USING ENTANGLEMENT AGAINST NOISE in quantum metrology

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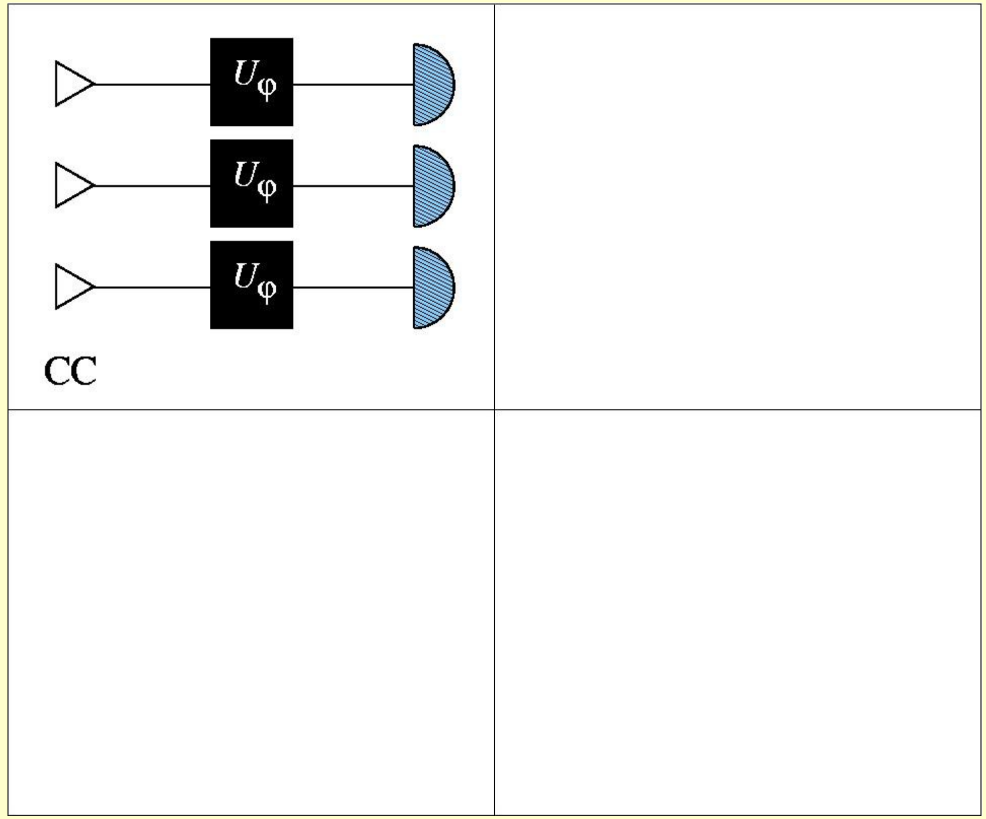
Parameter estimation, possible strategies

Parallel strategies

N
probes



Phys. Rev. Lett. **96**, 010401 (2006)

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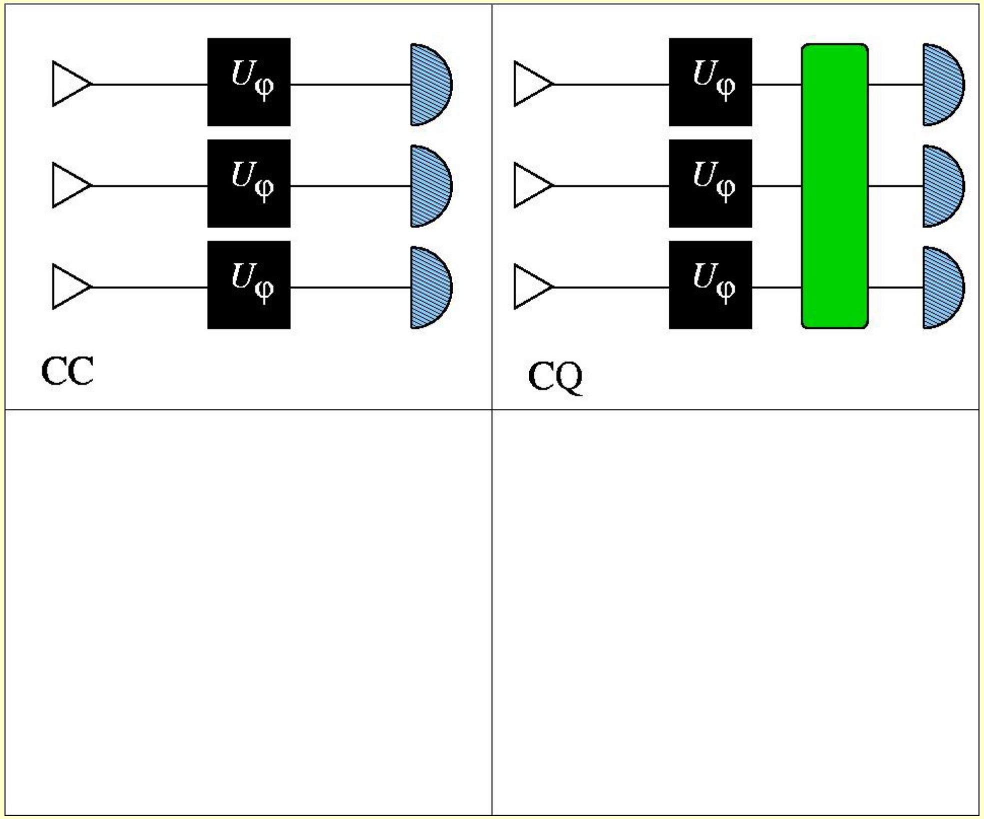

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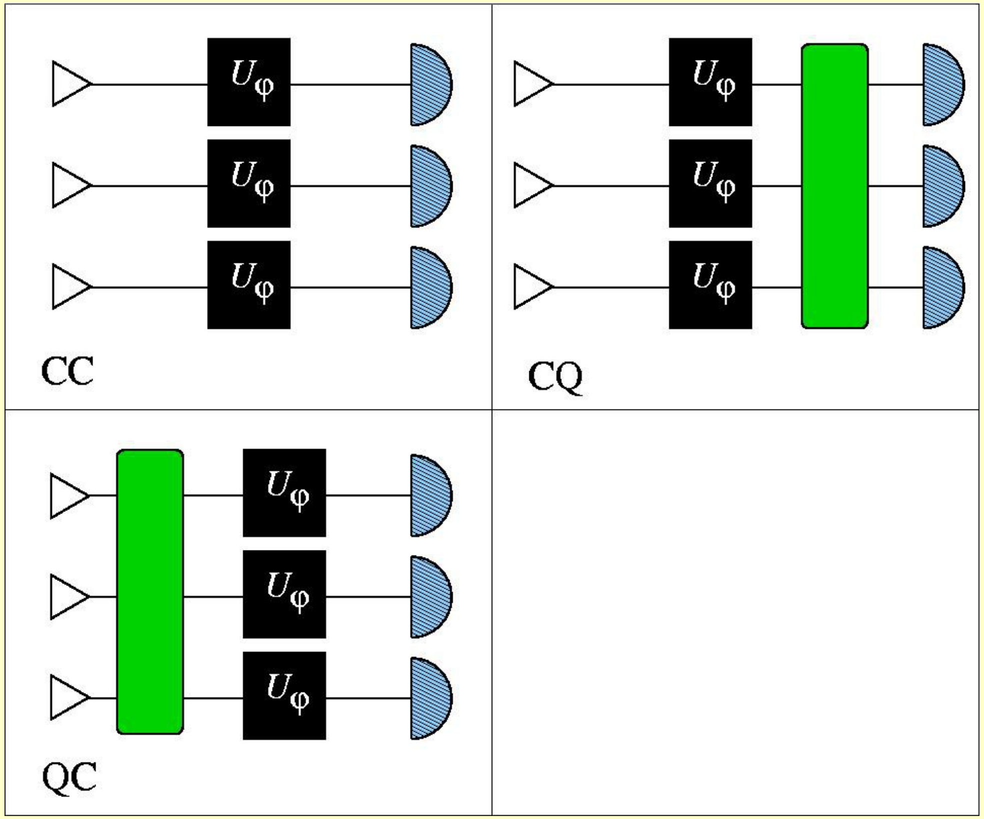

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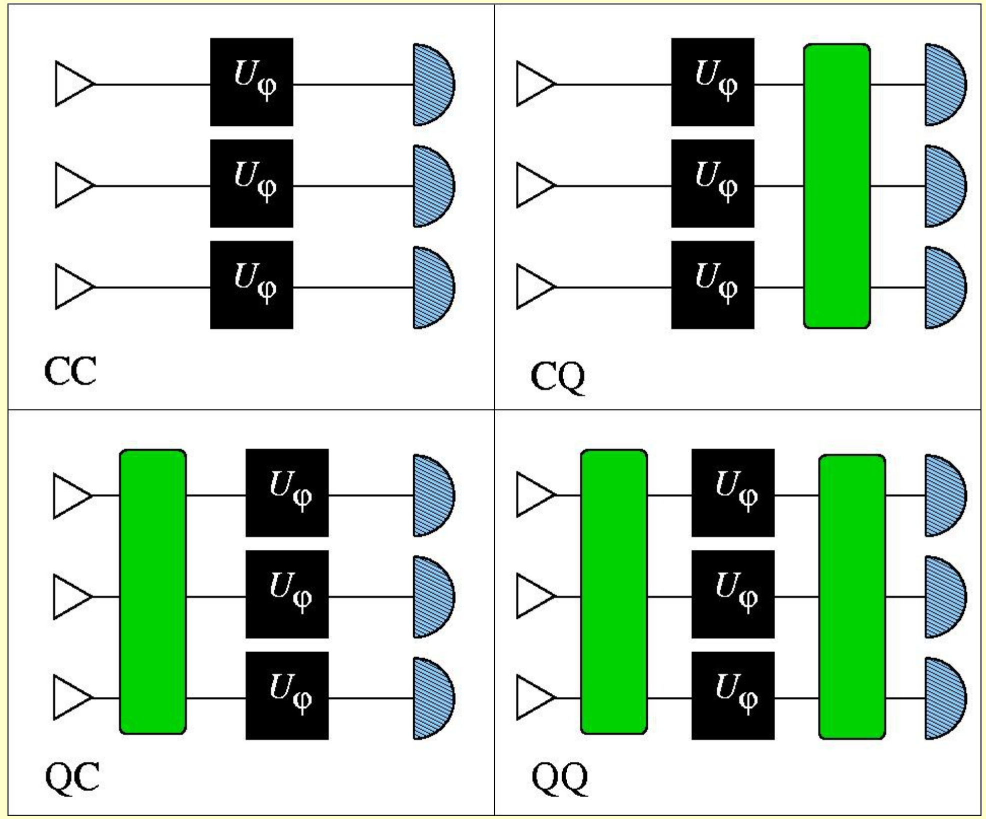

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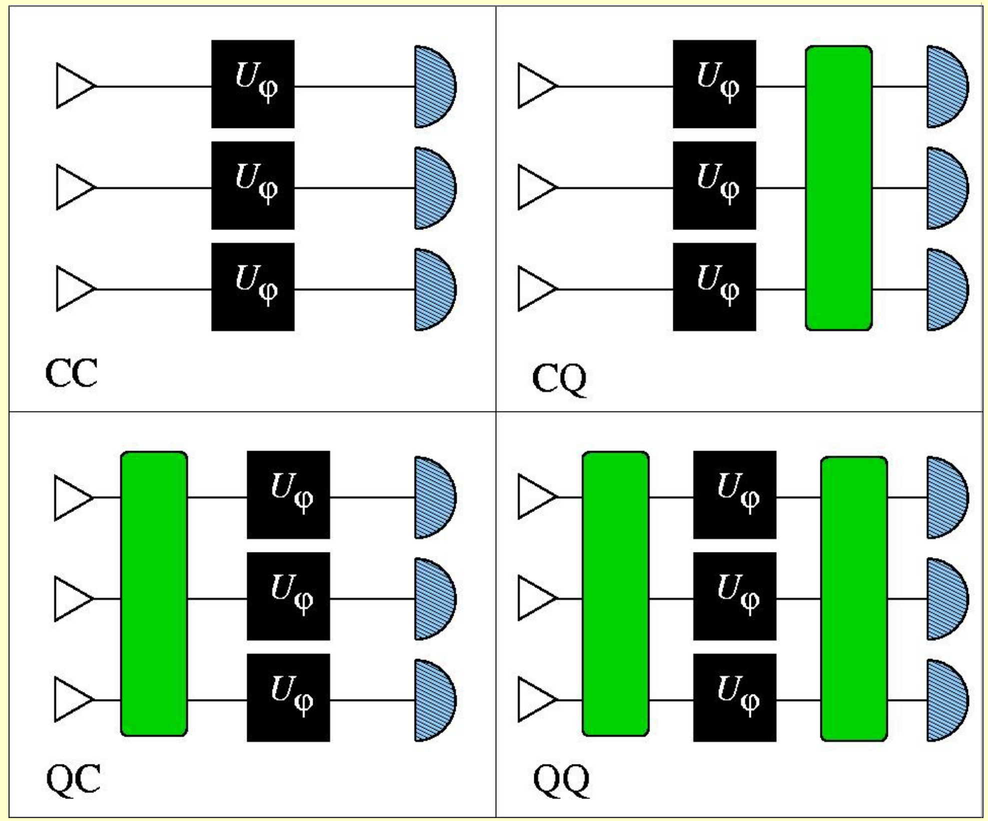
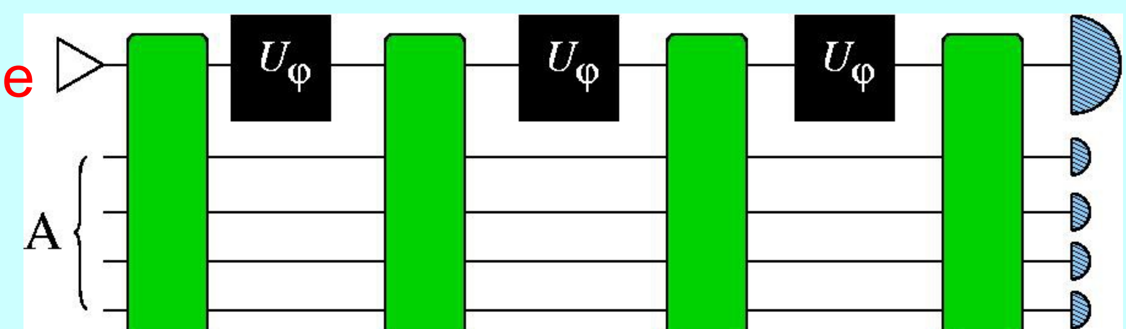


Sequential str.

