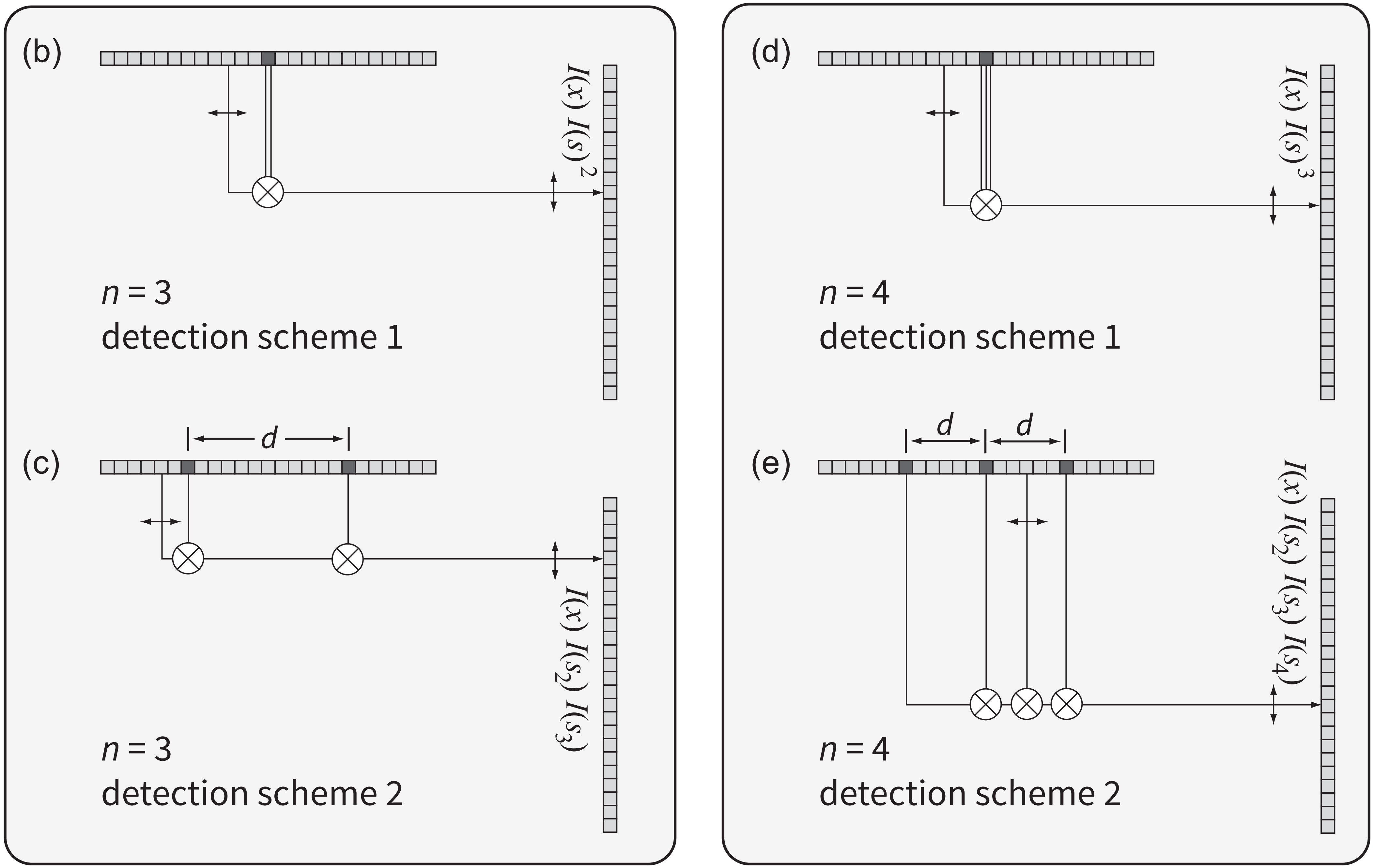




Pearce et al., *Phys Rev A* **92**, 043831 (2015).

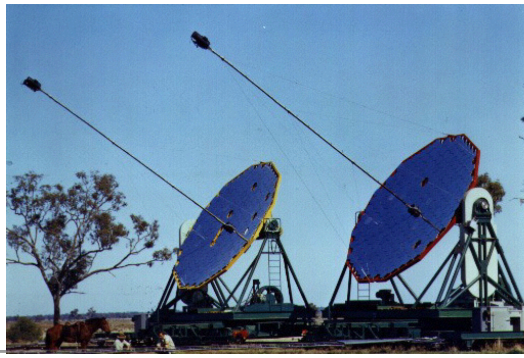
There are a number of methods for combining data of multiple pixels.





Pearce et al., *Phys Rev A* **92**, 043831 (2015).

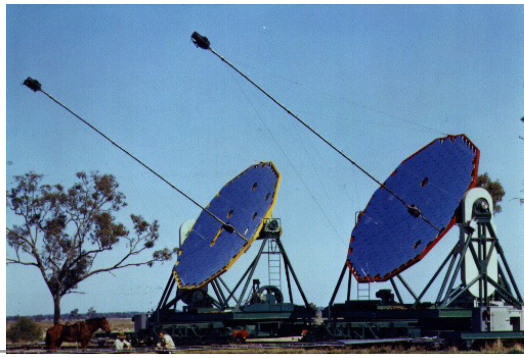
Image Data Processing



Calculate:

- n-point correlations functions $g^{(n)}$ for the disk;
- covariance matrix Cov[$g^{(n)}(x)$,$g^{(n)}(x')$] from $g^{(n)}$;
- Fisher information $I_n(a)$ from Cov[$g^{(n)}(x)$,$g^{(n)}(x')$];
- precision in the radius a from $I_n(a)$.

The *n*-point correlation functions are easily calculated.



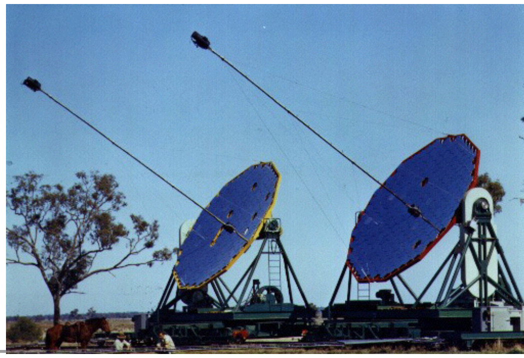
The n-point correlation function of a disk emitting pseudo-thermal (monochromatic) light is:

$$g^{(n)}(x) = (n-1)! + (n-1)(n-1)! \left(\frac{2J_1\left(\frac{ak|x|}{d}\right)}{\left(\frac{ak|x|}{d}\right)} \right)^2$$

The visibility is given by:

$$\mathcal{V} = \frac{g^{(n)}_{\max} - g^{(n)}_{\min}}{g^{(n)}_{\max} + g^{(n)}_{\min}} = \frac{n-1}{n+1}.$$

We must calculate the covariance matrix and the Fisher information



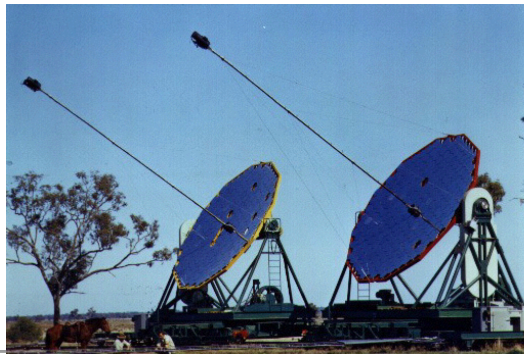
The covariance matrix between the n-point correlation functions at different positions is:

