(On the Practical Application of) Quantum Metrology for Frequency Standards

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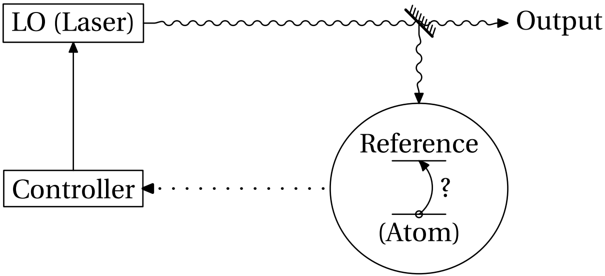
System



Outline

1. Motivation: Improving Stability in (Ion) Clocks
2. On the Importance of Non-Quantum Limits
3. Clock Controller Design

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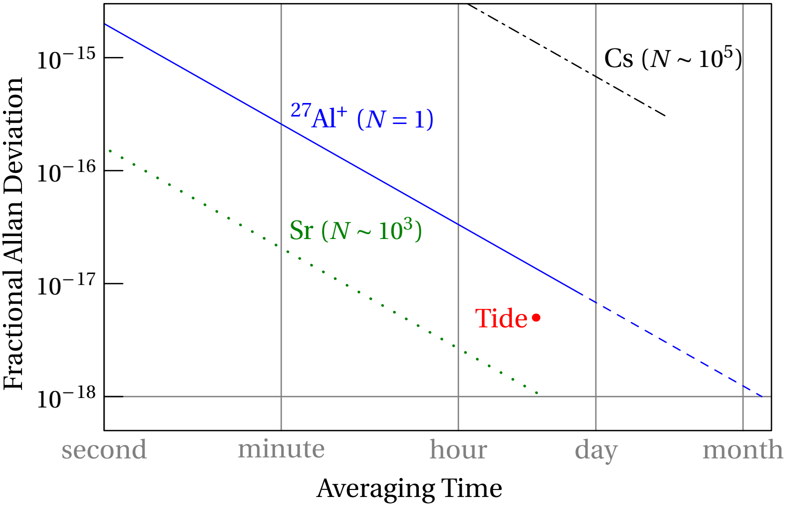
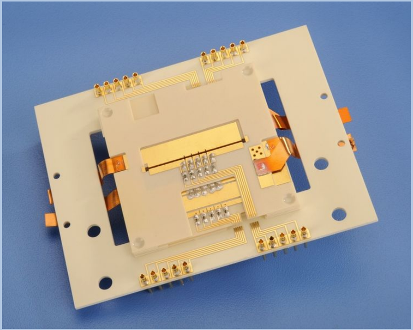
Stability Matters



Multi-Ion Clocks are Coming

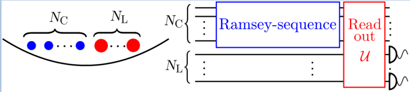
Traps



Appl. Phys. B **114**, 231 (2014)

We have low-micromotion traps suitable for 10^{-19} spectroscopy of ion ensembles

Protocols



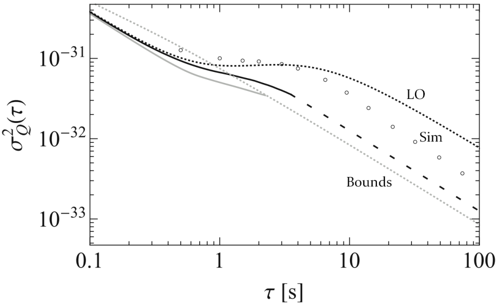
PRL **116**, 013002 (2016)

We have read-out protocols for chains of group-13 ions with sub-linear overhead

But will they be entangled?

We Need Credible Performance Promises

Eentanglement in a serious frequency standard will be hard work.
Will it bring greater benefits than other kinds of hard work?