1 probe



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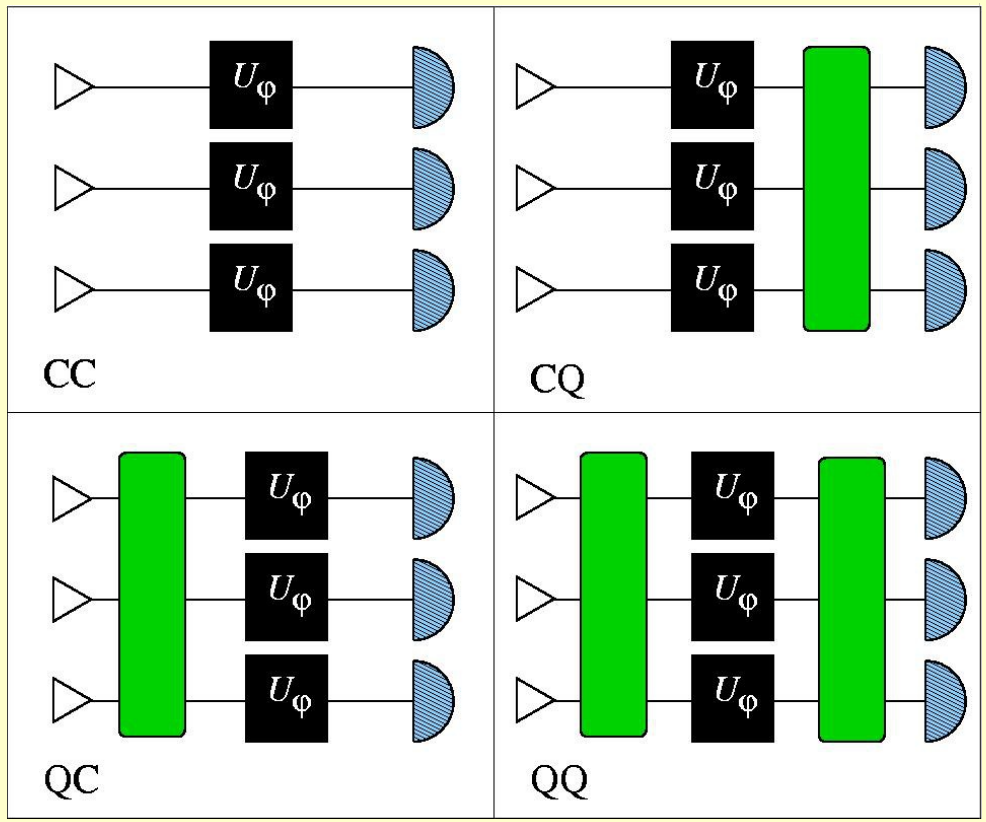

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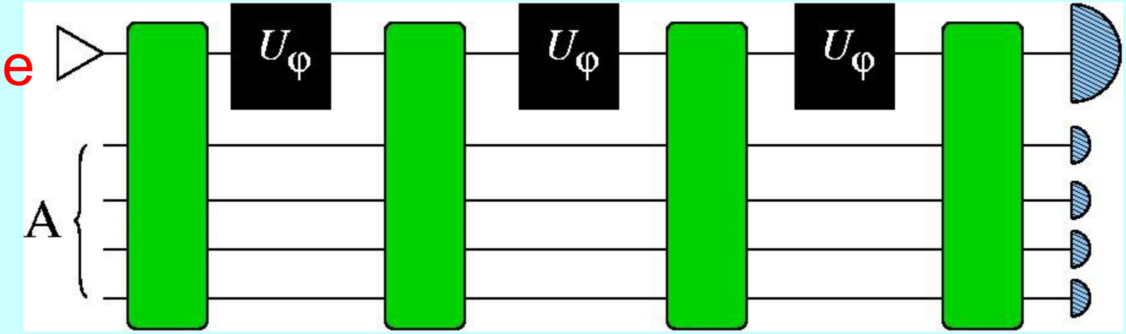

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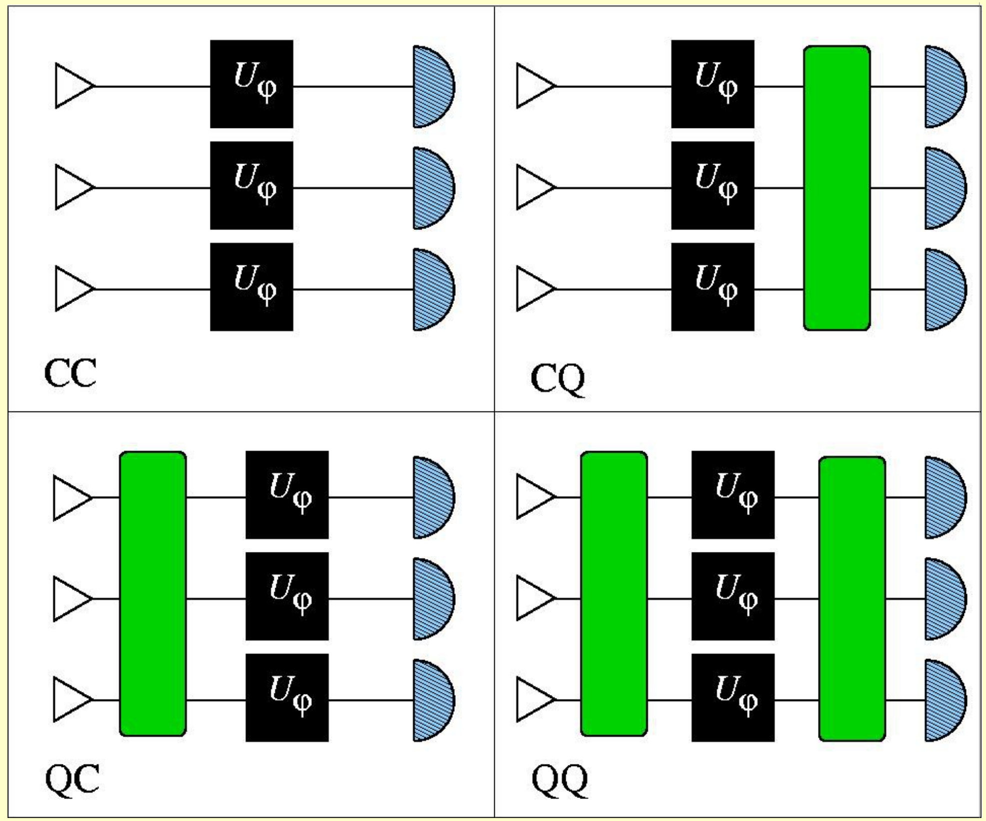

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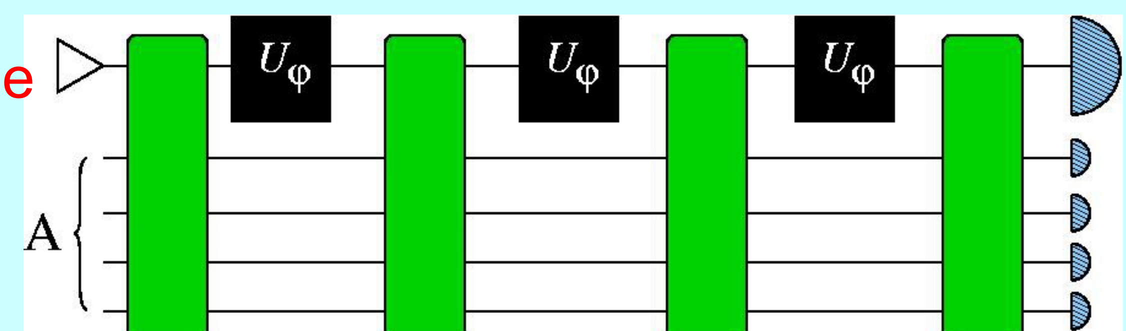

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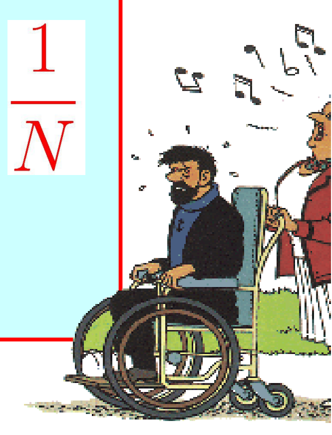
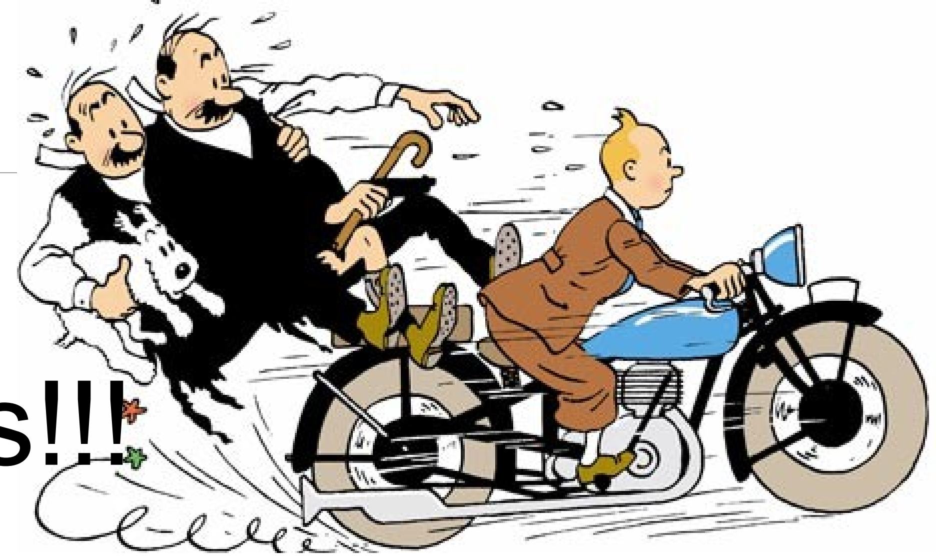
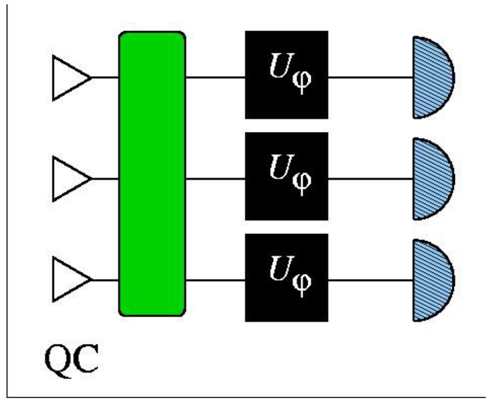


Sequential can simulate any parallel, but takes more time!

Parallel strategies



Ancillas are useless!!!



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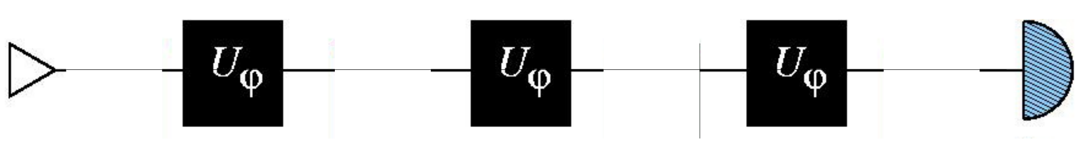

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Sequential and entangled-parallel are equivalent in the noiseless case...

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The unentangled strategy performs **WORSE!!!**

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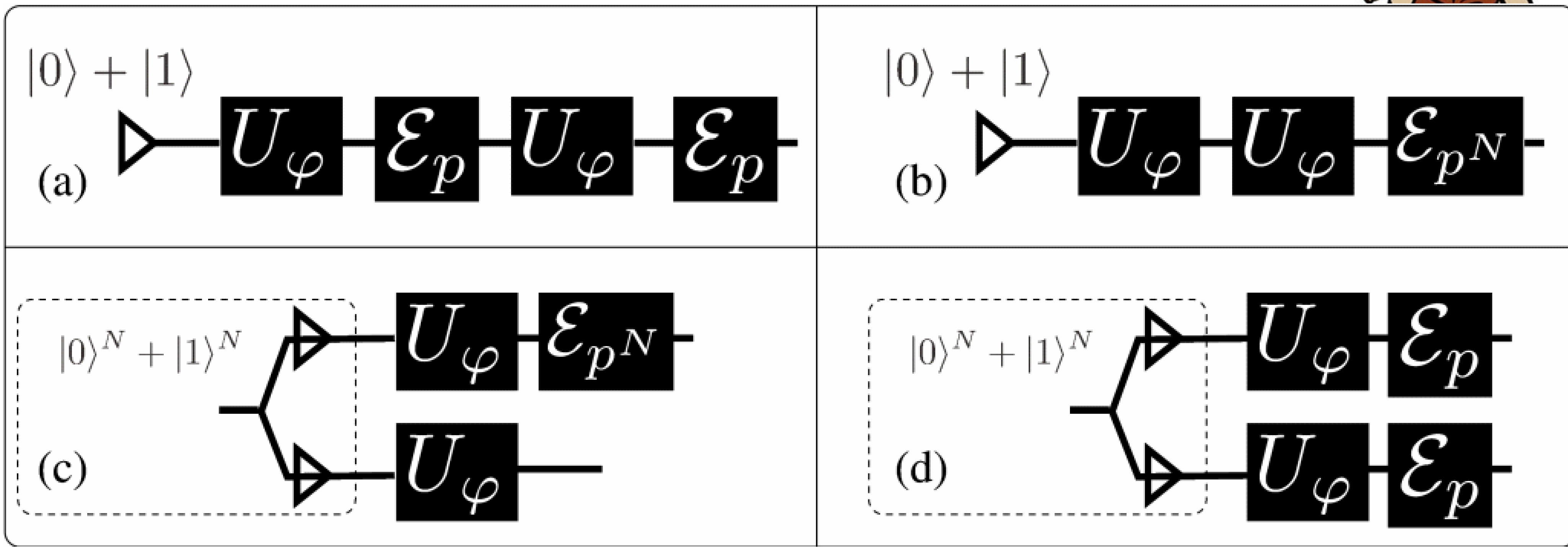
The unentangled strategy performs **WORSE!!!**

Then you should use entanglement to achieve higher sensitivity in the presence of noise!

very surprising!

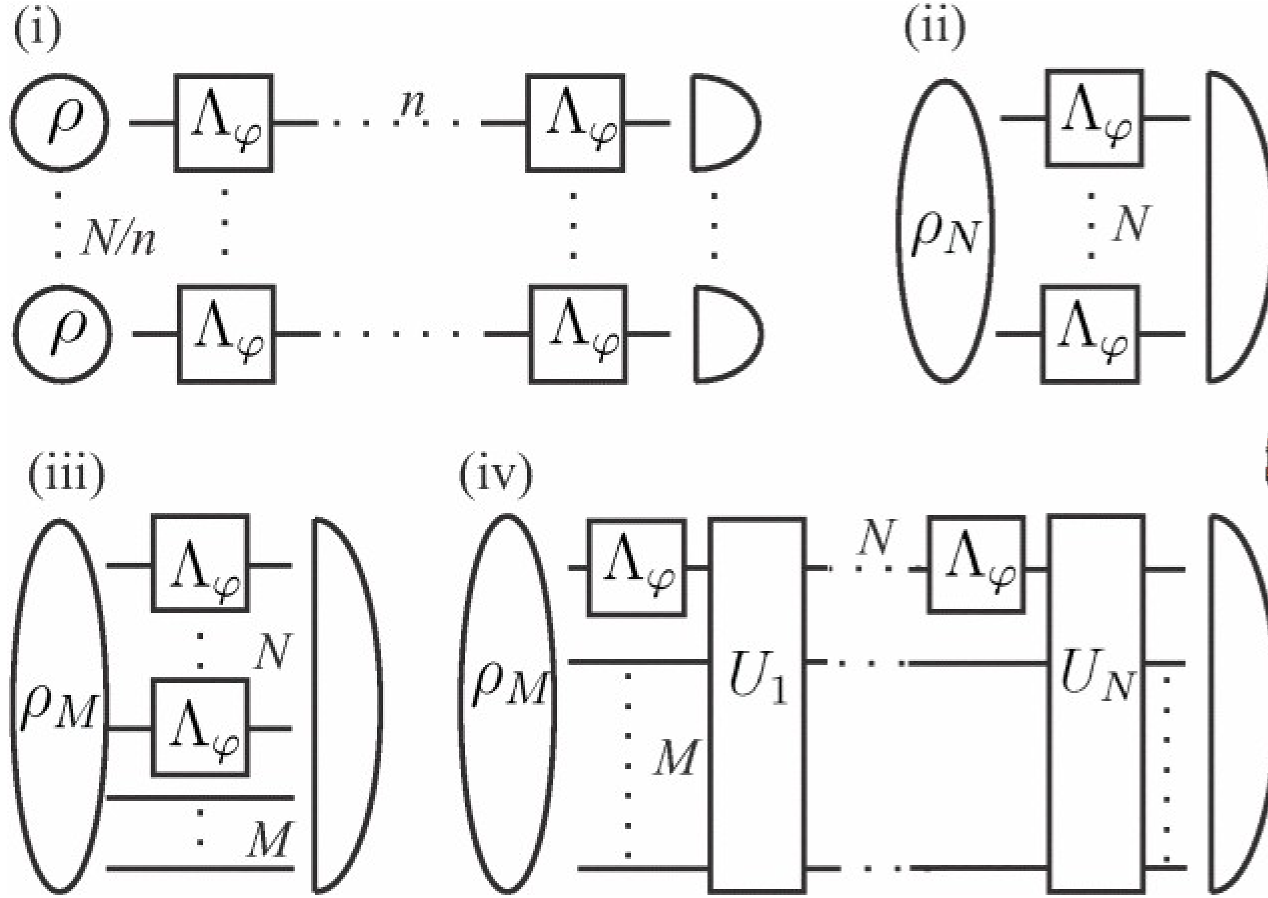
simple example: erasure noise.

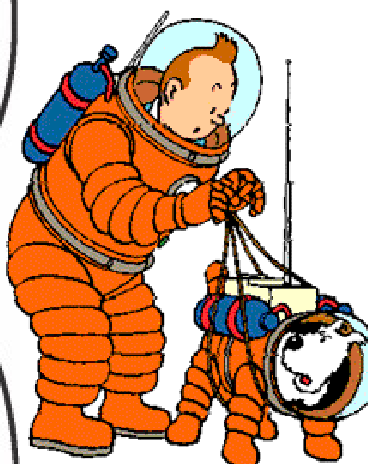
simple to see: noise and unitary commute!