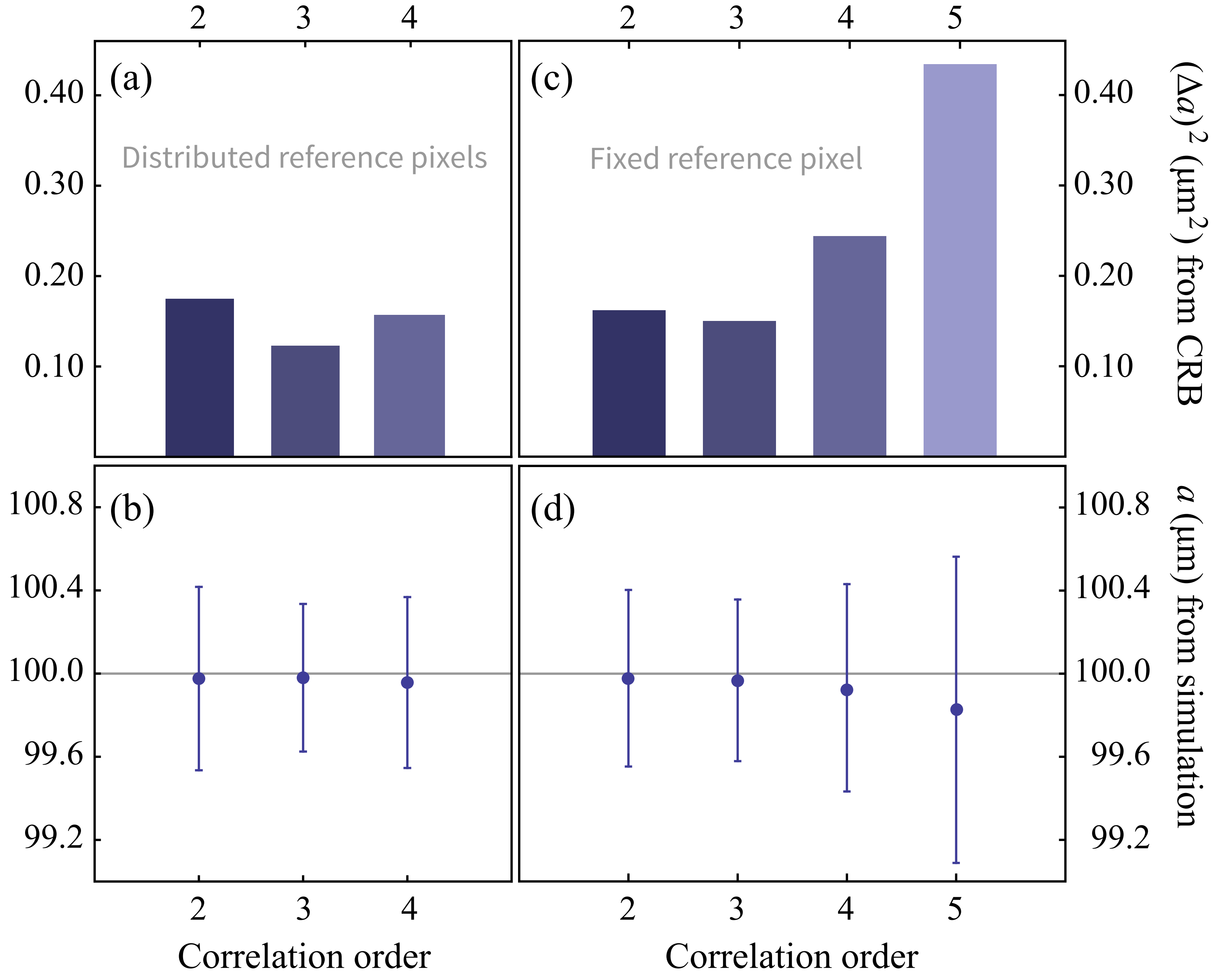
$$\mathbf{C}_{ij} = \frac{1}{N^2}\sum_k [g^{(2n)}(x_i, x_j) - g^{(n)}(x_i)g^{(n)}(x_j)]$$
$$= \frac{1}{N}[g^{(2n)}(x_i, x_j) - g^{(n)}(x_i)g^{(n)}(x_j)].$$

This is used to calculate the Fisher information:

$$I(a) = \left(\frac{\partial \boldsymbol{\mu}}{\partial a}\right)^T \mathbf{C}^{-1}\left(\frac{\partial \boldsymbol{\mu}}{\partial a}\right) + \frac{1}{2}\mathrm{tr}\left(\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial a}\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial a}\right)$$