We need

Pragmatically

Realistic analysis/simulation of specific, implementable schemes (so we know what to try)

Fundamentally

Credible, tight, general bounds on achievable performance
(so we know when to quit)

Outline

The Standard Argument

Phase estimation with N unentangled atoms has a minimum resolution

$$(\Delta\phi)_{\text{SQL}} = \frac{1}{\sqrt{N}}$$

due to the independent quantum projection noise (QPN) of each atom. For a single measurement of duration T, the resulting frequency resolution is

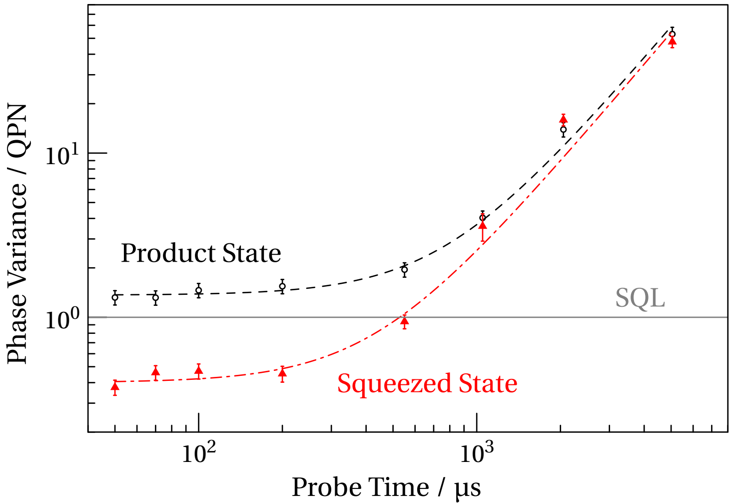
$$\Delta\omega = \frac{1}{T\sqrt{N}}$$

Phase estimation with N *entangled* atoms can reach a resolution

$$(\Delta\phi)_{\text{Heisenberg}} = \frac{1}{N},$$

so should allow an N-fold reduction in averaging times!

Are We Really QPN-Limited?



IDL, Monika H. Schleier-Smith, Vladan Vuletić, PRL **104**, 250801 (2010)
see also Hosten et al., Nature **529**, 505 (2016)
Treutlein & co., yesterday's talk.

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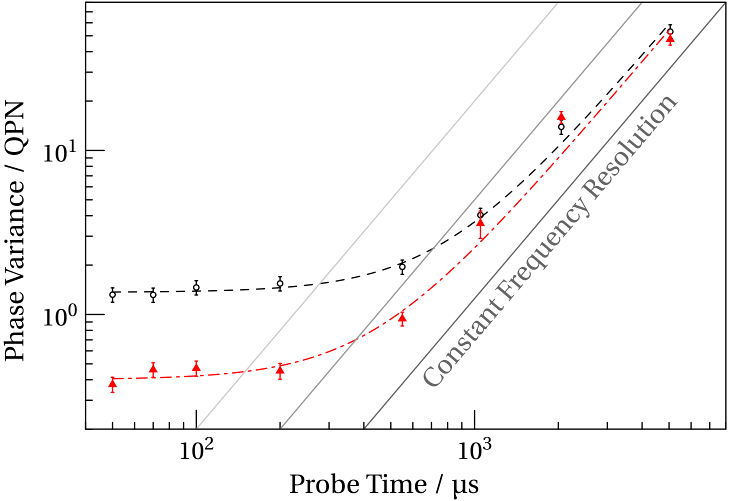

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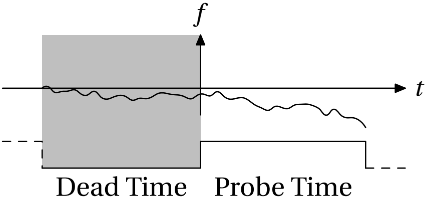
Aside: LO Noise Hurts in Two Ways