(a) Optimal sequential strategy, which is equivalent to a (b) sequential strategy where a (larger) erasure happens at the end, which is equivalent to a specific (c) parallel-entangled strategy where the erasure is only on the first probe, which is equivalent to a (d) parallel-entangled strategy in the presence of erasure on all probes. This last is weaker than the optimal parallel-entangled strategy, since the input state is not optimized

General hierarchy of metrology strategies





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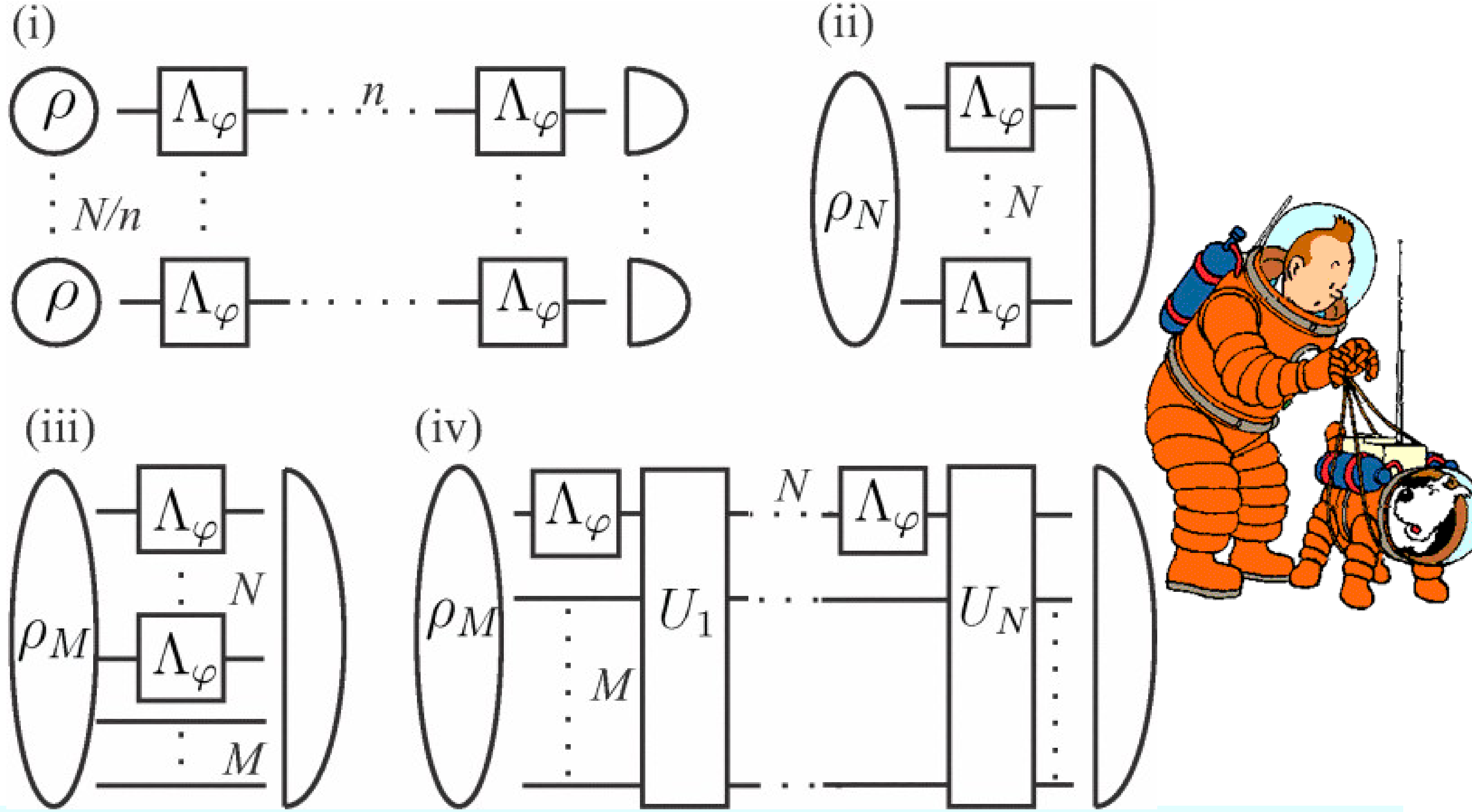
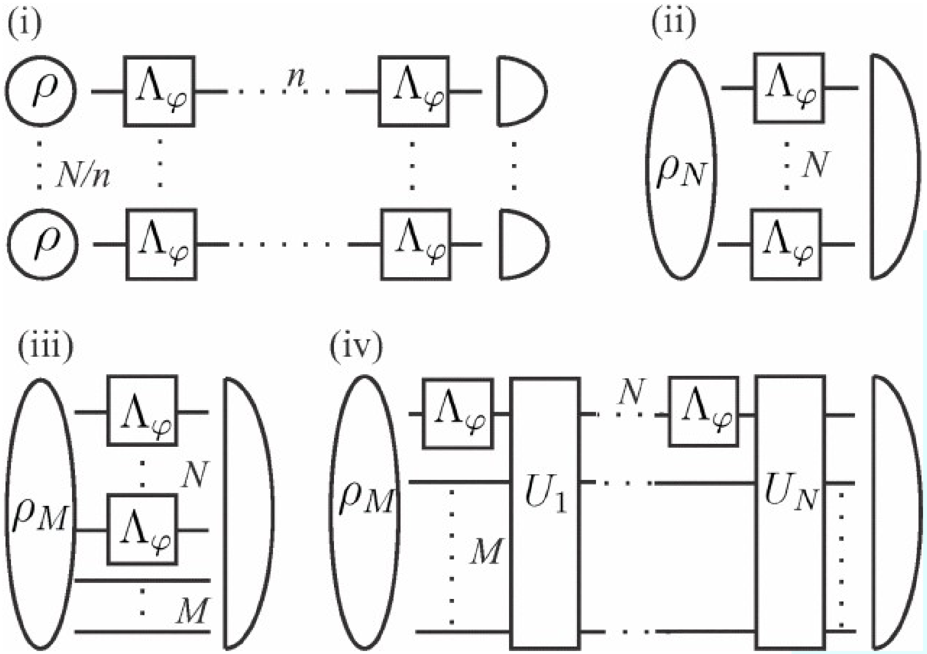


$$\text{(i)} = \text{(ii)} = \text{(iii)} = \text{(iv)} \qquad \text{decoherence free,}$$

$$\text{(i)} < \text{(ii)} = \text{(iii)} = \text{(iv)} \qquad \text{dephasing, erasure,}$$

$$\text{(i)} < \text{(ii)} < \text{(iii)} \stackrel{?}{=} \text{(iv)} \qquad \text{amplitude-damping,}$$

$$\textbf{(i)} \stackrel{?}{\leqslant} \textbf{(ii)} \leqslant \textbf{(iii)} \stackrel{?}{=} \textbf{(iv)} \qquad \textbf{general conjecture.}$$



Fisher info for these strategies:

$$F^{(\mathrm{i})} = \max_{\rho,n} F\{[\Lambda_\varphi^n(\rho)]^{\otimes N/n}\},$$

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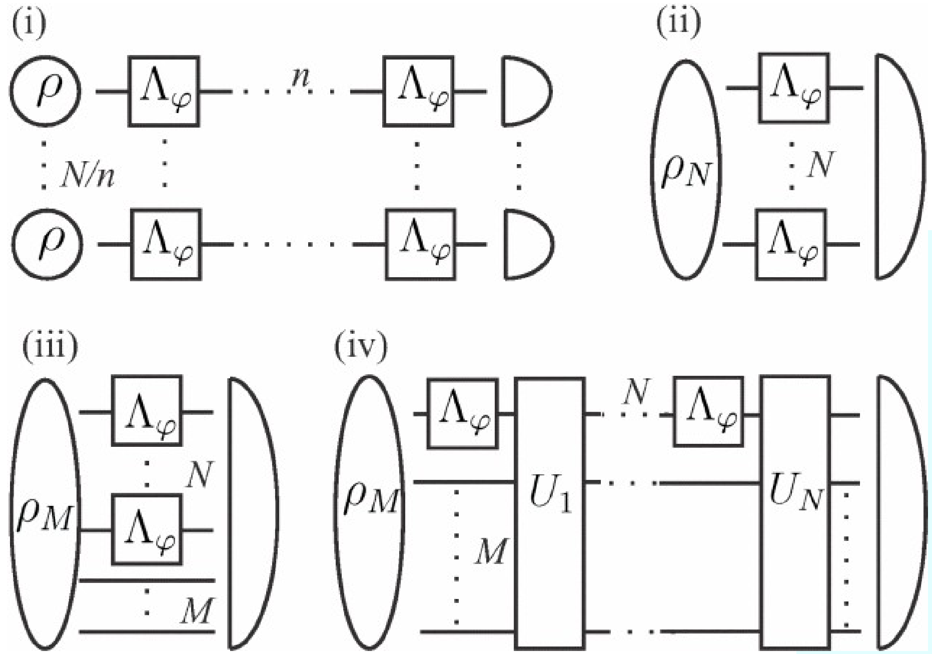

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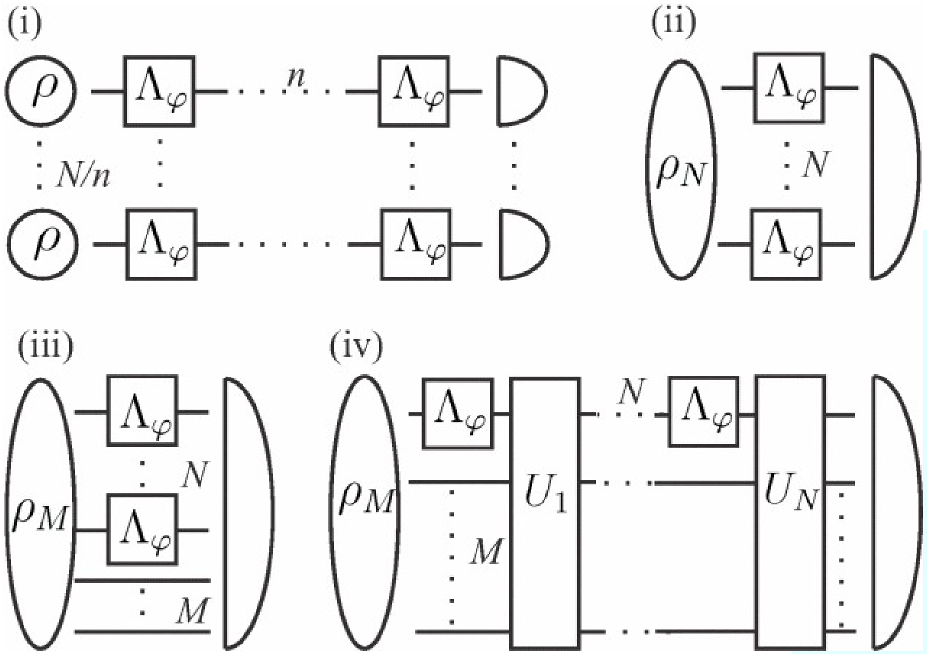
Obvious results:

$$F^{(\mathrm{ii})} \le F^{(\mathrm{iii})} \quad F^{(\mathrm{iii})} \le F^{(\mathrm{iv})}$$



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Other relations?

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using the parallel structure of the Kraus maps, we can prove:

$$F^{(\mathrm{ii/iii})} \le 4 \min_{K_k^\varphi} N\|\alpha\| + N(N-1)\|\beta\|^2$$

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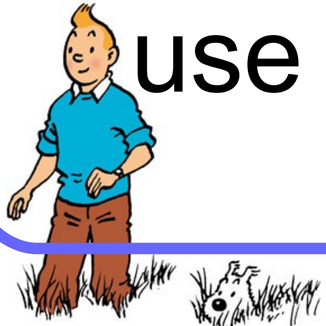

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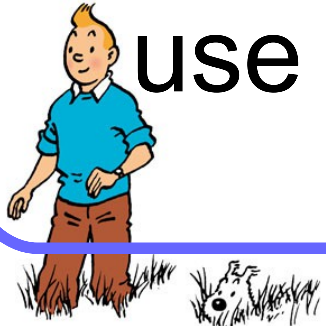

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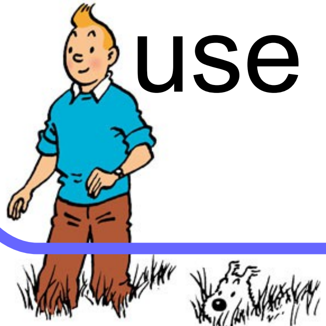

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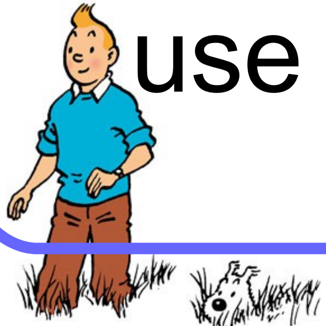

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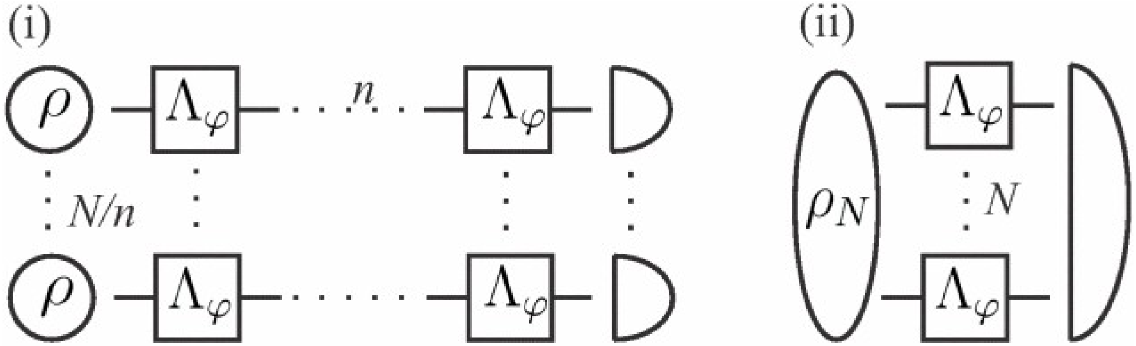
NO!! $\longrightarrow$ (**maybe** they're equivalent! CONJECTURE!)