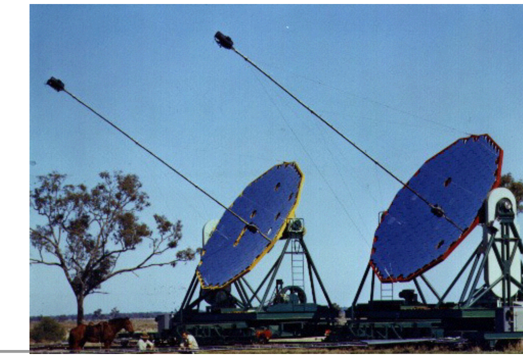
Resolution of the disk radius



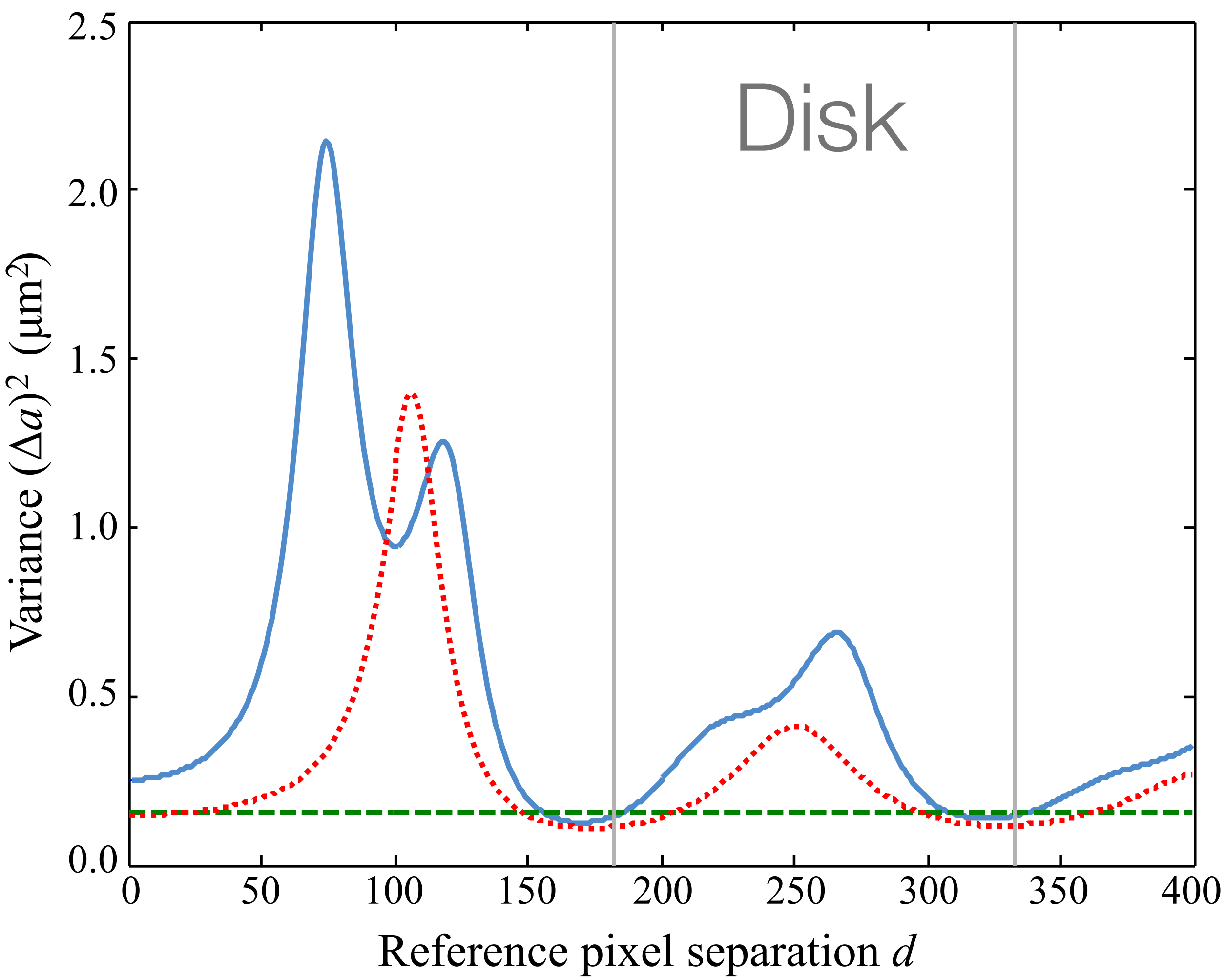


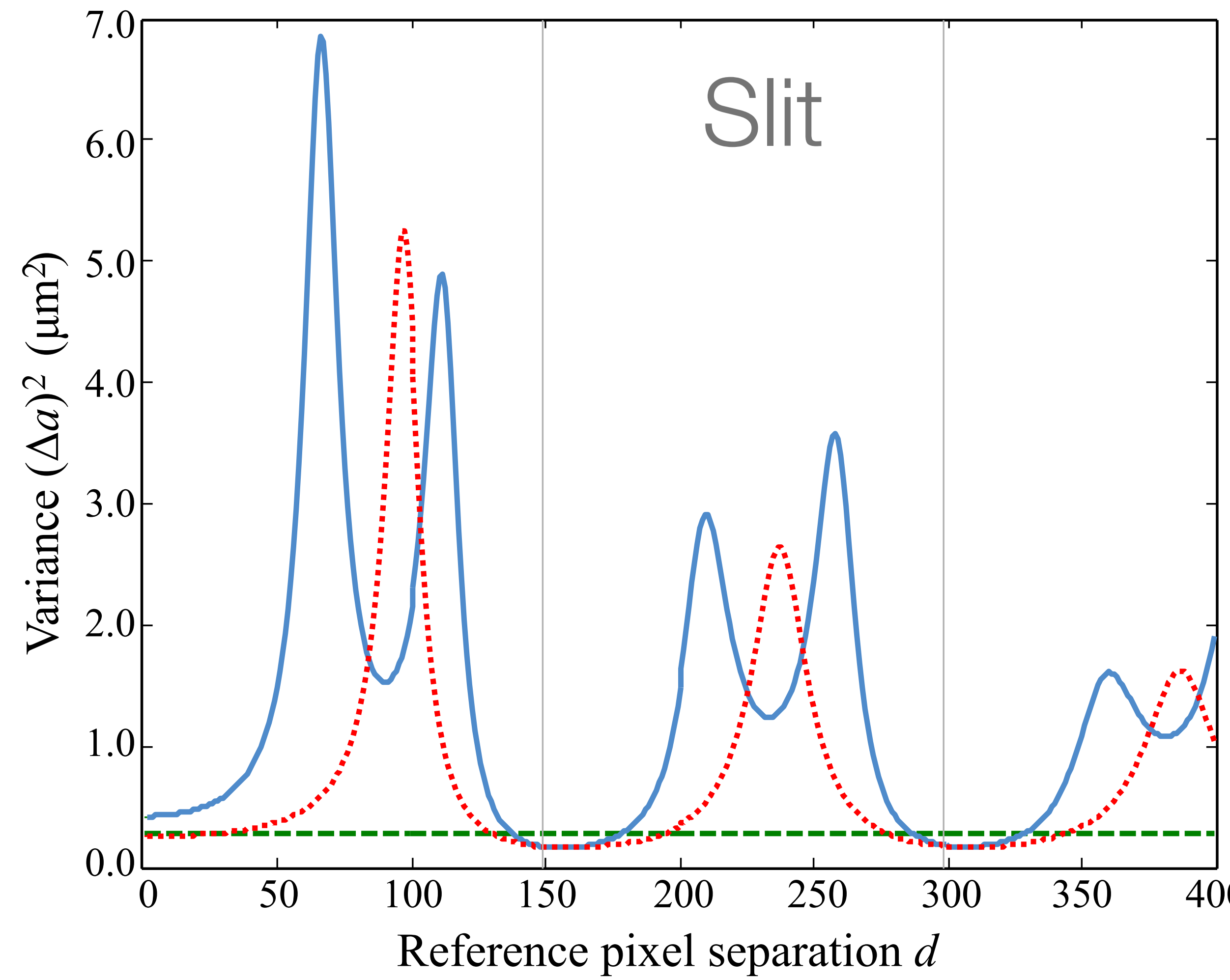
Pearce et al., *Phys Rev A* **92**, 043831 (2015).

Distributed reference pixels give better results than fixed reference pixels.



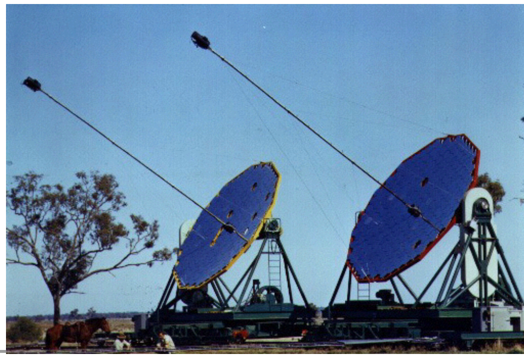
Pixel separation has a large effect on the variance:





Pearce et al., *Phys Rev A* **92**, 043831 (2015).

Remarks



- Higher-order correlations increase the imaging resolution, *even for classical light*.
- However, higher orders become increasingly computationally intensive.
- The ultimately achieved precision is quite sensitive to the type of imaging considered, and to correlations in the data (such as optical coherences).
- What is the *quantum* Fisher information?

Quantum description of black body sources.

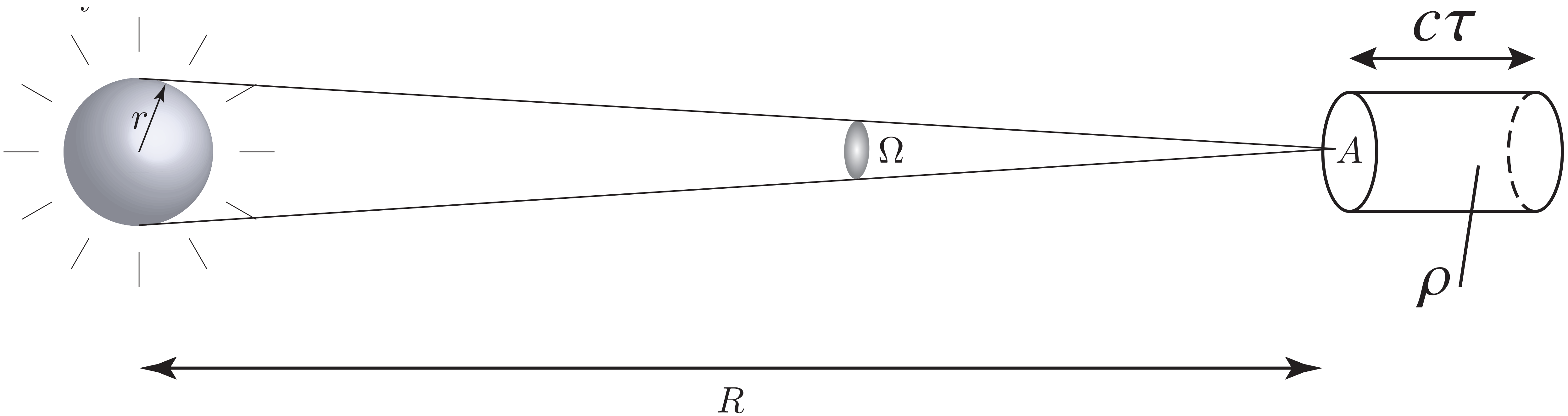
- Black body sources are described by the density matrix

$$\rho = \otimes_{\mathbf{k}} \rho_{\mathbf{k}} \qquad \rho_{\mathbf{k}} = \sum_{m_{\mathbf{k}}=0}^{\infty} \frac{\langle \hat{n}_{\mathbf{k}} \rangle^{m_{\mathbf{k}}}}{(1 + \langle \hat{n}_{\mathbf{k}} \rangle)^{m_{\mathbf{k}}+1}} |m_{\mathbf{k}}\rangle\langle m_{\mathbf{k}}|$$

- The expectation values $\langle \hat{n}_{\mathbf{k}} \rangle$ are completely determined by the temperature of the source.

- Our aim is to find the measurement that achieves the quantum Cramer-Rao bound for temperature measurements.

The state observed in the far field depends on the detection volume.



- Suppose we observe the state $\boldsymbol{\rho}$ in the far field of the radiating object.

The state observed in the far field depends on the detection volume.

- We can divide this spectrum up into independent spectral modes of spectral width $1/\tau$.
- The transverse coherence area of each spectral mode is proportional to $\Omega\nu$.
- Below a certain frequency, every spectral mode has a coherence area larger than A.

The state observed in the far field depends on the detection volume.

- We define mode operators $\hat{a}_i, \ \hat{a}_i^\dagger$ for each frequency mode.

- Each mode is Gaussian so the state is completely characterised by the first and second moments,

$$\boldsymbol{a} = (\hat{a}_1, \hat{a}_1^\dagger, \ldots, \hat{a}_n, \hat{a}_n^\dagger)^{\mathrm{T}}$$

$$\boldsymbol{\mu} = \langle \boldsymbol{a} \rangle \qquad \boldsymbol{\lambda} = \boldsymbol{a} - \boldsymbol{\mu}$$

$$\Sigma_{ij} = \frac{1}{2}\left[\langle a_i a_j \rangle + \langle a_j a_i \rangle\right]$$

Moments of the black body radiation field.

- For thermal modes the first moments are all 0:

$$\boldsymbol{\mu} = 0$$

- Due to the independence of each spectral mode, the covariance matrix has the form

$$\Sigma = \oplus_\nu \begin{pmatrix} 0 & \langle \hat{n}_\nu \rangle + \frac{1}{2} \\ \langle \hat{n}_\nu \rangle + \frac{1}{2} & 0 \end{pmatrix}$$

- The average number of photons in the frequency mode centred on ν depends on the fraction of that mode volume inside the state ρ .

Average photons per mode

- The average number of photons in spectral mode ν is given by

$$\langle \hat{n}_\nu \rangle = \frac{2\Omega_S A \nu^2}{c^2} \left(\frac{1}{\mathrm{e}^{\beta h \nu} - 1} \right) \qquad \Omega_S = \frac{A_S}{R^2}$$

- If Ω_S is also unknown, we must attempt to estimate this simultaneously.

- If we are uninterested in Ω_S, we must treat it as a nuisance parameter.