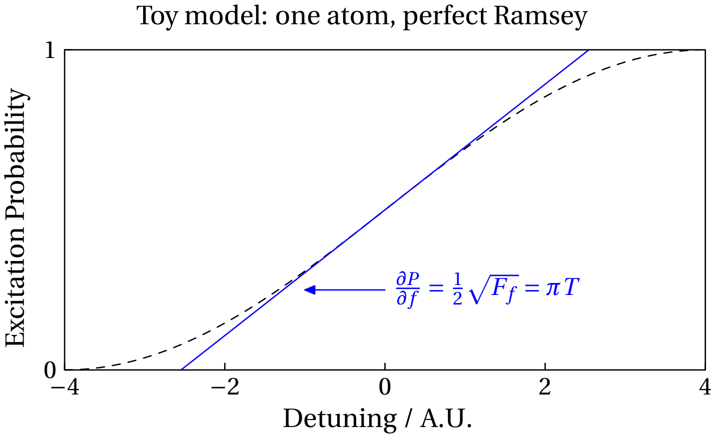
Dick Effect

- Unobserved frequency fluctuations during dead time cannot be corrected
- Limits mostly lattice clocks
- Fixed by continuous or simultaneous interrogation

Probe Time/Resolution

- Probe response function must be informative at all plausible LO frequencies
- Limits mostly ion clocks
- *Not* fixed by simultaneous interrogation

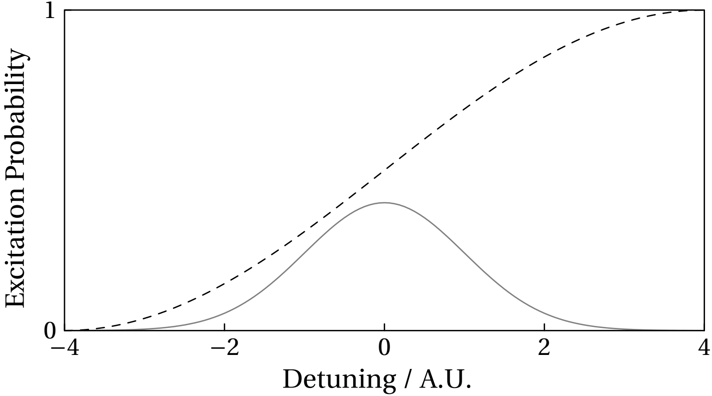
There Is No Quantum Limit to Frequency Resolution



The frequency resolution limit must come from another timescale, which must be understood if we're going to optimize.
The extra timescale generically comes from non-Markovian, correlated noise.

The Importance of the Prior

Toy model: Gaussian prior LO frequency uncertainty



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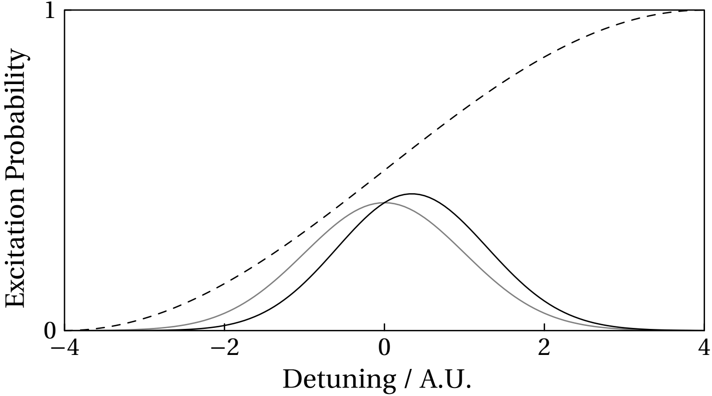