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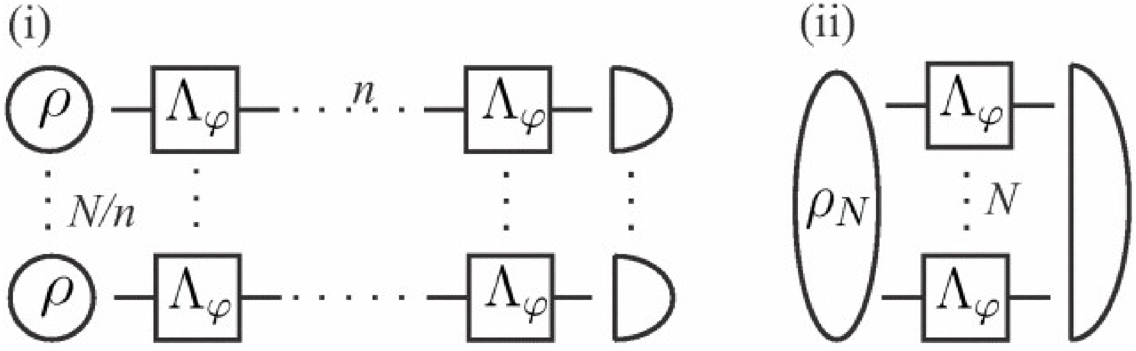

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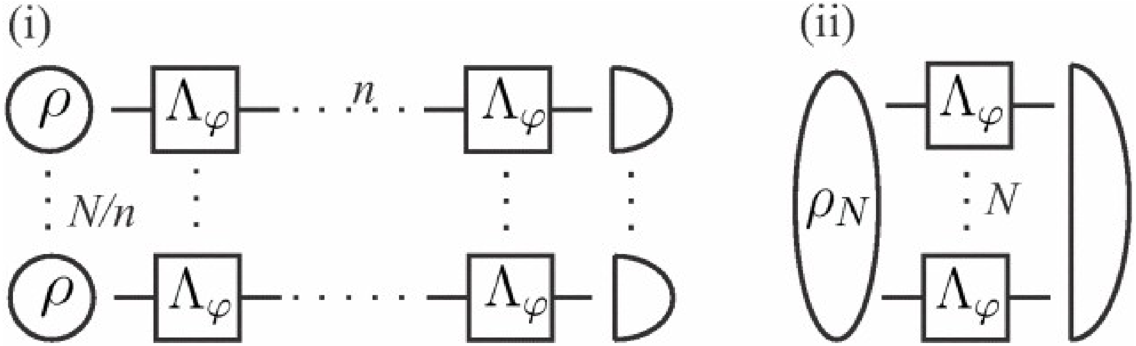

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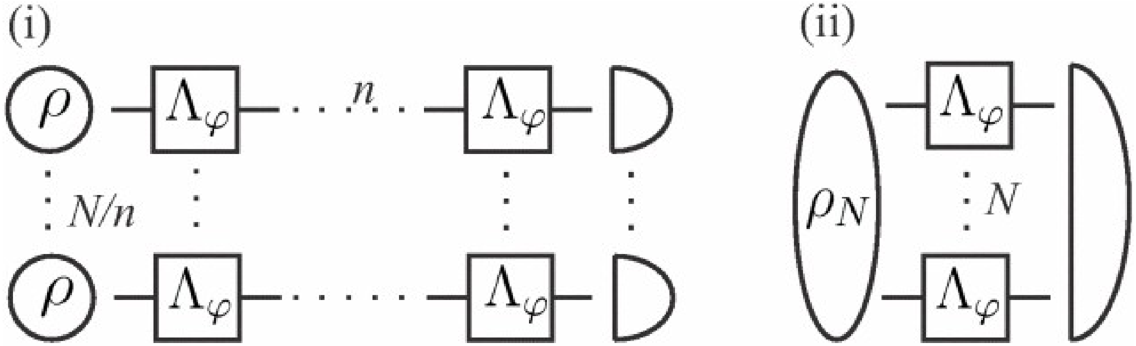

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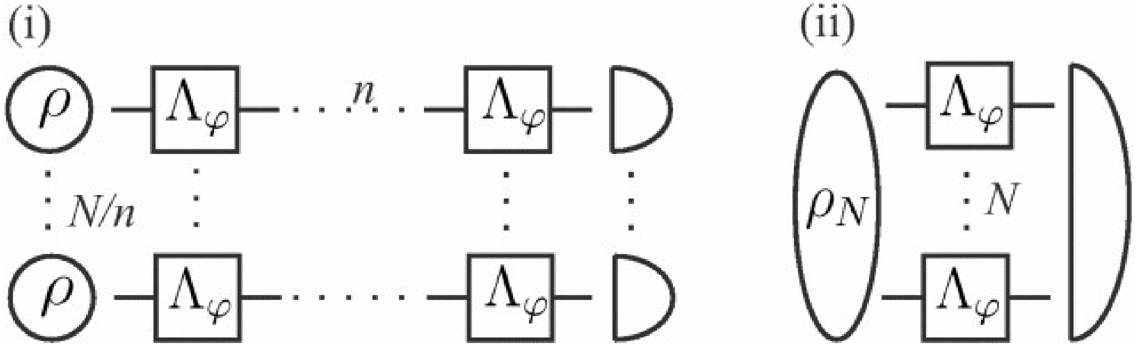

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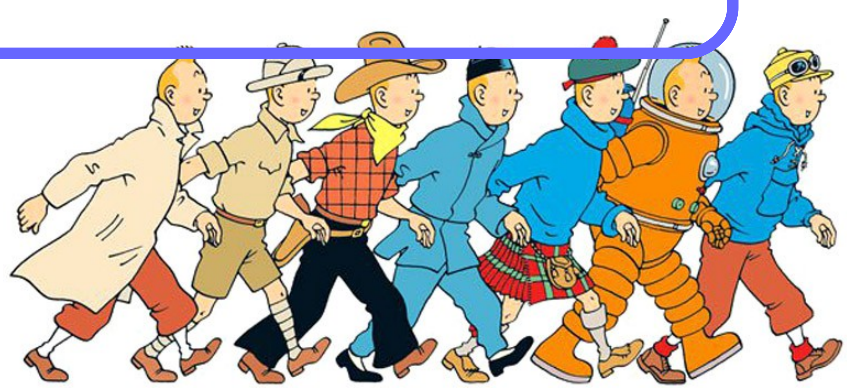

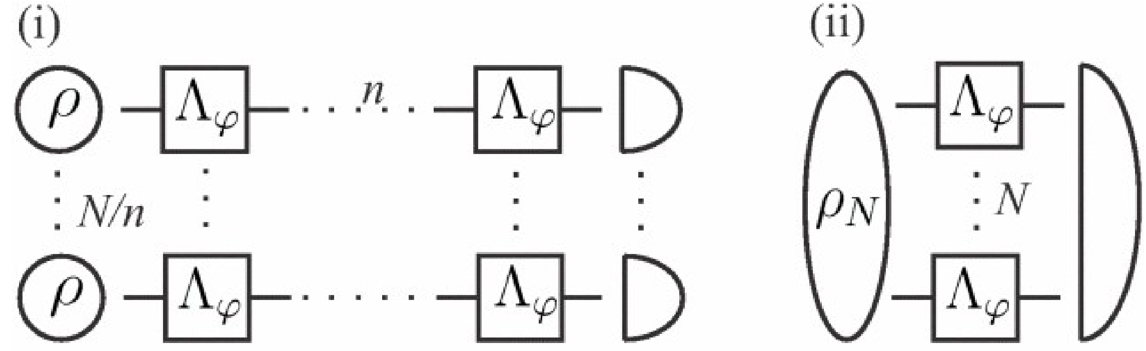

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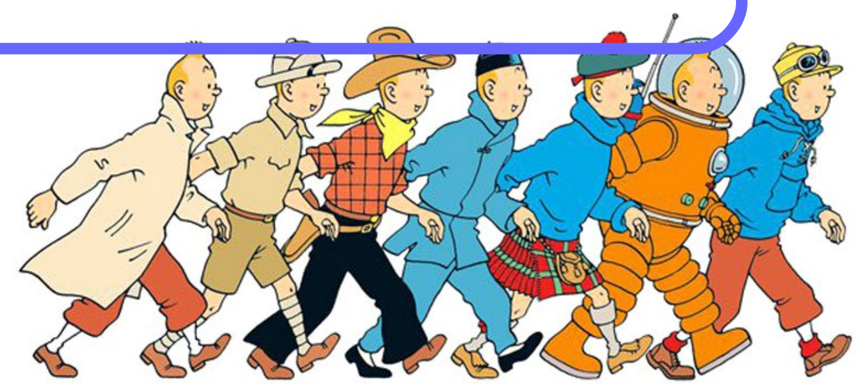
NO!!! (conjecture)

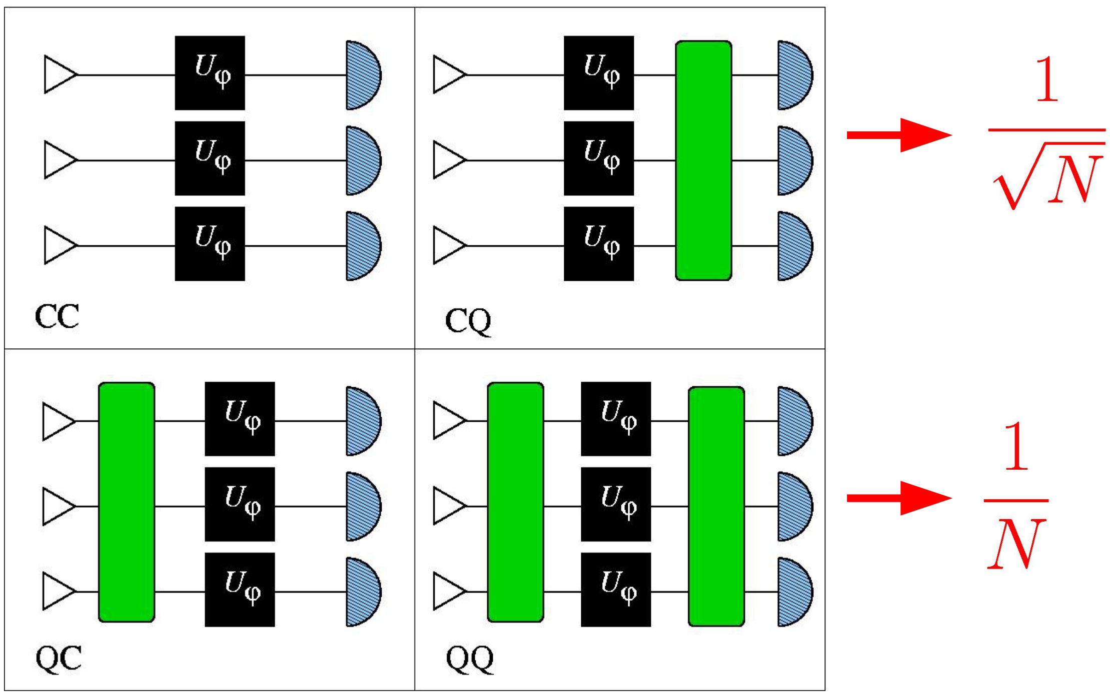


Open question!

is entanglement at the measurement stage useful!??

(it's useless in the noiseless case!)

[recent work by Kavan Modi?]



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Take two strategies (an **unentangled** and an **entangled** one) that are **equivalent** without noise. Which one is better when you add noise?

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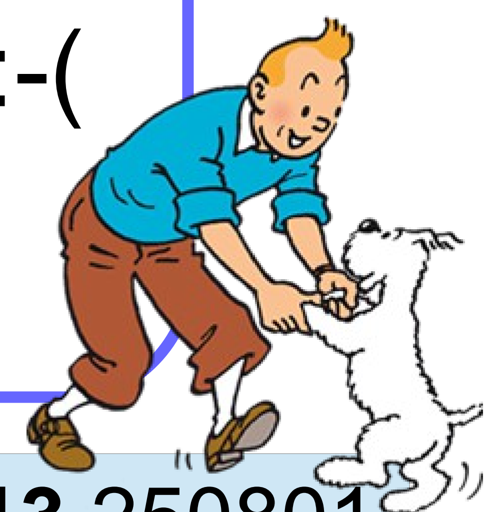
  - Useless in the noiseless case :-(



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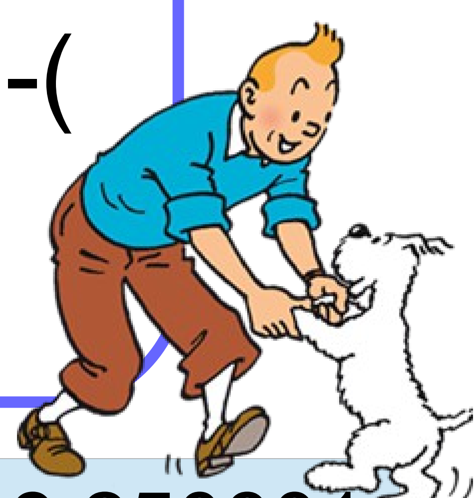
The entangled one!! **WHAT!?**

- the role of **ancillas** in q metrology?
  - Useless in the noiseless case :-(
  - Useful in the noisy case :-)



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Take home message

Entangled protocols may be more robust to noise than unentangled ones which are equivalent without noise!

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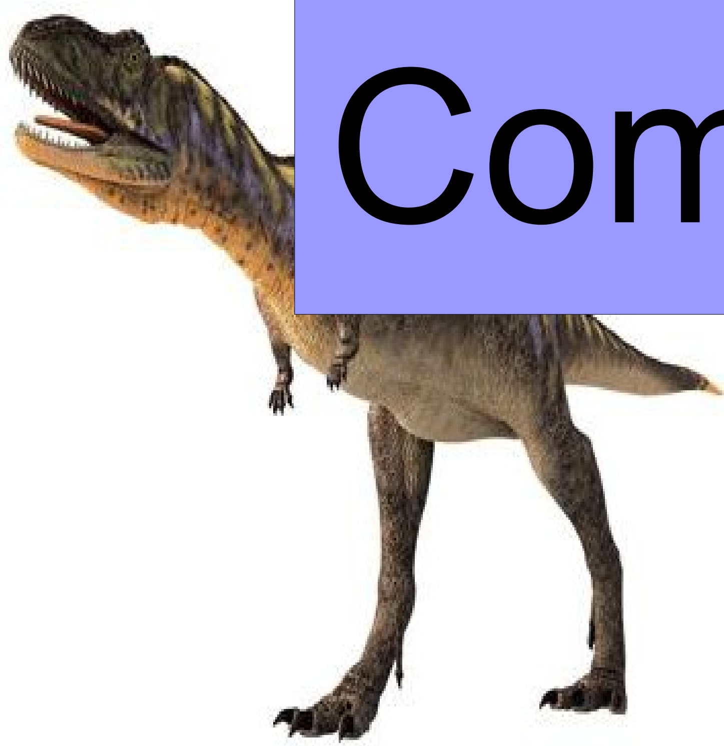
Entanglement and Complementarity



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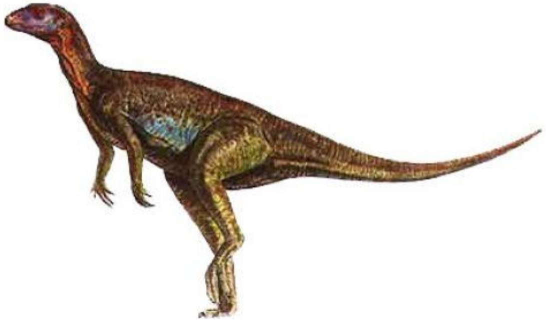
www.qubit.it

What I'm going to talk about

We always say that entangled states are more correlated... WHAT DOES IT MEAN exactly?



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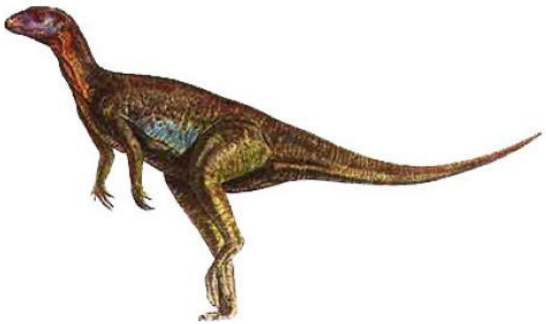
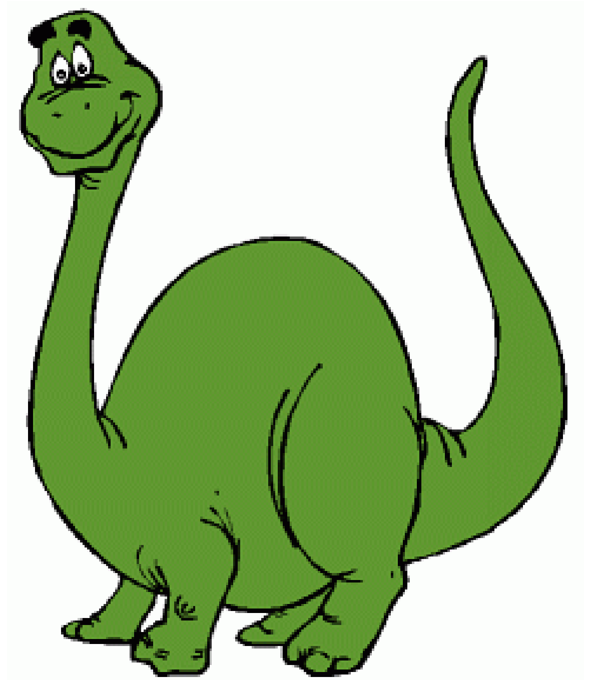
they have more correlations among complementary observables than separable ones



Usual approaches to study entanglement



- Non locality
- LOCC (?!?!)
- Bell inequality violations
- Enhanced precision in measurements
- etc.

Here: we use correlations among two (or more) COMPLEMENTARY PROPERTIES

different way to think about entanglement, as correlations among complementary properties

Remember: Complementary properties.