Gaussian quantum optimum estimators

- the QFI for a Gaussian state is given by [1]

$$\left[\mathbf{I}_Q\right]_{ij} = \frac{1}{2}\mathfrak{M}^{-1}_{\alpha\beta,\gamma\kappa}\partial_i\Sigma^{\alpha\beta}\partial_j\Sigma^{\gamma\kappa} + \Sigma^{-1}_{\alpha\beta}\partial_i\mu^{\alpha}\partial_j\mu^{\beta}$$

$$\mathfrak{M} = \Sigma\otimes\Sigma + \frac{1}{4}\Omega\otimes\Omega \qquad \Omega = \bigoplus_{j=1}^{n} i\sigma_y$$

- The SLD for parameter θ_i is given by

$$\mathcal{L}_i = \frac{1}{2}\mathfrak{M}^{-1}_{\gamma\kappa,\alpha\beta}\partial_i\Sigma^{\alpha\beta}(\lambda^{\gamma}\lambda^{\kappa} - \Sigma^{\gamma\kappa}) + \Sigma^{-1}_{\gamma\kappa}\partial_i\mu^{\kappa}\lambda^{\gamma}$$

[1] Y. Gao and H. Lee, The European Physical Journal D **68**, 347 (2014)

Quantum estimation for black bodies

- The SLD and QFI for single mode black bodies are therefore given by

$$\mathcal{L}_i = \sum_{l=1}^{m} \frac{(\partial_i \langle n_{\nu_l} \rangle)(\langle n_{\nu_l} \rangle - \hat{n}_{\nu_l})}{\langle n_{\nu_l} \rangle + \langle n_{\nu_l} \rangle^2}$$

$$\mathbf{I}_Q^{(\nu)} = \sum_{l=1}^{m} \frac{1}{\langle n_{\nu_l} \rangle + \langle n_{\nu_l} \rangle^2} \nabla_{\boldsymbol{\theta}} \langle n_{\nu_l} \rangle (\nabla_{\boldsymbol{\theta}} \langle n_{\nu_l} \rangle)^{\mathrm{T}}$$

- The summation is due to the independence of modes.

Estimation from a single spectral mode

- For a single spectral mode, the QFI matrix for parameters $\boldsymbol{\theta} = (\Omega_S, T)^{\mathrm{T}}$ is non-invertible

$$\mathbf{I}_Q^{(\nu)} = \frac{1}{\langle n_\nu \rangle + \langle n_\nu \rangle^2} \nabla_{\boldsymbol{\theta}} \langle n_\nu \rangle (\nabla_{\boldsymbol{\theta}} \langle n_\nu \rangle)^{\mathrm{T}}$$

- We cannot distinguish between large cold objects and small hot objects

Multi-parameter estimation

- To estimate $\boldsymbol{\theta} = (\Omega_S, T)^{\mathrm{T}}$ we must use at least two spectral modes.

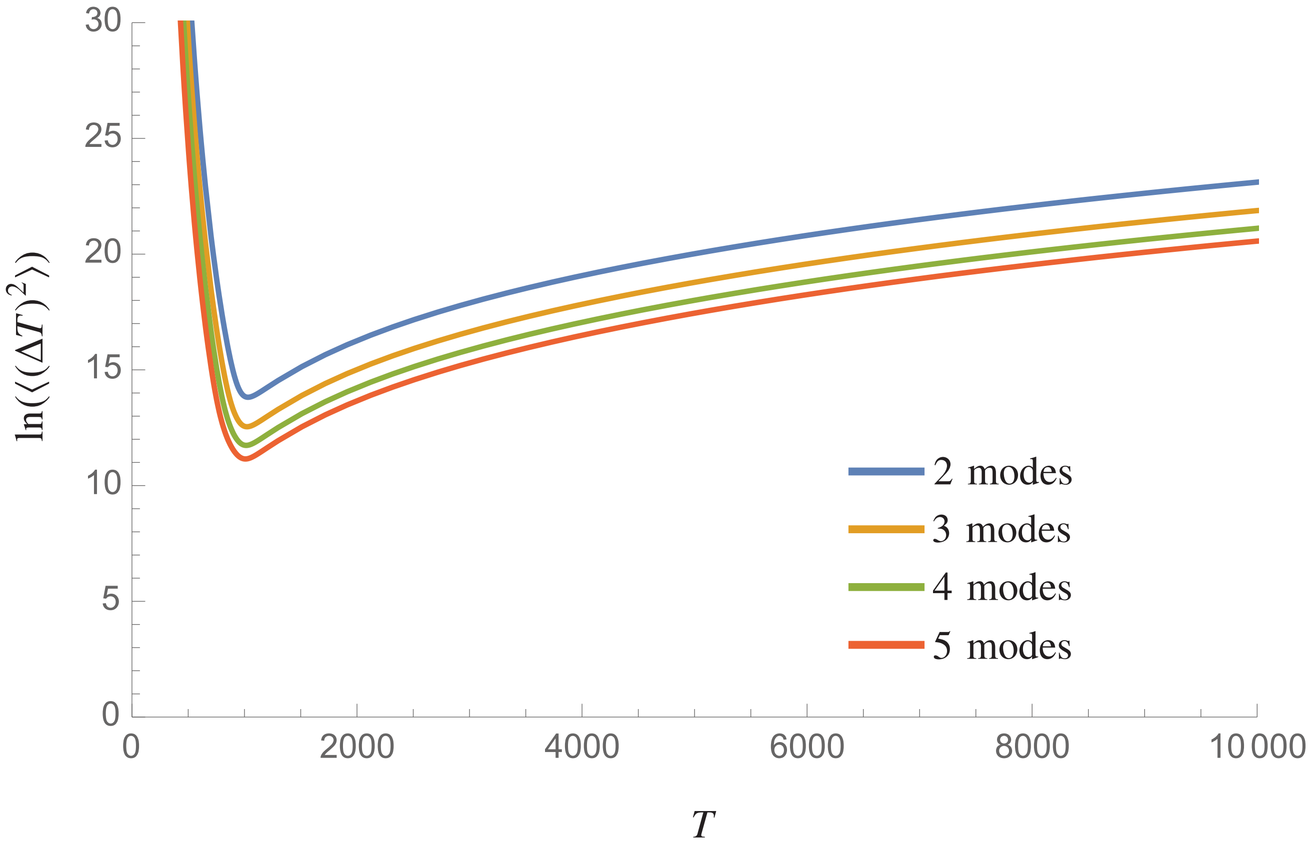
- The extension to multiple parameters generally increases the variance of a parameter estimation problem.

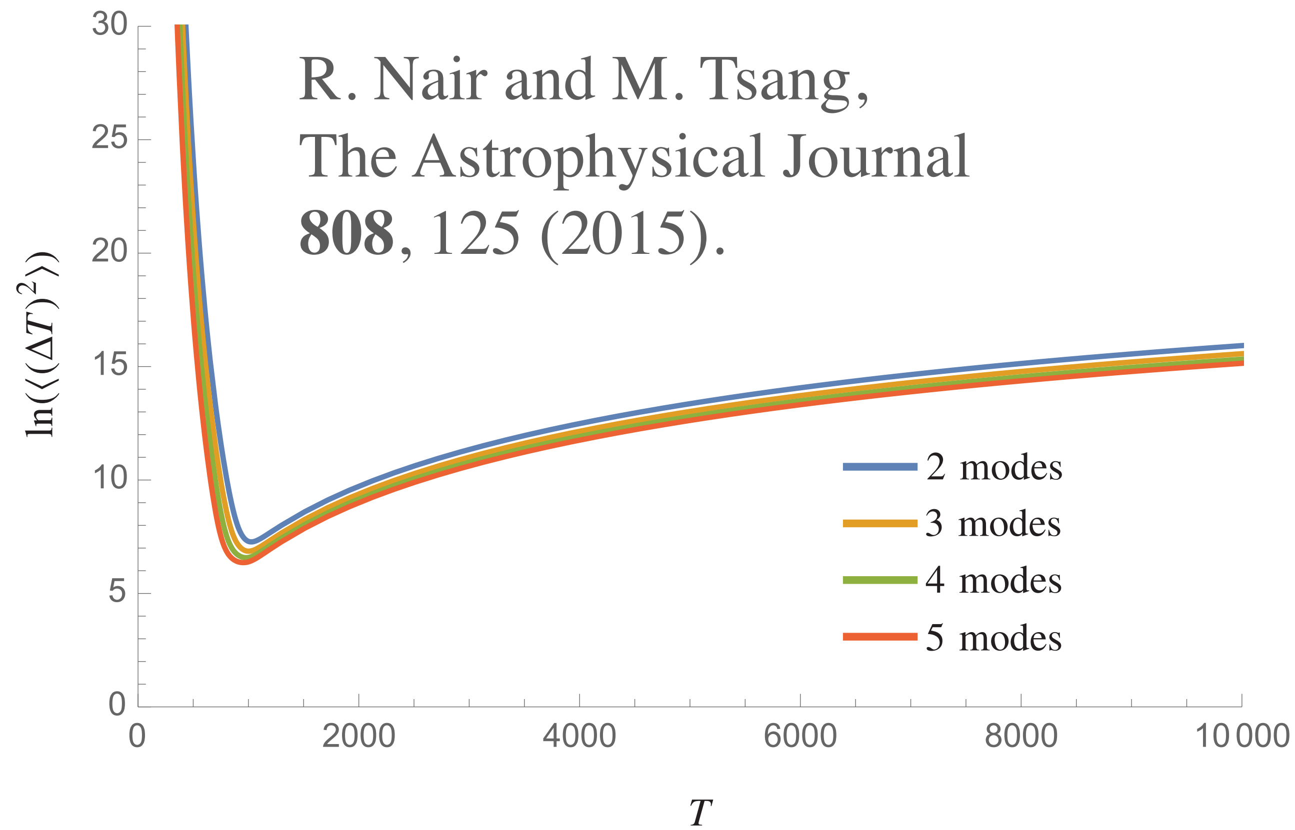
- To make a fair comparison between the two strategies we must also assume that the single parameter problem utilises two spectral modes

$$\langle(\Delta\theta_i)^2\rangle \geq \frac{1}{\sum_{l=1}^{m} \mathrm{I}_Q^{(\nu_l)}} \qquad \langle(\Delta\theta_i)^2\rangle \geq \left[\sum_{l=1}^{m} \mathbf{I}_Q^{(\nu_l)}\right]^{-1}$$