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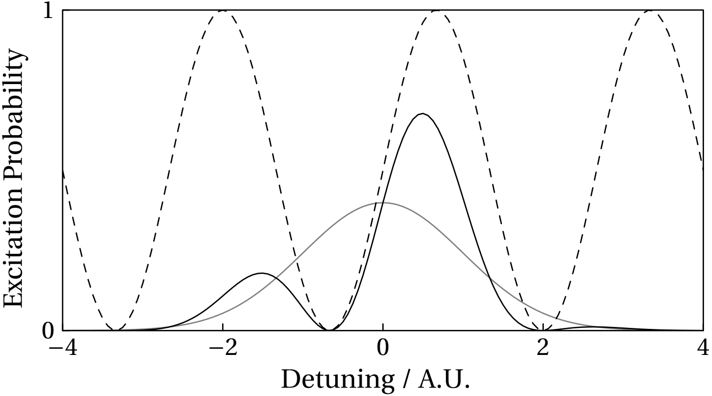

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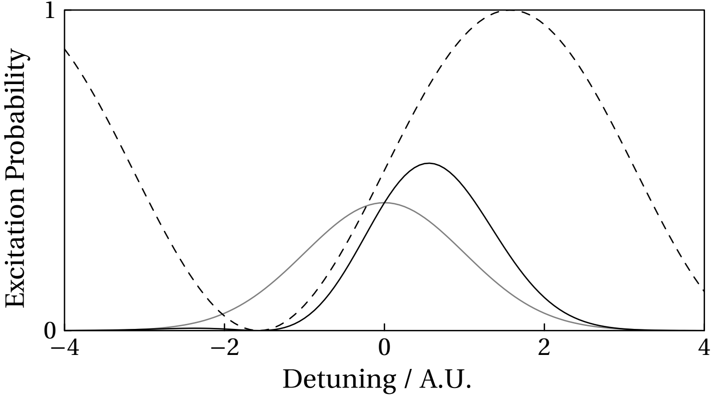
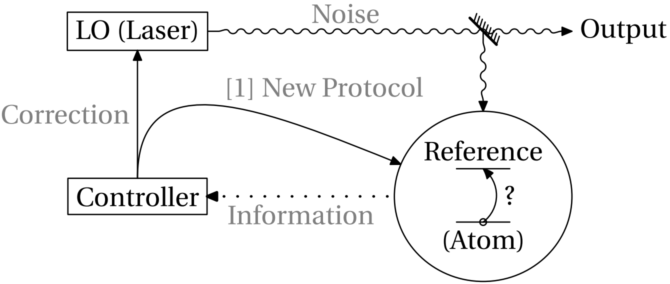


Fisher information and linearization describe suboptimally short probe times. Need a *global* figure of merit, e.g. posterior variance.

Whence the Prior?



Optimal probe protocol depends on prior, which depends on controller performance, which depends on probe protocol, …

Fortunately, LO noise dominates, so iterative optimization converges quickly.

[1] General, sadly impractical solution: Mullan and Knill, Phys. Rev. A **90**, 042310 (2014)

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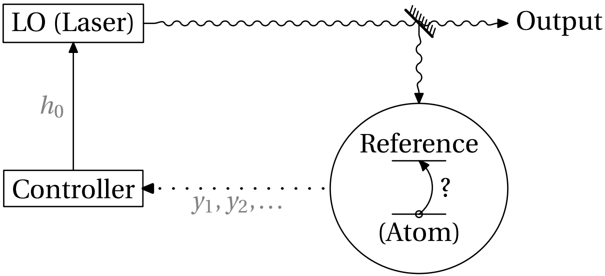
The Controller's Job



Given a history $\ldots, y_4, y_3, y_2, y_1$ of *estimates* of the LO frequency, make a prediction h_0 of the next estimate y_0.

We restrict ourselves to controllers that make frequency corrections.

General Linear Controllers

General linear controllers (GLC) make predictions of the form

$$h_0 = \sum_k w_k y_k \qquad\qquad \sum_k w_k = 1.$$

For instance, the standard integrating controller is

$$h_0 = h_1 + g(y_1 - h_1) = \sum_{k=1}^{\infty} g(1-g)^k \, y_k$$

How *Should* We Choose the Weights?