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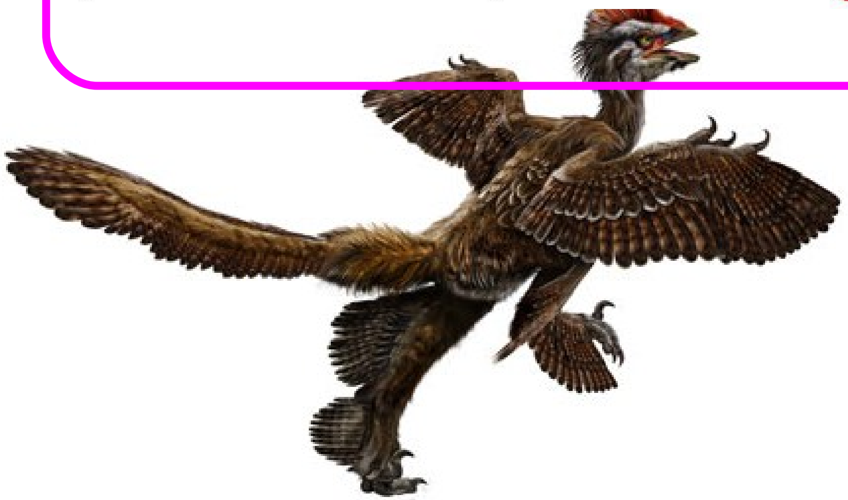
Two observables: the knowledge of one gives no knowledge of the other

$$A = \sum_a f(a)|a\rangle\langle a|$$

$$C = \sum_c g(c)|c\rangle\langle c|$$

$$\boxed{\left|\langle a|c\rangle\right|^2 = \frac{1}{d}}$$



simplest example:

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$$\frac{|00\rangle + |11\rangle}{\sqrt{2}} = \frac{|++\rangle + |--\rangle}{\sqrt{2}}$$

Maximally entangled state: perfect correlation BOTH on 0/1 and on +/-

$$|\pm\rangle \equiv \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

simplest example:

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}} = \frac{|++\rangle + |--\rangle}{\sqrt{2}}$$

Maximally entangled state: perfect correlation BOTH on 0/1 and on +/-

$$(|00\rangle\langle 00| + |11\rangle\langle 11|)/2$$

simplest example:

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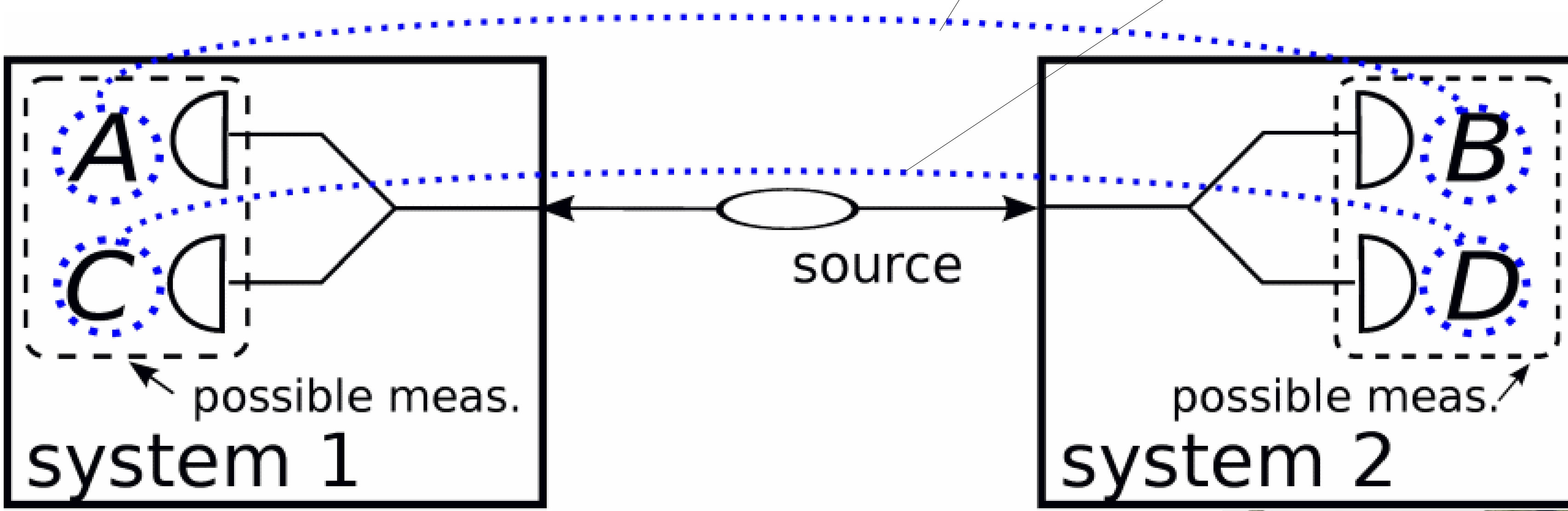
Maximally entangled state: perfect correlation BOTH on 0/1 and on +/-

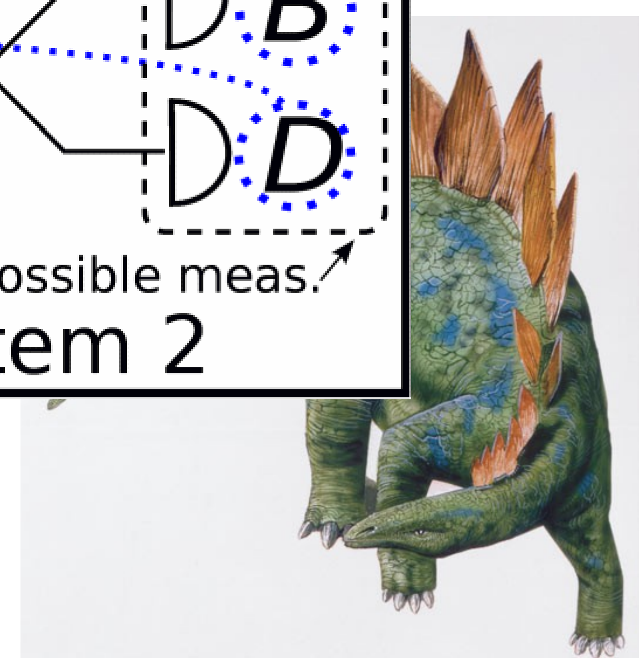
$$(|00\rangle\langle 00| + |11\rangle\langle 11|)/2 =$$
$$(|+\rangle\langle +| + |-\rangle\langle -|)/2 \otimes (|+\rangle\langle +| + |-\rangle\langle -|)/2$$

separable state: perfect correlation for 0/1, no correlation for +/-

Simple experiment

- On system 1 measure either *A* or *C*
- On system 2 measure either *B* or *D*
- Calculate correlations *A-B* and *C-D*





How to measure correlation?

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- Mutual information

$$I_{AB} = H(A) + H(B) - H(A,B)$$

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$$I_{AB} = H(A) + H(B) - H(A,B)$$

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$$\mathcal{C}_{AB} \equiv \frac{\langle AB\rangle - \langle A\rangle\langle B\rangle}{\sigma_A \sigma_B}$$

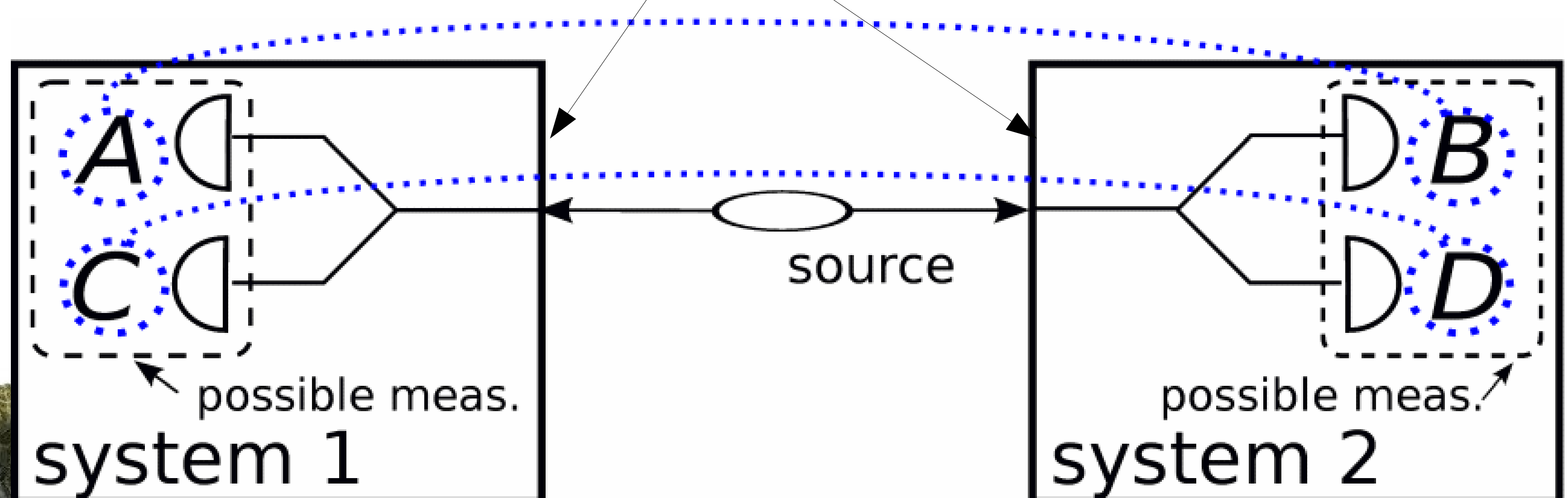
$|\mathcal{C}_{AB}| = 1 \Rightarrow$ perfect correlation or anticorrelation

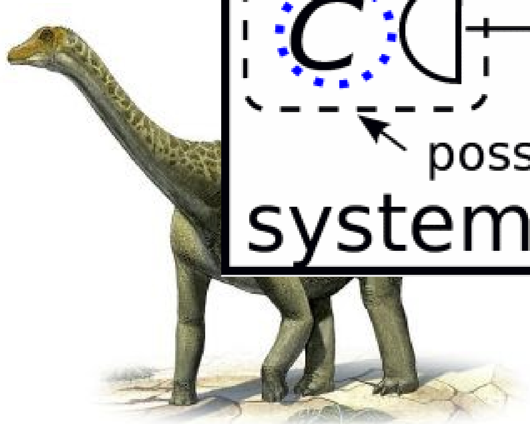
Use these to measure correlations among

2 complementary properties

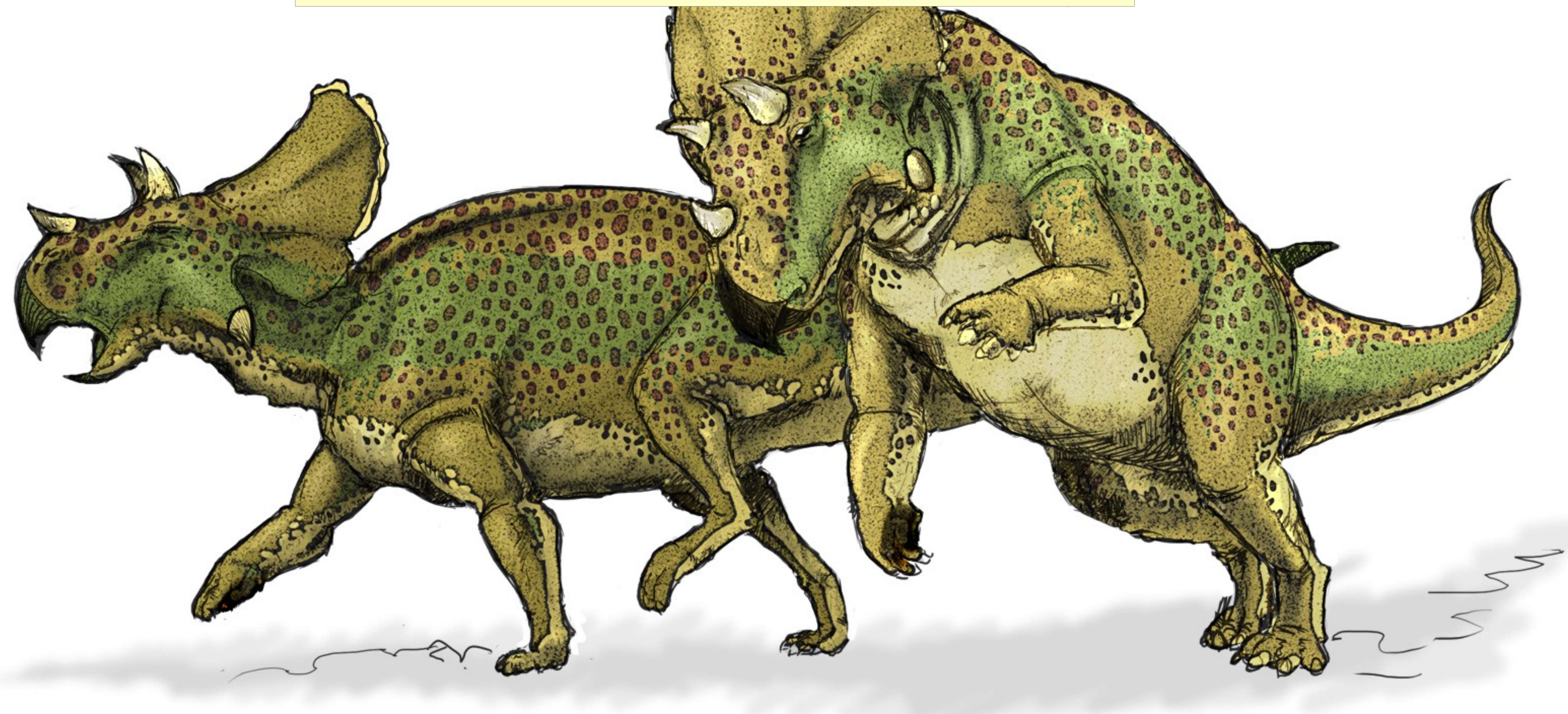
$$A \otimes B \xleftrightarrow{\text{complem to}} C \otimes D$$

of 2 systems





Some results...