Single parameter vs. multi-parameter

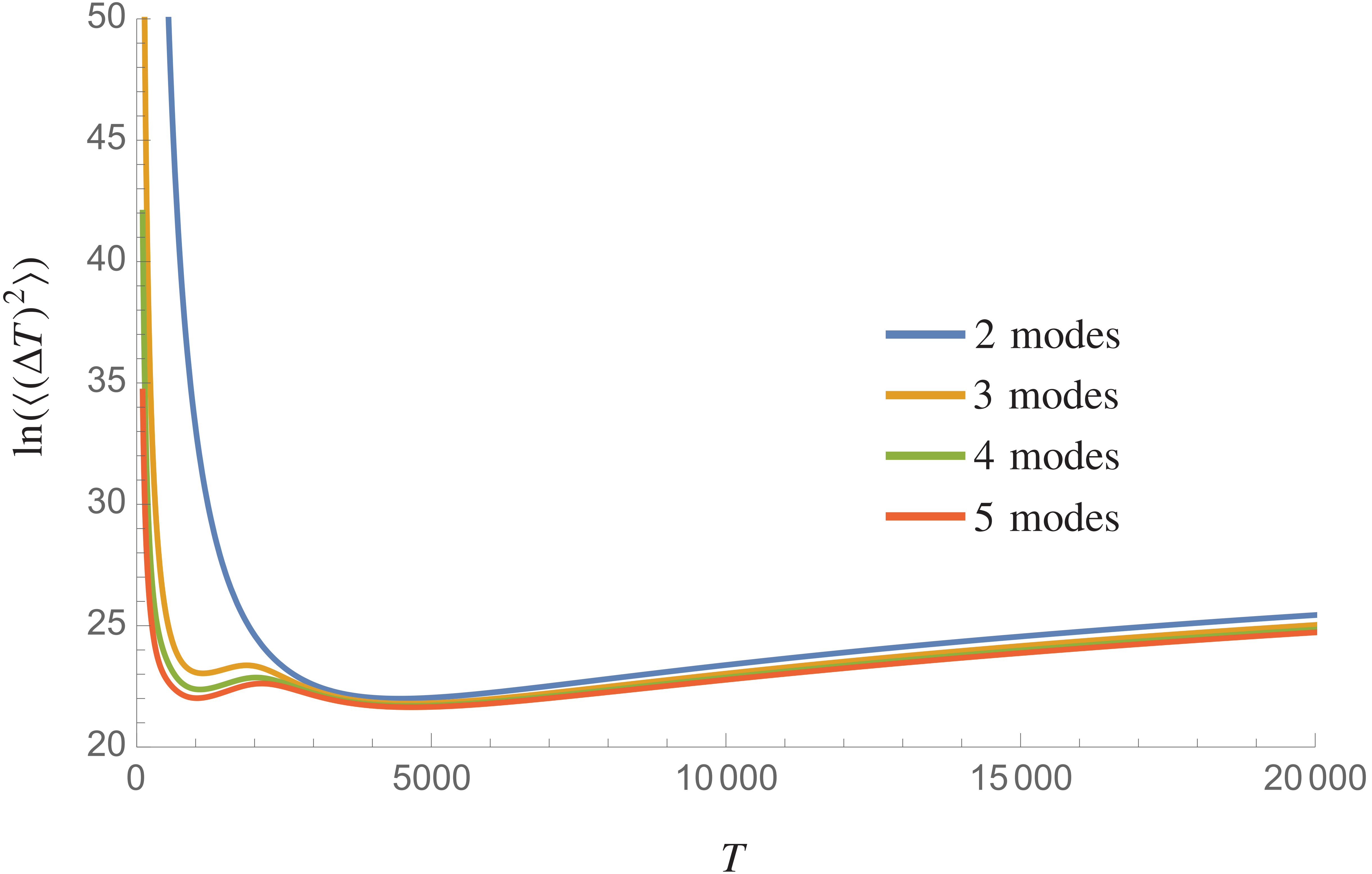
- We plot the single and multi-parameter variance of T for 2, 3, 4, and 5 spectral modes.





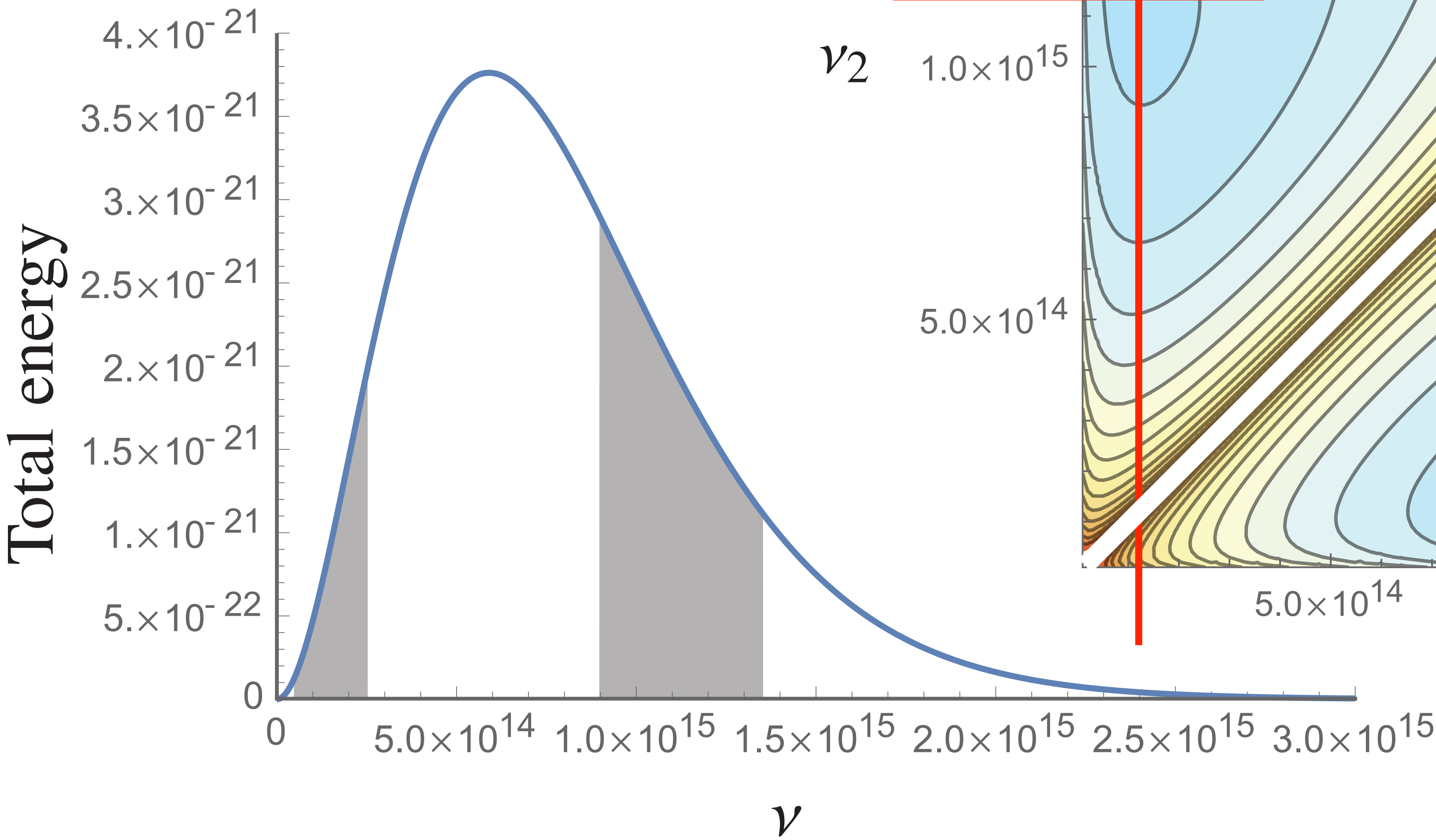
- The optimal measurement is still photon counting.