The mean-squared error of the prediction

$$h_0 = \sum_k w_k y_k$$

is

$$\langle (h_0 - f_0)^2 \rangle = \boldsymbol{w}^\top \boldsymbol{C} \boldsymbol{w},$$

using the covariance matrix

$$C_{jk} = \langle (y_0 - y_j)(y_0 - y_k) \rangle.$$

The MSE is minimized when

$$\boldsymbol{C}\boldsymbol{w} = \begin{pmatrix} \lambda \\ \lambda \\ \lambda \\ \vdots \end{pmatrix} \qquad \lambda \in \mathbb{R} \ni \sum_k w_k = 1$$

Computing the Weights from Available Information

From spectrum:

$$C_{jk} = 4\int_0^\infty S(f)\,\mathrm{sinc}^2(\pi f T)\sin(j\pi f T)\sin(k\pi f T)\cos((j-k)\pi f T)\,\mathrm{d}f$$

From phenomenological model:

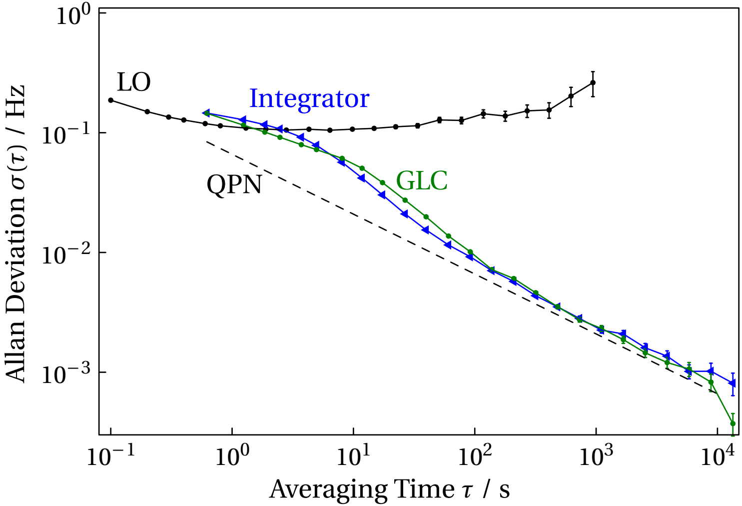
$$\boldsymbol{C} = \sigma^2_{\text{white}}(T)\begin{pmatrix} 2 & 1 & 1 & \cdots \\ 1 & 2 & 1 & \cdots \\ 1 & 1 & 2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \sigma^2_{\text{flicker}}\begin{pmatrix} 2 & 1.57 & 1.30 & \cdots \\ 1.57 & 3.13 & 2.43 & \cdots \\ 1.30 & 2.43 & 3.74 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \sigma^2_{\text{r.walk}}(T)\begin{pmatrix} 2 & 5/2 & 5/2 & \cdots \\ 5/2 & 5 & 11/2 & \cdots \\ 5/2 & 11/2 & 8 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \cdots$$

From clock data:

$$C_{jk} = \langle (y_0 - y_j)(y_0 - y_k)\rangle$$

The Impact of Controller Performance

10 ions, 0.6 s Ramsey, 20 ms dead time



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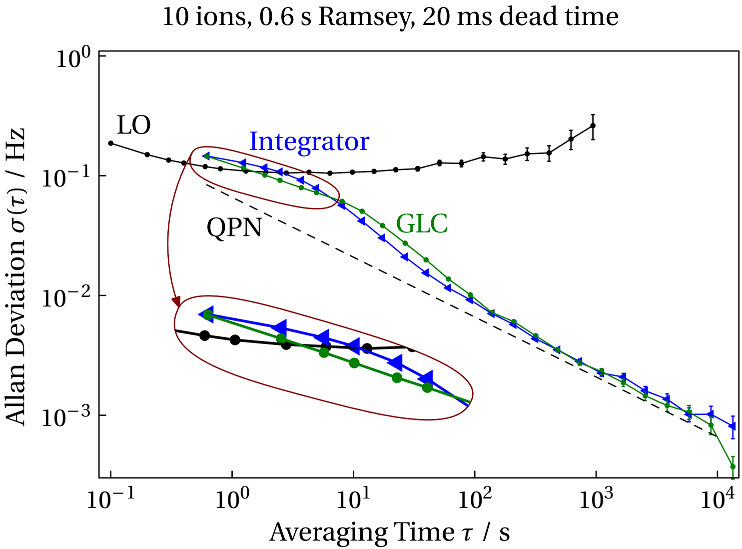
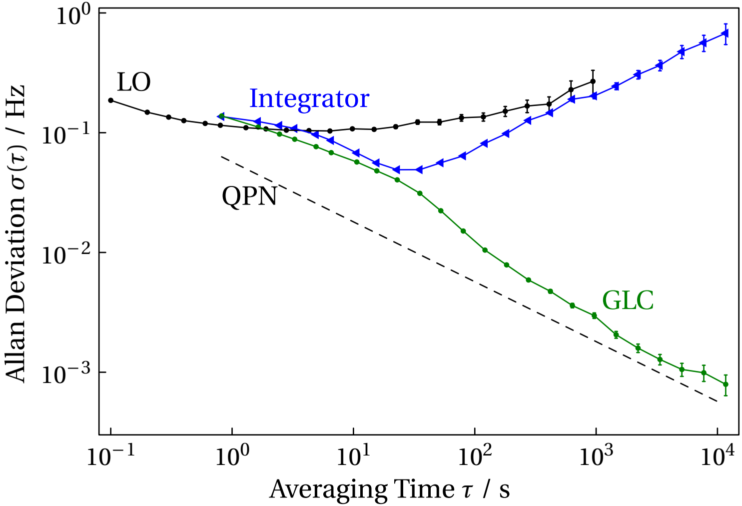
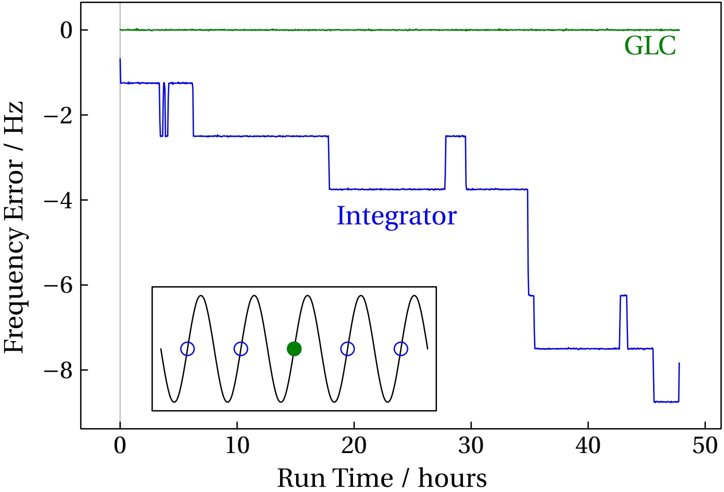


The Impact of Controller Performance

10 ions, 0.8 s Ramsey, 20 ms dead time



The Impact of Controller Performance



Summary and Open Questions

I've argued that:

- Multi-ion clocks are coming and their stability matters.
- Clock measurement protocols should optimize for gain of frequency information, not phase resolution. This optimization depends on the prior knowledge of LO frequency available before the measurement.
- The servo controller maintains this prior knowledge, and is worth optimizing.

I wonder:

- Does the frequency error distribution acquire heavy tails? Should we optimize for kurtosis as well as variance?
- Can *non*-linear control algorithms better prevent fringe hops, especially when facing non-Gaussian priors?

Dziękuję wam

Collaborators:

- Krzysztof Chabuda
- Rafał Demkowicz-Dobrzański

Current students:

- Nils Scharnhorst
- Stephan Hannig
- Johannes Kramer
- Lennart Pelzer

Former students:

- Jannes Wübbena
- Sana Amairi
- Kornelius Jakobsen

PI:

- Piet O. Schmidt

geo Q

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