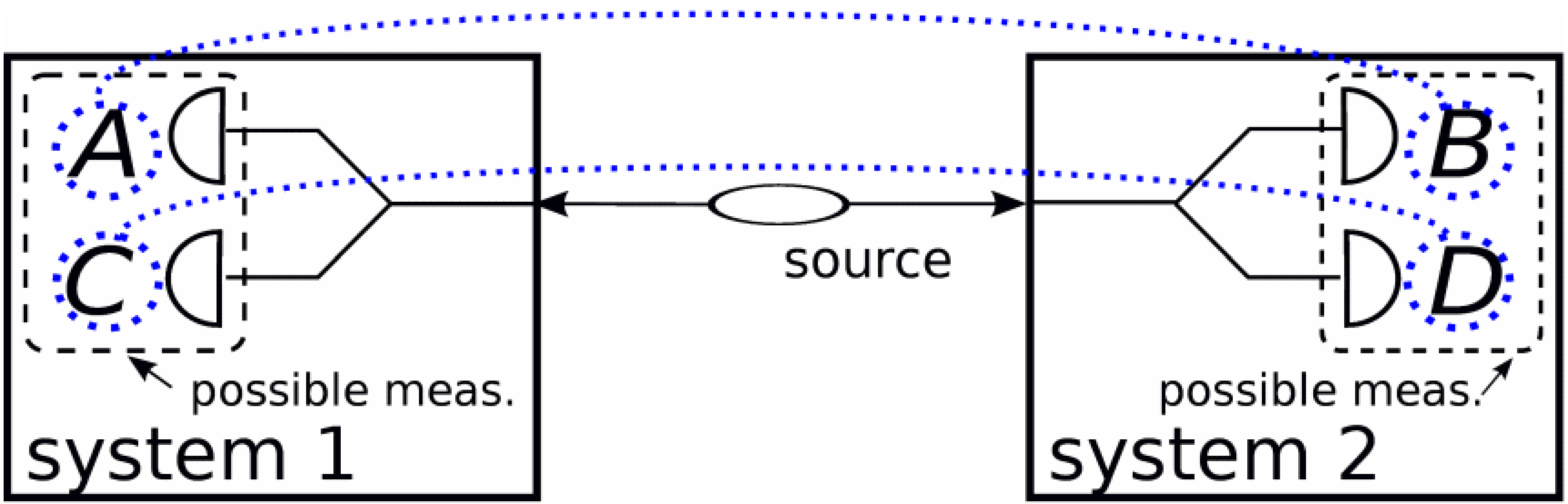


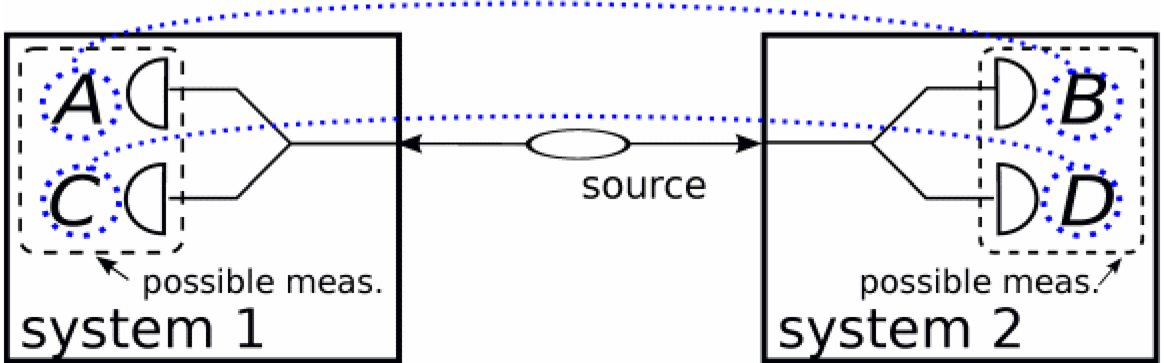
Start with mutual information

$$I_{AB} = H(A) + H(B) - H(A,B)$$

“total” correlation given by the sum

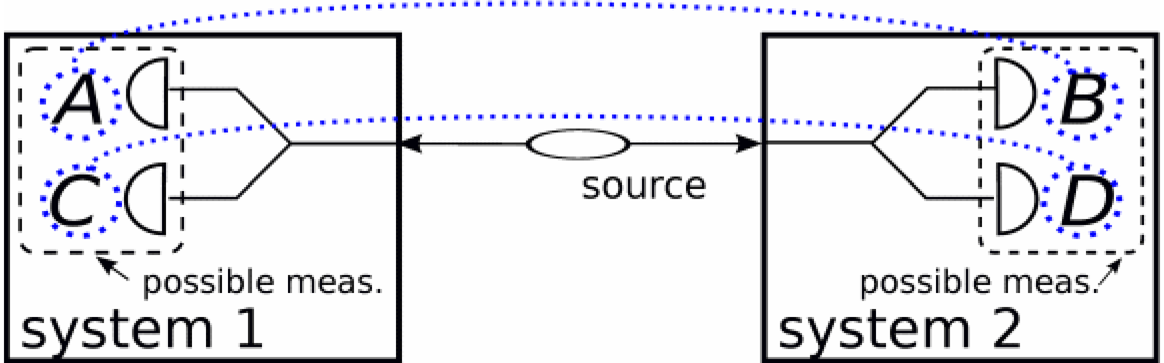
$$I_{AB} + I_{CD}$$





$A \otimes B \xleftrightarrow{\text{complem to}} C \otimes D$

The system state is **maximally entangled** iff perfect correlation on **both** *A-B* and *C-D*



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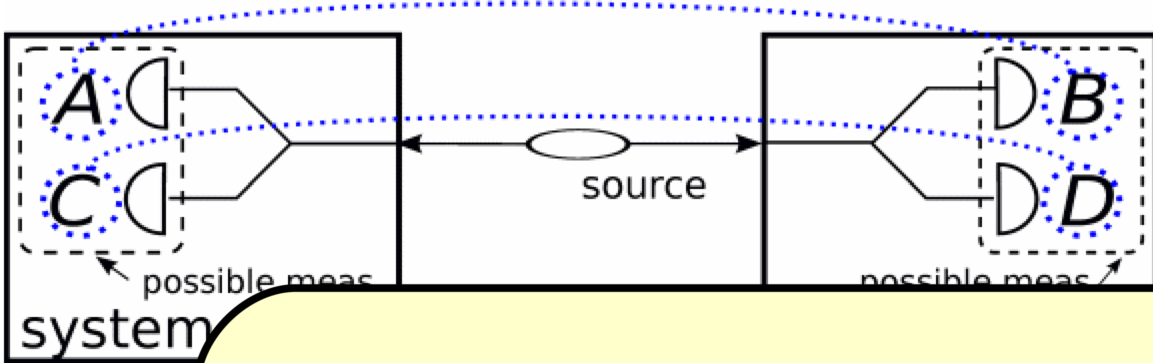
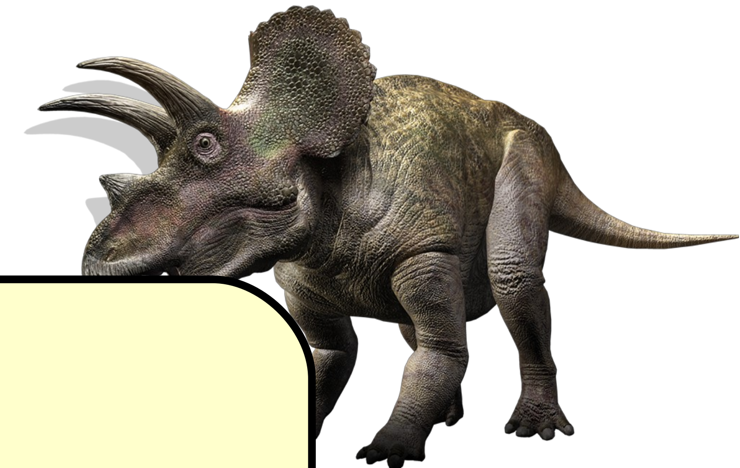
The system state is maximally entangled iff perfect correlation on both *A-B* and *C-D*

$$I_{AB} + I_{CD} = 2 \log d$$

(for some observ *ABCD*)

$$\Leftrightarrow \quad |\Psi_{12}\rangle \text{ maximally entangled}$$





$A \otimes$

$$I_{AB} \leqslant \log_2 d$$

$$I_{CD} \leqslant \log_2 d$$

T… **…mally**
e… …lation
on **both** *A-B* and *C-D*

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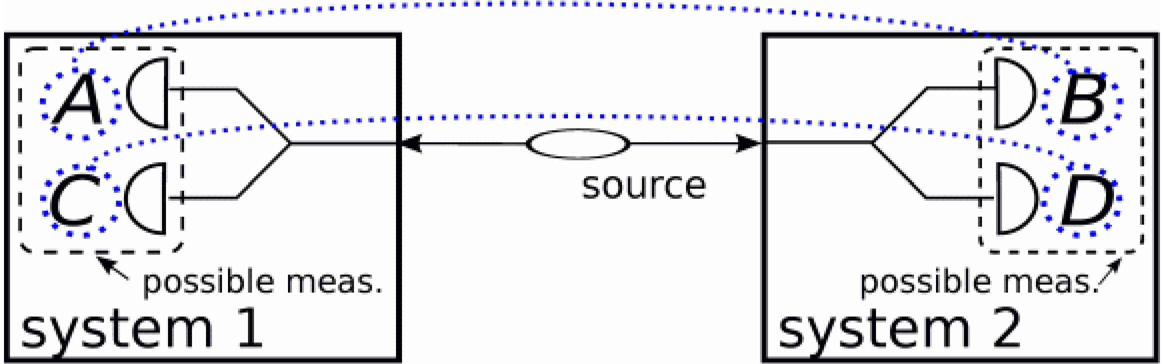
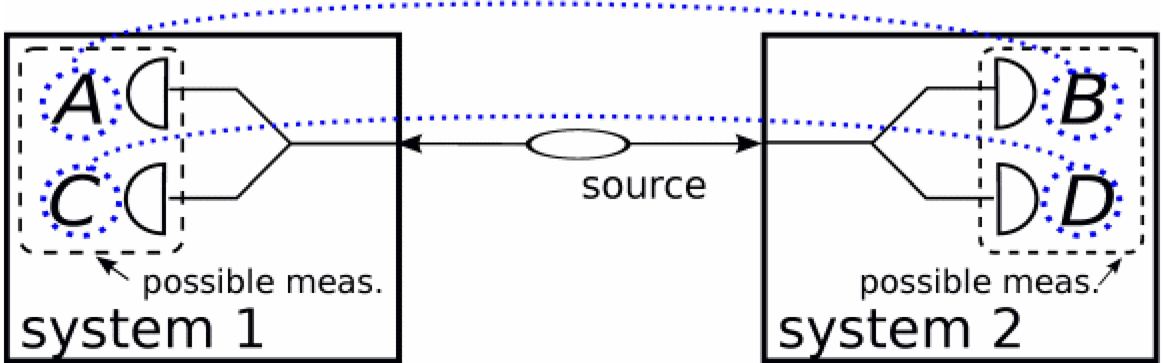


$$A \otimes B \xleftrightarrow{\text{complem to}} C \otimes D$$

The system state is **entangled** if correlations on **both** *A*-*B* and *C*-*D* are large enough



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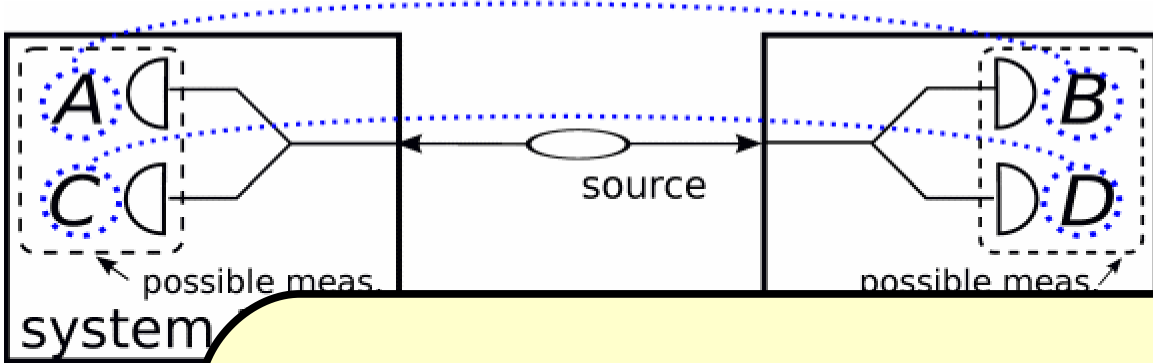
$$I_{AB} + I_{CD} > \log d \quad \Rightarrow$$

$$\rho_{12} \text{ ent}$$



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T… **…gled** if
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ρ_{12} ent

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Can the bound be made **tighter**?

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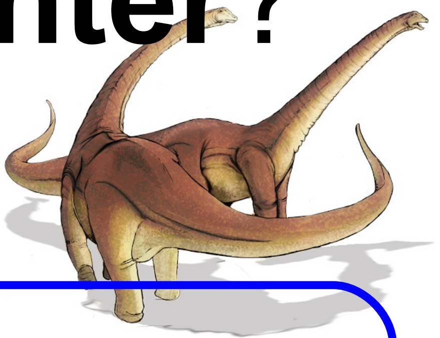


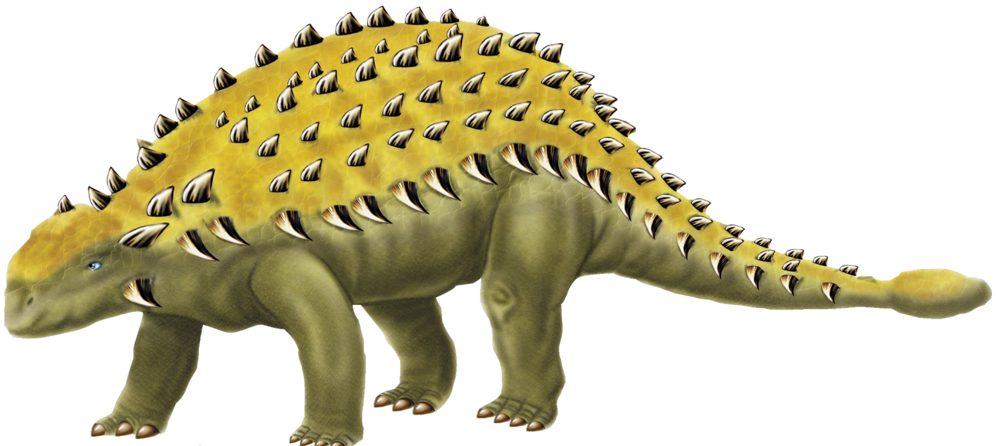
the **separable** state

$$\frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$$

saturates it: $I_{AB} + I_{CD} = \log d$

The systen state is **entangled** if correlations on **both** *A*-*B* and *C*-*D* are large enough

is the converse true?



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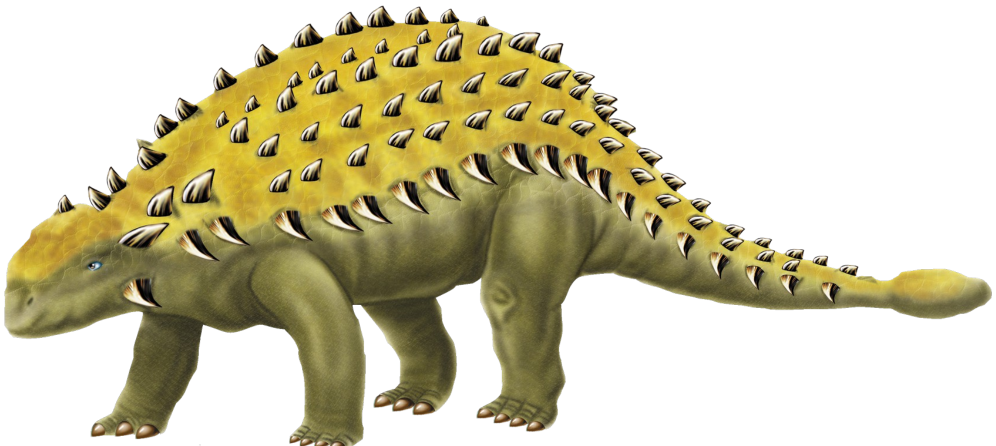

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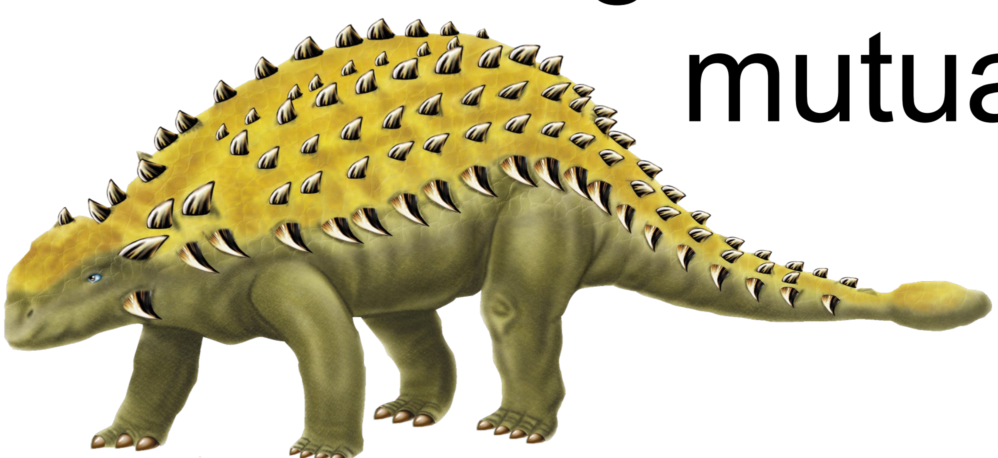
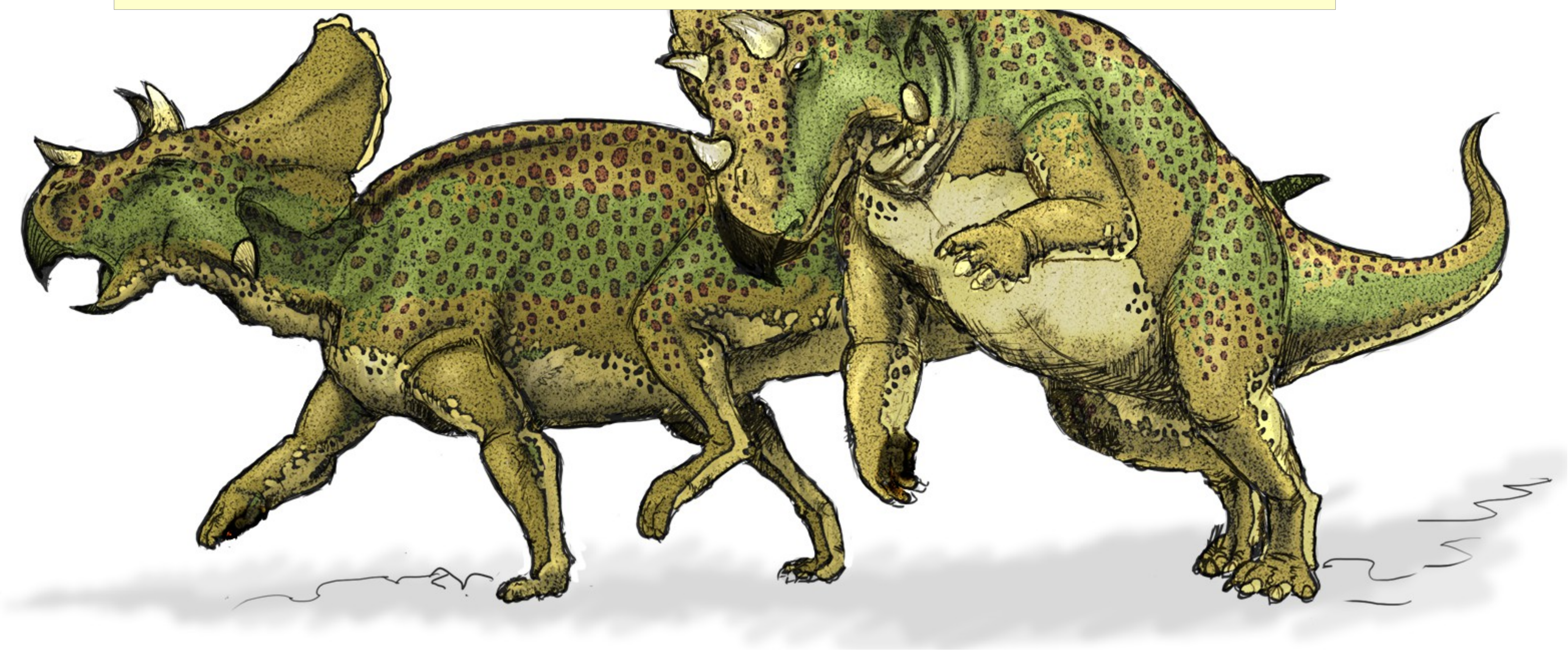
$$|\psi_{\epsilon}\rangle = \epsilon|00\rangle + \sqrt{1-\epsilon^2}|11\rangle$$