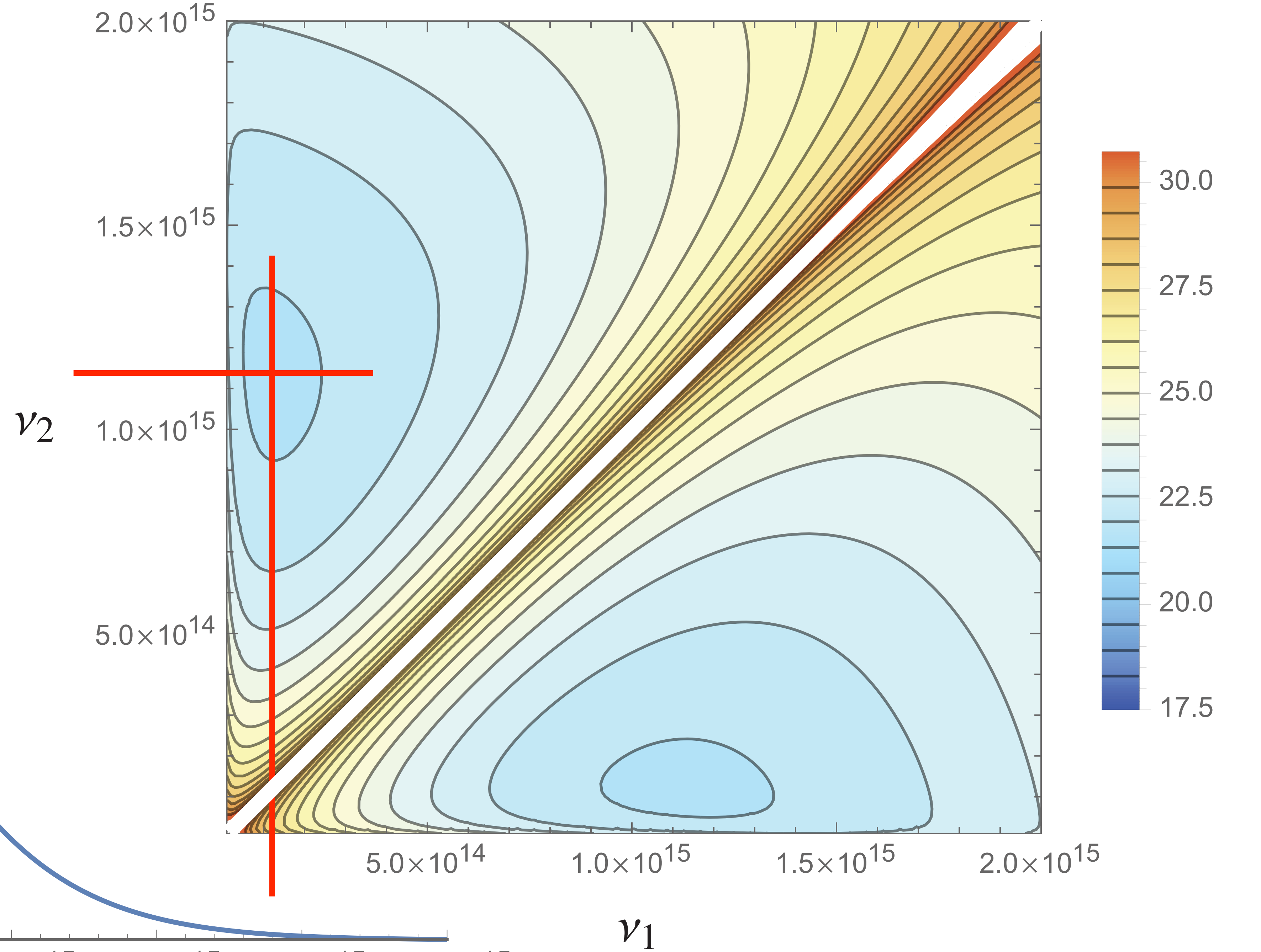
The temperature precision is a complicated function of the measured frequencies.



We must choose the frequency modes that optimise the precision.

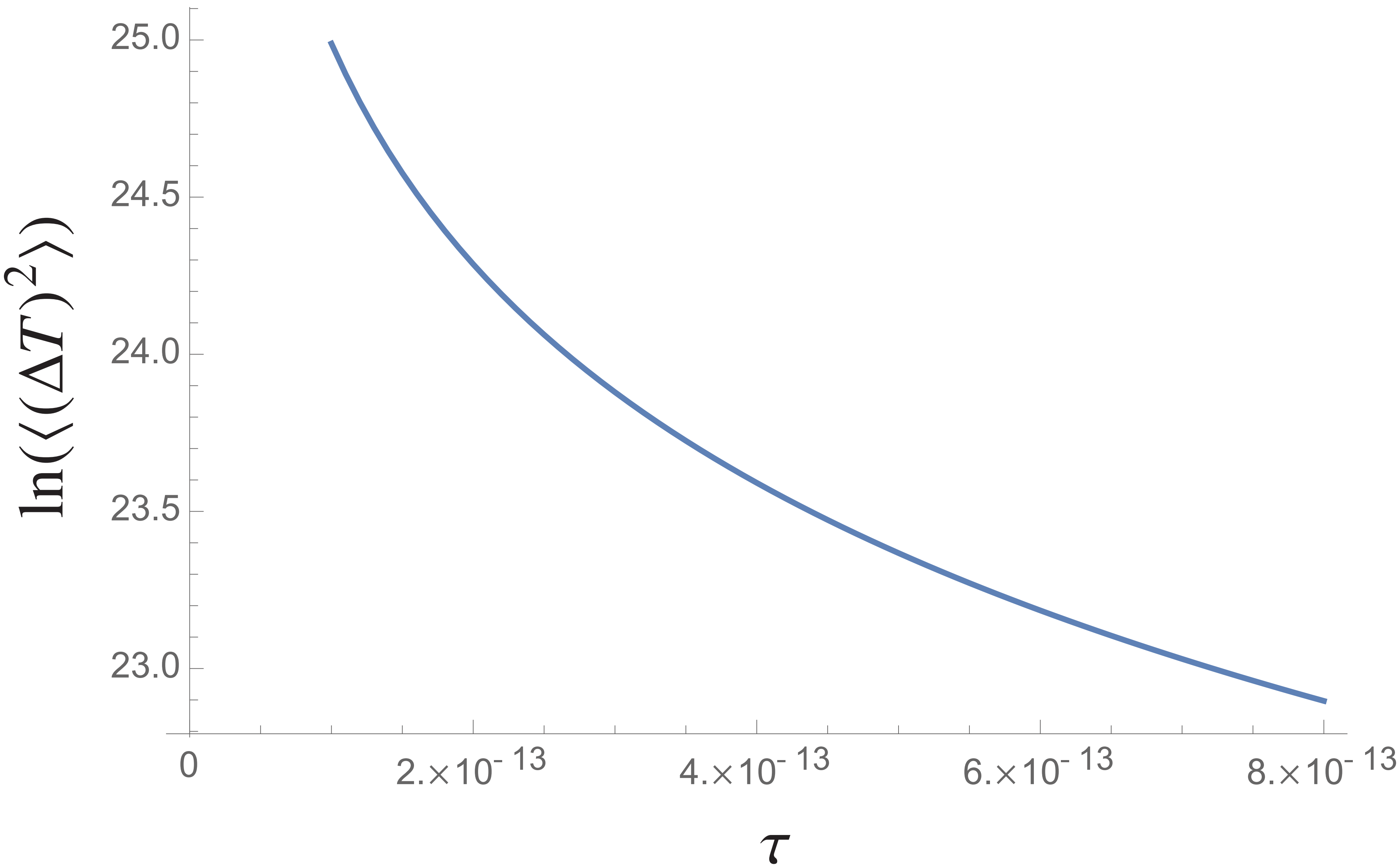
- Two frequencies:





Increasing the measurement time increases the precision of our estimates.

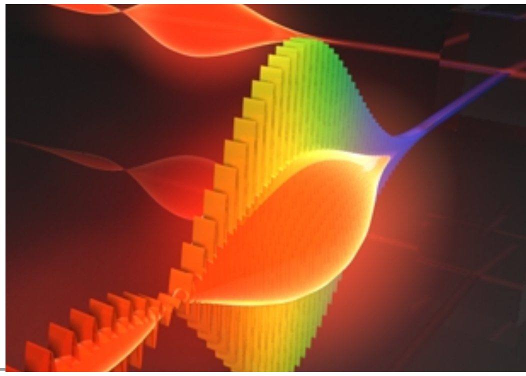
- By increasing the observation time τ, we observe more spectral modes in a given frequency interval.

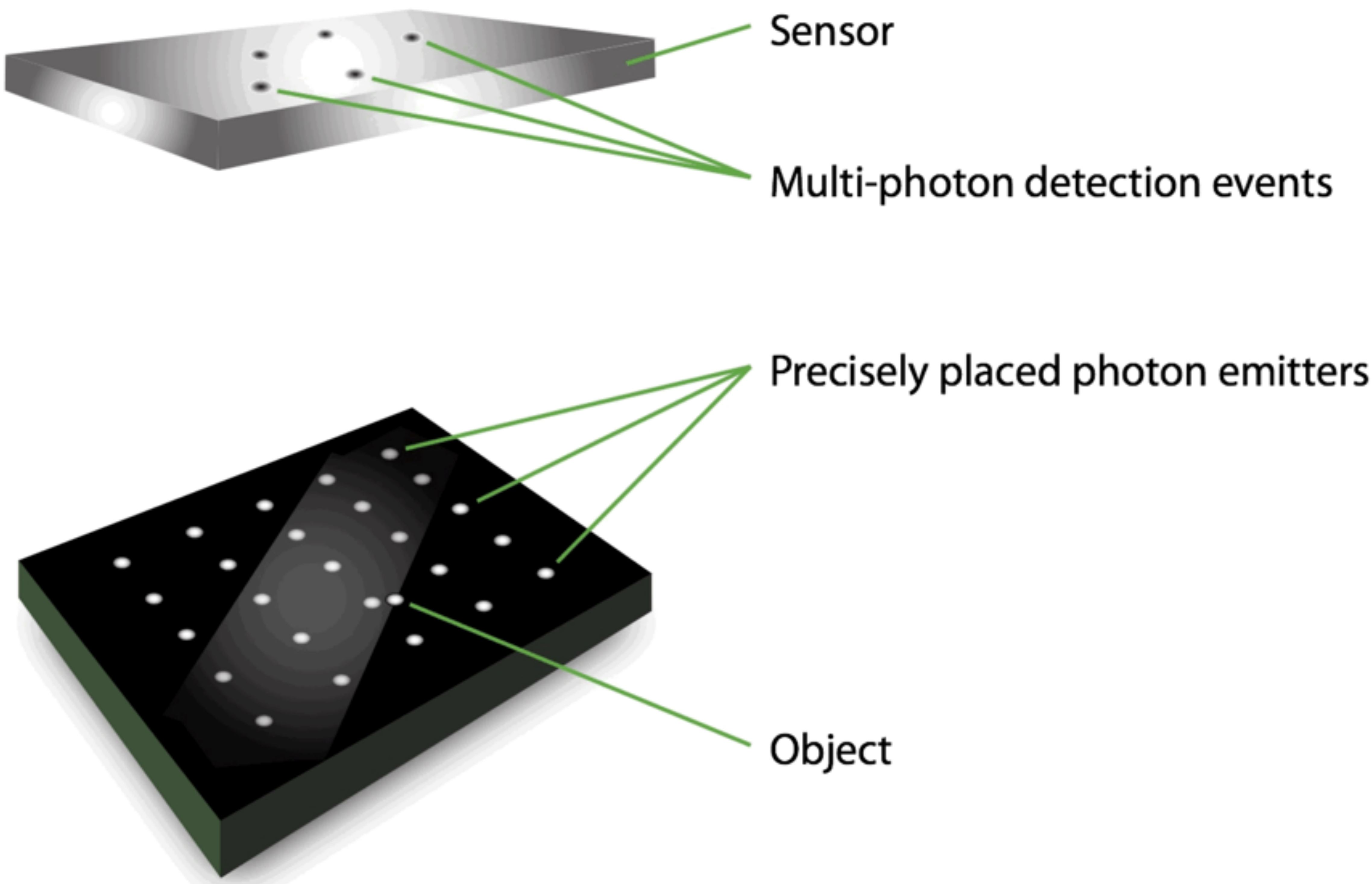


Remarks

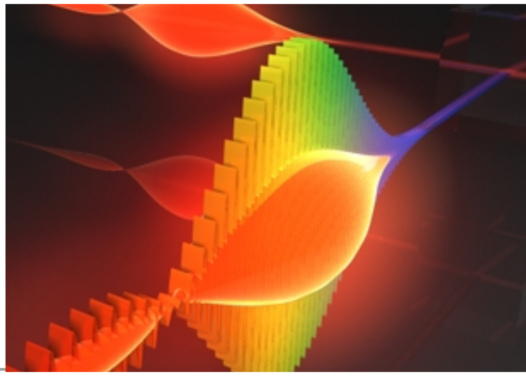
- Thermometry involves nuisance parameters that need to be taken into account when calculating the QFI.
- This requires measurements in multiple frequency modes.
- There is a broad range of frequency modes that give near optimal precision.
- Photon counting is optimal for thermometry, even in the presence of nuisance parameters.

Use multi-photon correlations to obtain increased sensing resolution.

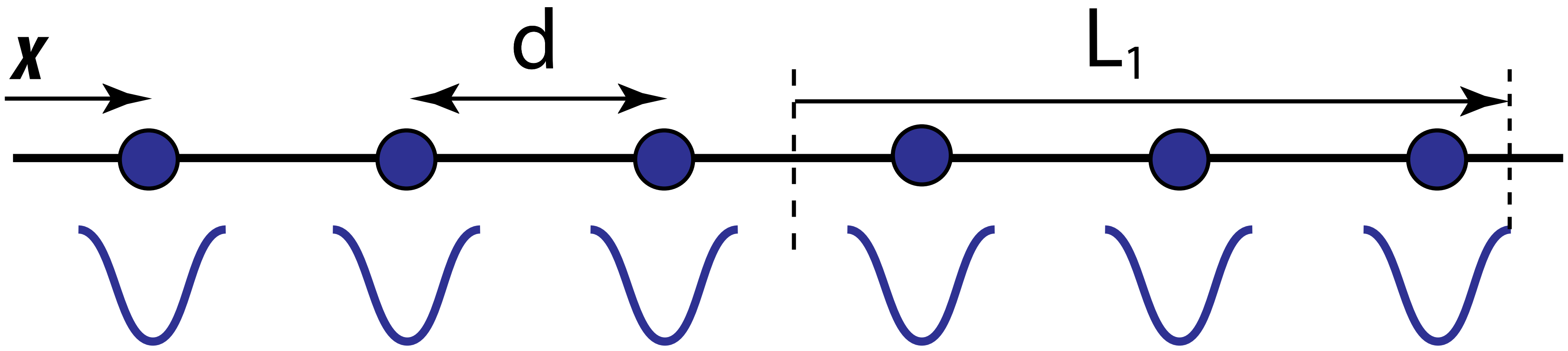




A one-dimensional array of single photon sources.



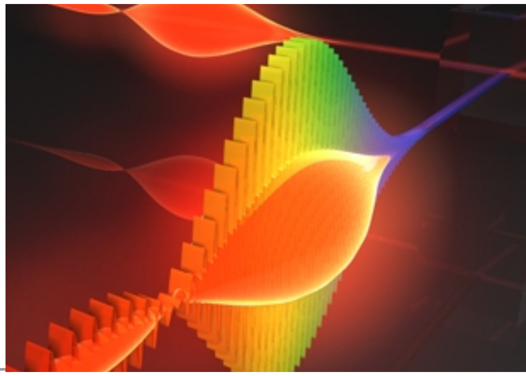
- Consider a string of equidistant single photon sources.



- The sources have an intrinsic Gaussian uncertainty in position:

$$|\psi\rangle = \frac{1}{\mathcal{N}} \int_{-\infty}^{\infty} d\boldsymbol{x}\, e^{\left[-\frac{1}{4s^2}\sum_{i=1}^{N}(x_i-\mu_i)^2\right]} \hat{\boldsymbol{a}}^{\dagger}(\boldsymbol{x}) \cdot |\mathbf{0}\rangle$$

We can calculate the quantum Fisher information for the lattice constant *d*.



- The QFI is given by

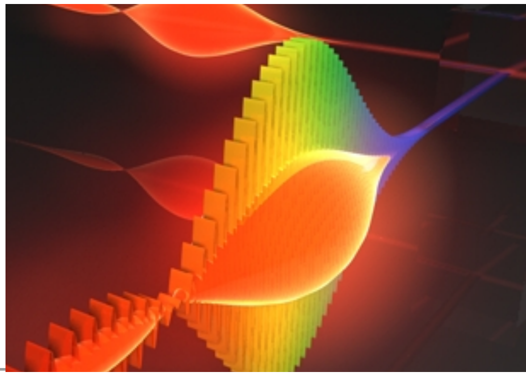
$$\mathcal{I}_Q \leq 4\left\{\langle\psi'(d)|\psi'(d)\rangle - |\langle\psi(d)|\psi'(d)\rangle|^2\right\}.$$