is entangled but has negligible mutual info for $\epsilon \to 0$



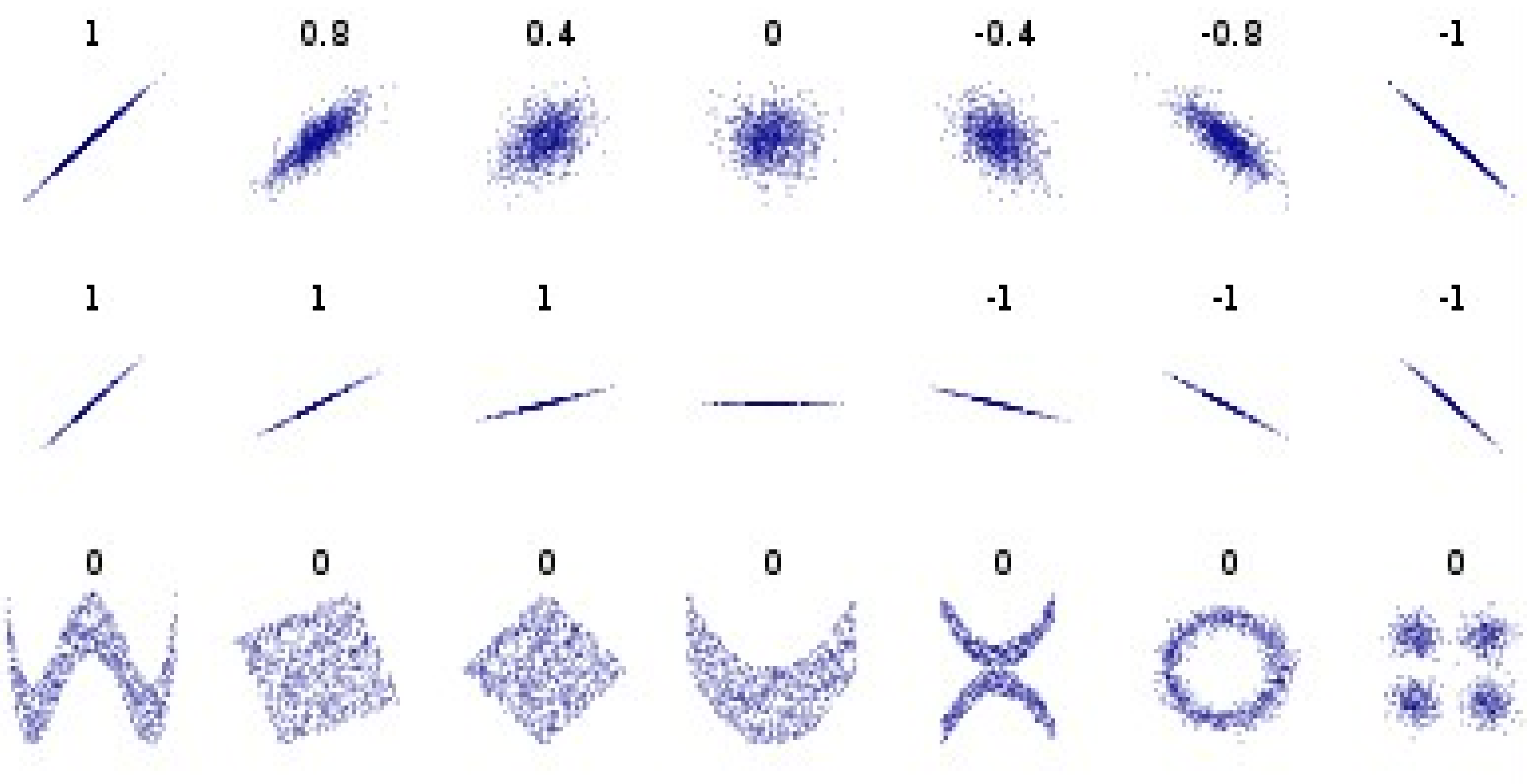
Another measure of correlation...



Pearson correlation coefficient

$$\mathcal{C}_{AB} \equiv \frac{\langle AB\rangle - \langle A\rangle\langle B\rangle}{\sigma_A \sigma_B}$$

$|\mathcal{C}_{AB}| = 1 \Rightarrow$ perfect correlation or anticorrelation



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... but its modulus is still $\leqslant |1|$:

Pearson correlation coefficient

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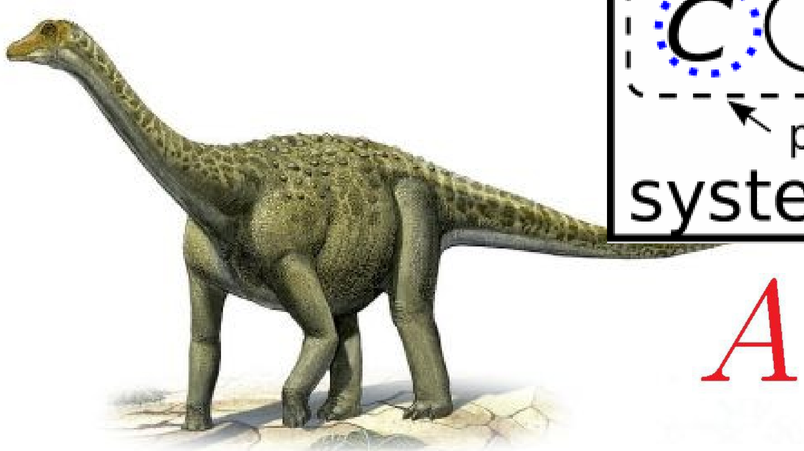
it can be **complex** for quantum expectation values

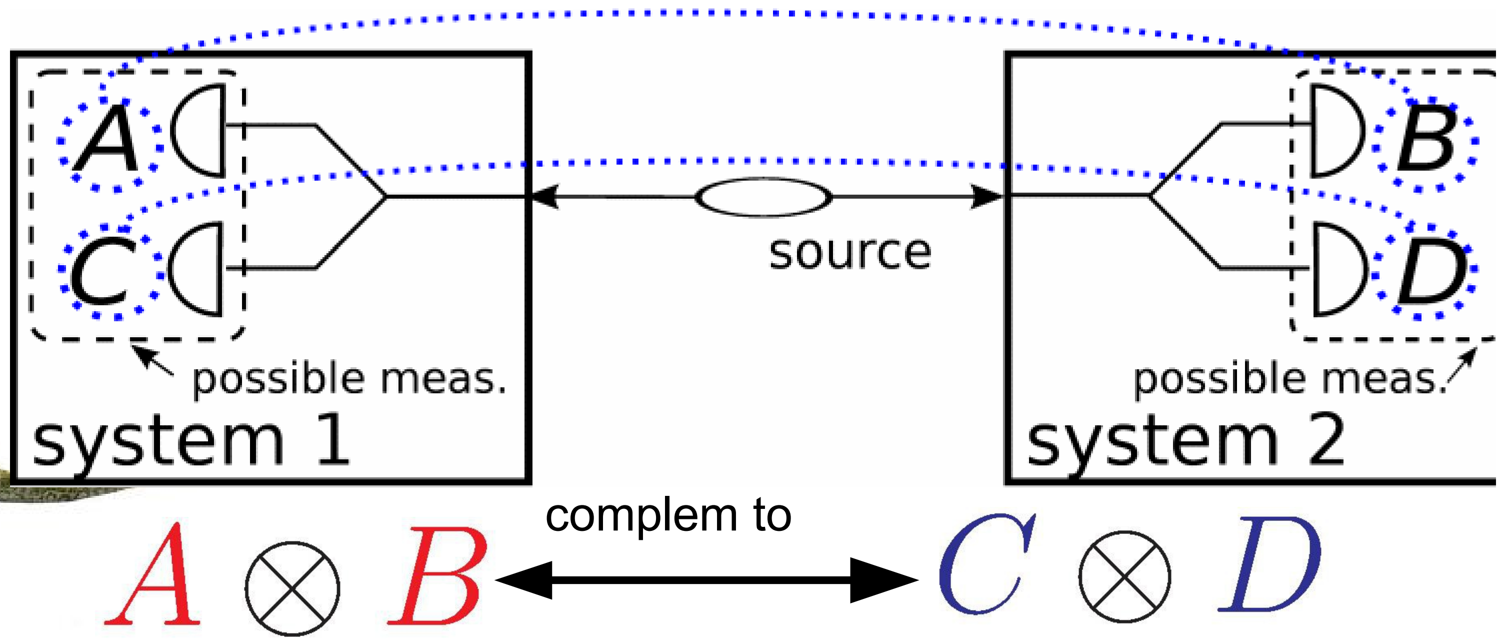
not a problem for us: *A* and *B* commute, so it's **REAL**

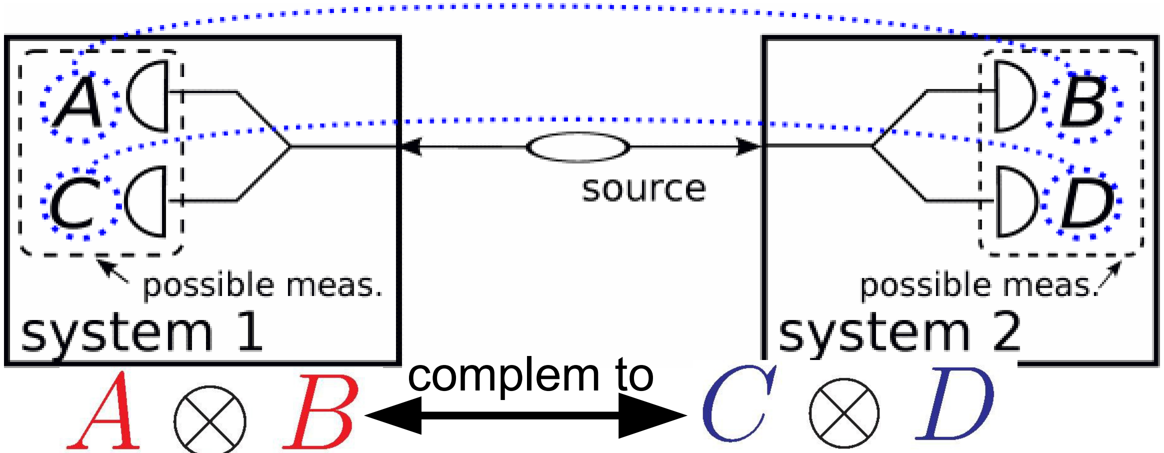
$$A \otimes B = A \otimes \mathbb{1} + \mathbb{1} \otimes B$$

Total correlation: again use the sum

$$|C_{AB}| + |C_{CD}|$$







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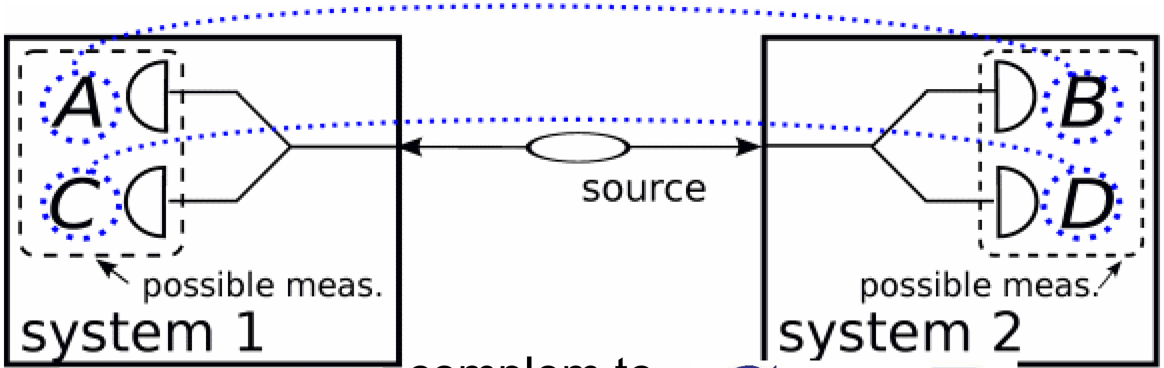

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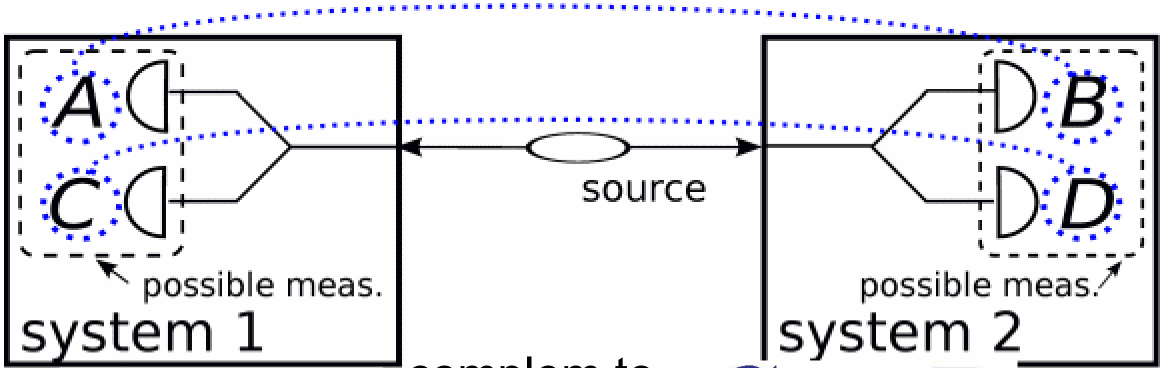
True also using Pearson! (for linear observables: Pearson measures only linear correl)