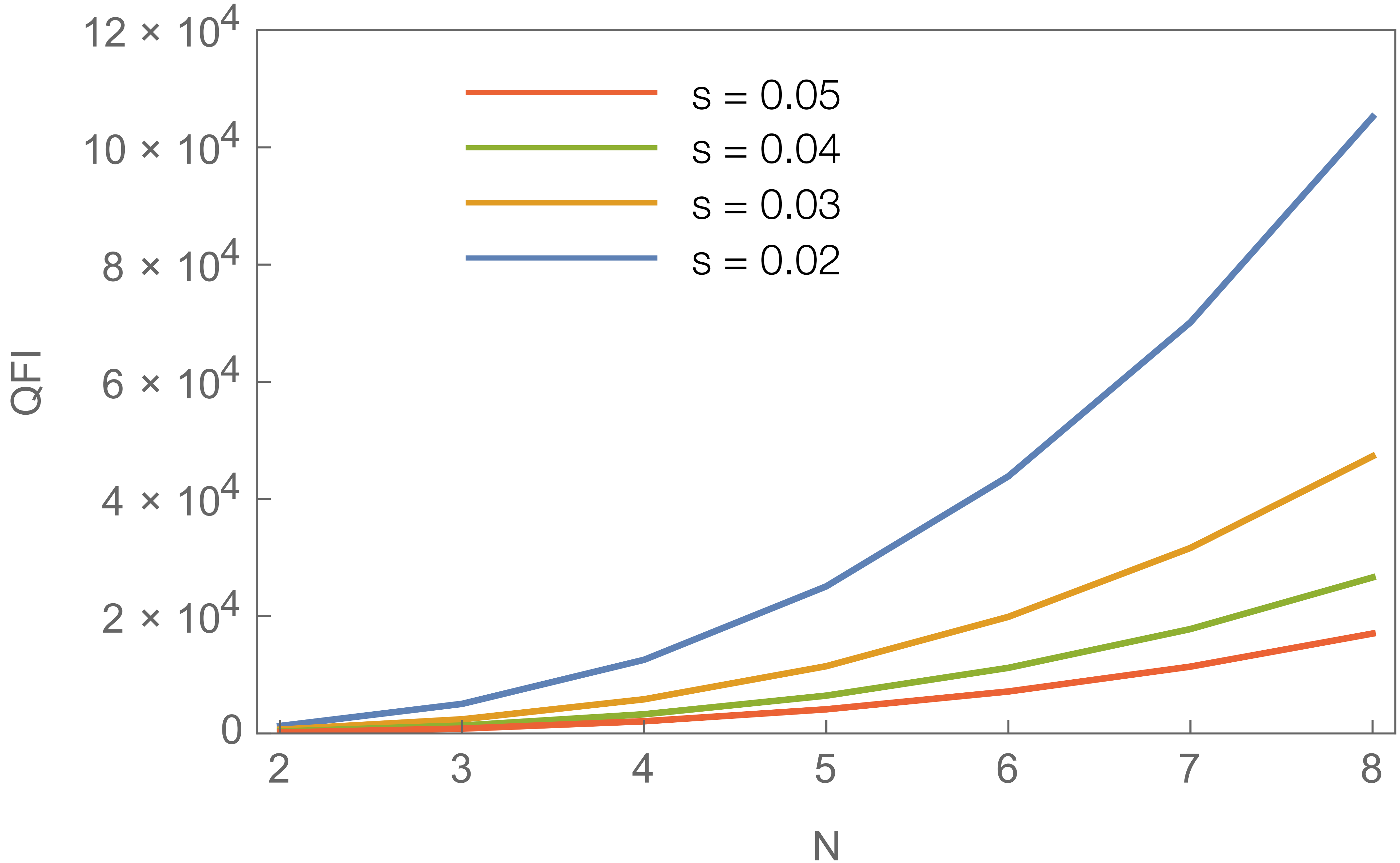
- and the derivative of the state with respect to *d* is

$$|\psi'\rangle = \frac{1}{2s^2\mathcal{N}}\int_{-\infty}^{\infty} d\boldsymbol{x}\left(\sum_{k=1}^{N} c_k(x_k-\mu_k)\right) e^{\left[-\frac{1}{4s^2}\sum_{i=1}^{N}(x_i-\mu_i)^2\right]}\hat{\boldsymbol{a}}^\dagger(\boldsymbol{x})\cdot|\mathbf{0}\rangle,$$

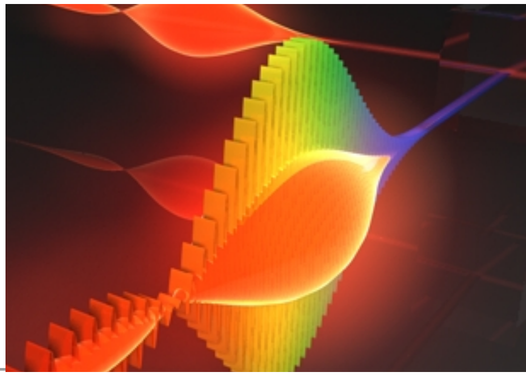
$$|\mathcal{N}|^2 = e^{dA/2s^2}\sum_\sigma\int_{-L_1}^{L_1} d\boldsymbol{x}\, e^{-\frac{1}{2s^2}\sum_{i=1}^{N}\left[x_i^2 - x_i(\mu_{\sigma(i)}+\mu_i)\right]}.$$

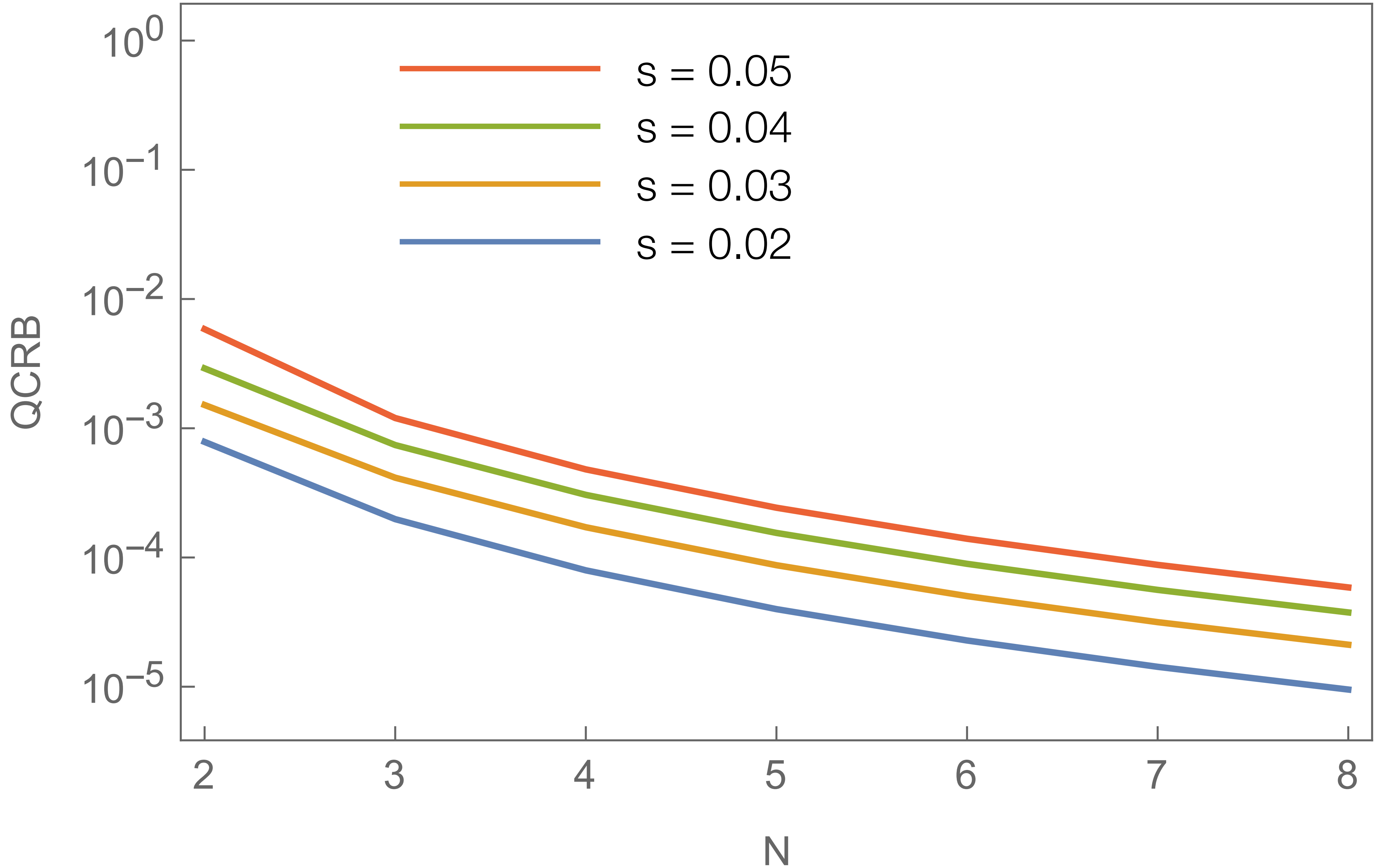
The QFI increases steadily with larger numbers of single photon sources *N*.



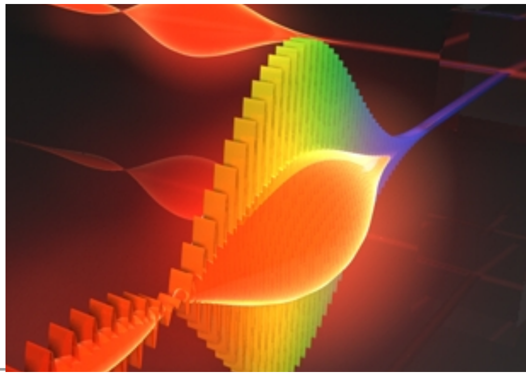


The quantum Cramér-Rao bound.





How can we turn this large QFI into a practical probe system?



- There is a lot of information about the lattice distance *d* in the state of *N* photons.

- In the far field we have access to only a fraction of this information.

- What is the QFI in the far field and what is the corresponding optimal measurement?

- Stretch and skew generators?

Conclusions

- There is extra information in higher-order correlations, even in classical light.
- To get this information, we must carefully model the nuisance parameters.
- Adding single photon sources in an array gives a large increase in the QFI.
- The question is how to get it out.