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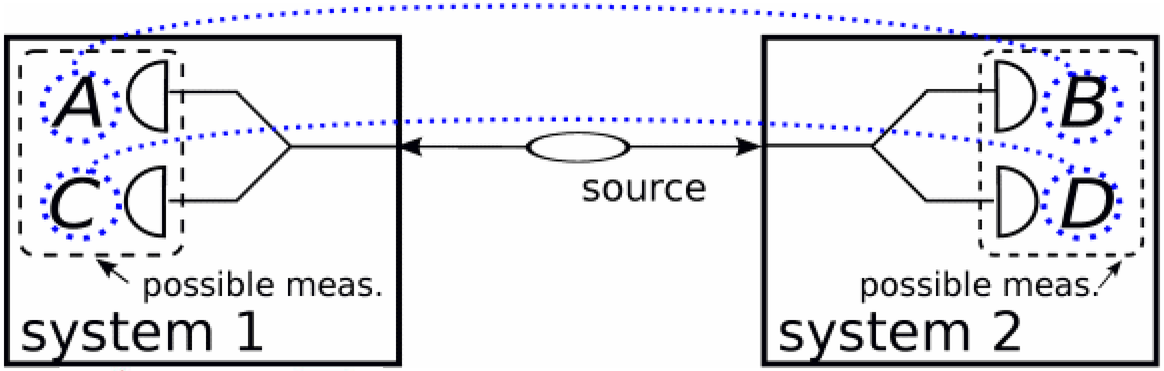
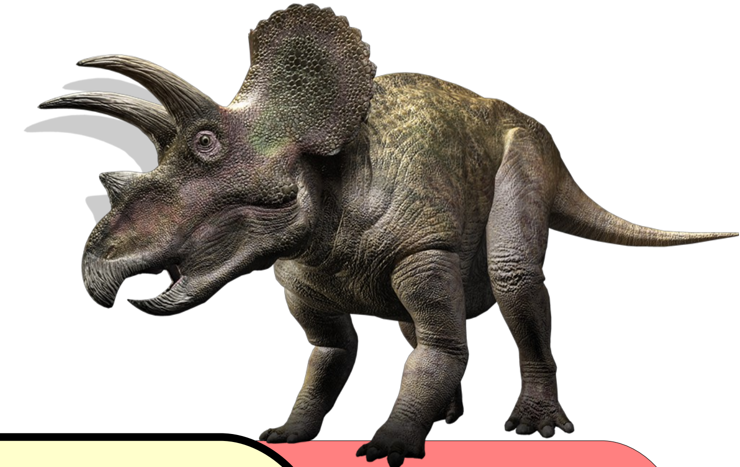


$A \otimes B$



T ... mally

e ... ation

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Tru ... linear

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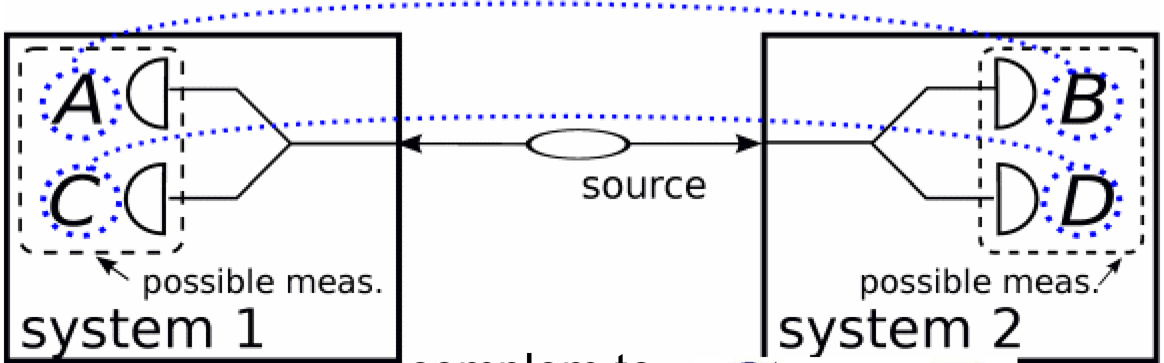

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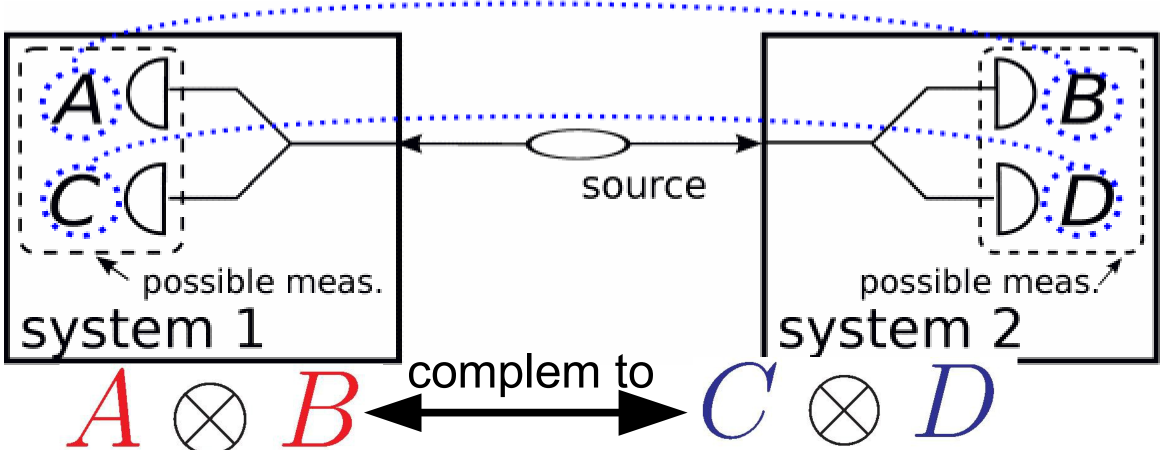

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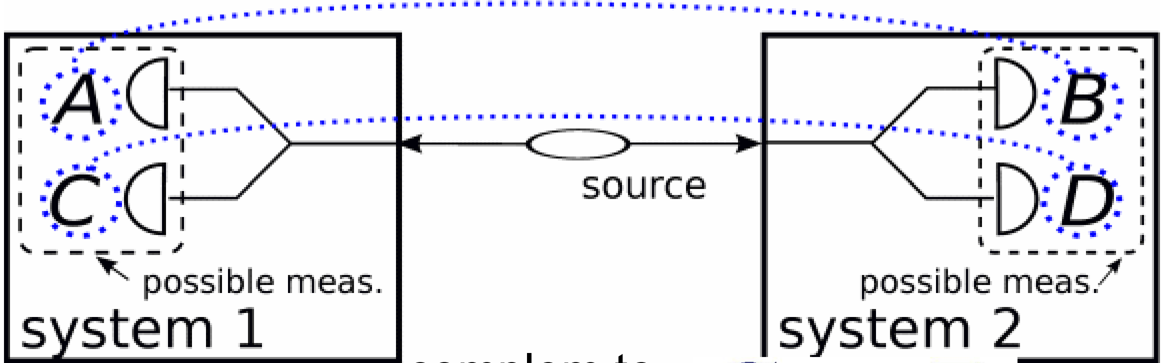
CONJECTURE: we don't know if it's true also using Pearson!



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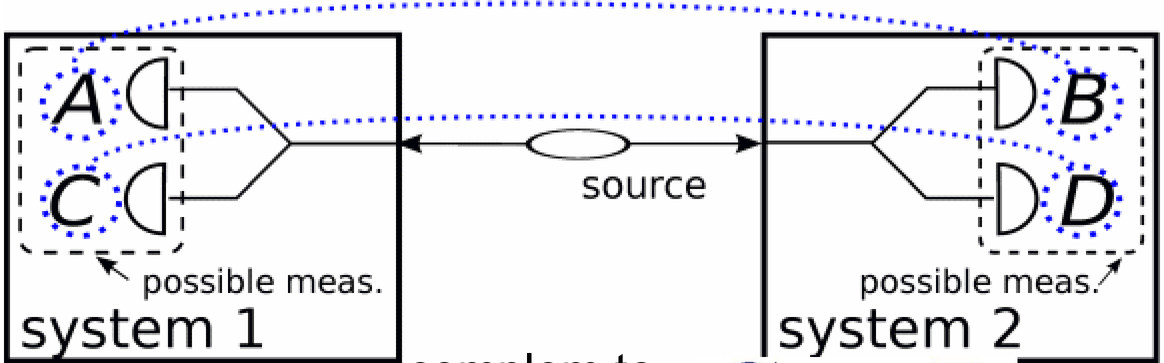

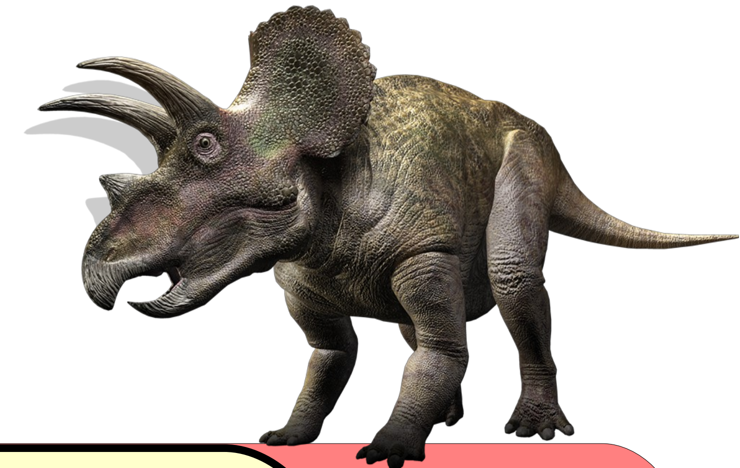

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Th... **...gled** if
co... ...d *C*-*D*

$$|\mathcal{C}_{AB}| \leqslant 1$$

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co... ...ow if
it's true also using Pearson!

$$|\mathcal{C}_{AB}| + |\mathcal{C}_{CD}| > 1 \Rightarrow \rho_{12} \text{ ent}$$

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Conjecture: $|\mathcal{C}_{AB}| + |\mathcal{C}_{CD}| > 1 \;\Rightarrow$ state is ent.

Again, the inequality is tight:

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Again, the inequality is tight:

separable state $\dfrac{|00\rangle\langle 00| + |11\rangle\langle 11|}{2}$

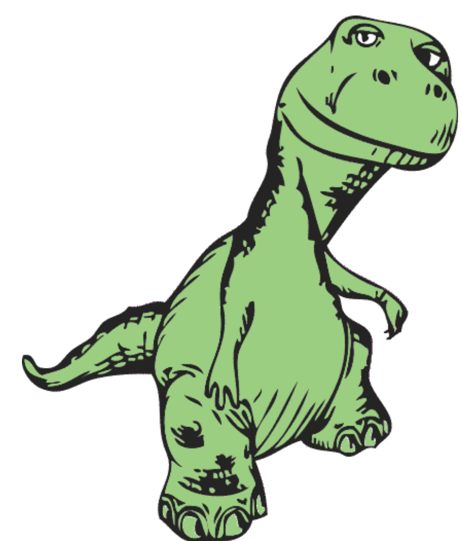
$$|\mathcal{C}_{AB}| + |\mathcal{C}_{CD}| = 1$$

(perfect correl on one basis,
no correl on the complem)

Is the Pearson correlation weaker than the mutual info?

only linear correlations

all correlations



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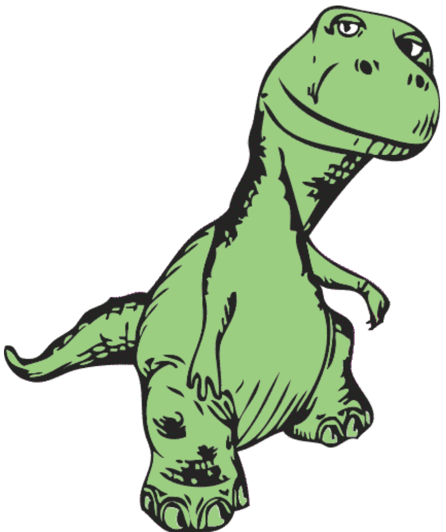
Has negligible mutual info for $\epsilon \to 0$



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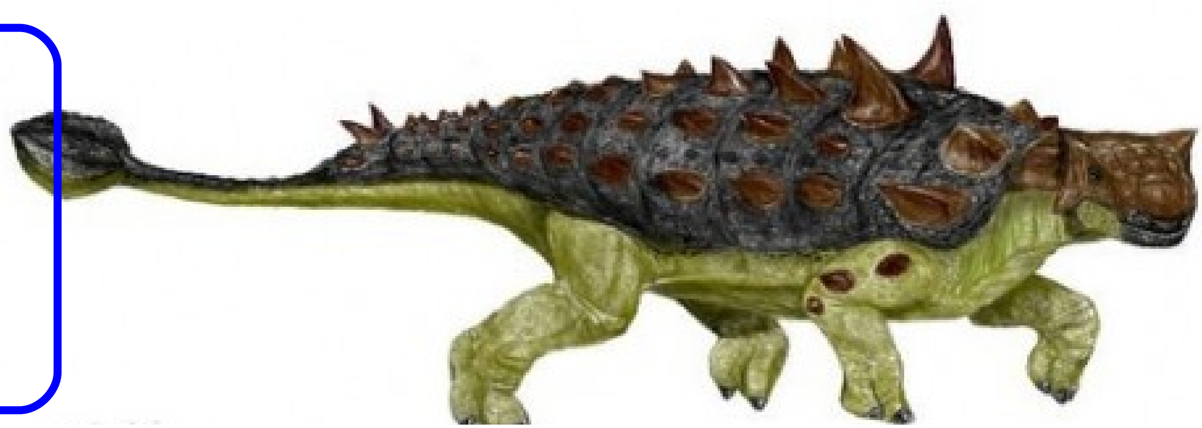
Has negligible mutual info for $\epsilon \to 0$

but Pearson correlation **always >1!**

Simple criterion for entanglement detection!!

Just measure two complementary properties. Are the correlations greater than perfect correlation on one?

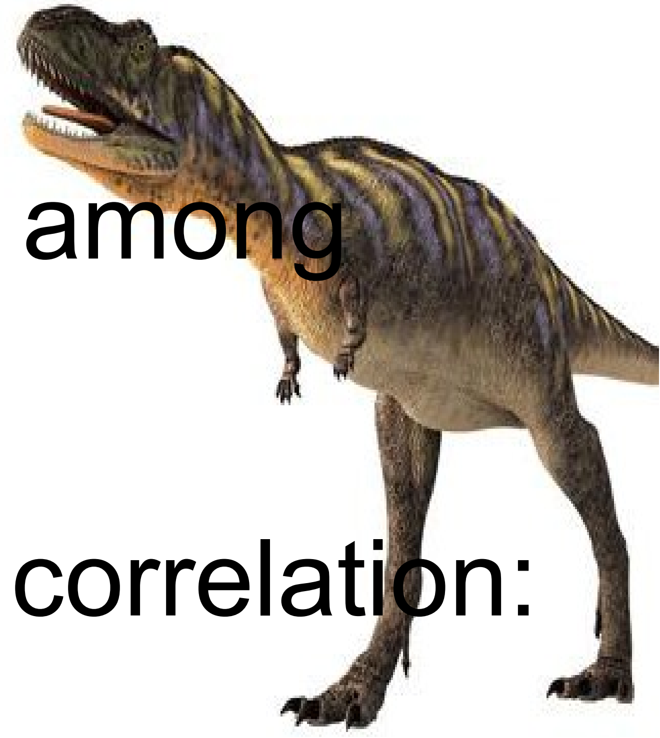
$\Rightarrow$ The state is entangled!



Simple to measure and simple to optimize.

Unfortunately: not very effective in finding entanglement in random states

What did I say?!?

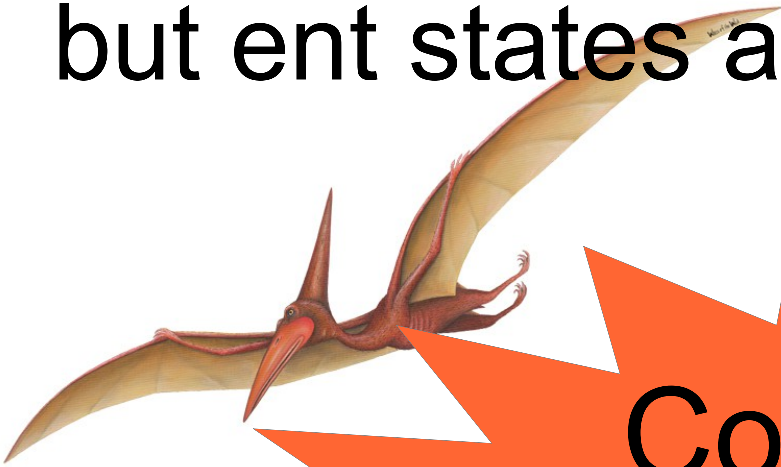


- Entanglement as correlation among complementary observables

- Using different measures of correlation:
    - Mutual info
    - Pearson correlation

- Some theorems and some conjectures

The most correlated states are entangled but ent states are not the most correlated



Correlations on complementary prop. help understanding entanglement

Lorenzo Maccone
maccone@unipv.it

PRL **114** 130401