Weak Measurements

From incompatible observables measurement to contextuality tests

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MIQC2

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Weak measurements [Aharonov et al., PRL 60 (1988)]: little information is extracted from a single measurement, but the state does NOT collapse.

Weak value: $$\langle \widehat{A} \rangle_w = \frac{\langle \psi_f | \widehat{A} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}$$

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$\widehat{X}$ and $\widehat{P}$ canonically conjugated

In the weak interaction regime approximation:

$$\langle \widehat{X} \rangle = \frac{\langle \phi_{out} | \widehat{X} | \phi_{out} \rangle}{\langle \psi_i | \widehat{\Pi}_f | \psi_i \rangle} = g \ \mathrm{Re}[\langle \widehat{A} \rangle_w]$$

Weak measurements

Some interesting properties:

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- $\mathrm{Im}[\langle\widehat{A}\rangle_w]$: arising from disturbance related to the von Neumann coupling [Dressel and Jordan, PRA 85 (2012)]

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- Expectation values as averages of weak values [Aharonov and Botero, PRA 72 (2005)]

$$\langle A\rangle_i = \sum_f |\langle\psi_i|\psi_f\rangle|^2\ \langle\widehat{A}\rangle_w$$

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  $\langle A\rangle_i = \sum_f |\langle\psi_i|\psi_f\rangle|^2 \; \langle\widehat{A}\rangle_w$
- POVMs can be realized as a sequence of weak values [Oreshkov and Brun, PRL 95 (2005)]

Possible applications

- **Metrology**:

- **Foundations of Quantum Mechanics**:

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- Amplification of measurement of coupling strength:
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  - Angular deflection [Dixon et al.]
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- Tests of quantum contextuality [Pusey]
- Hints on Quantum Mechanics interpretations [TSVF, Aharonov et al., …]

"Sharp" measurement in QM

Standard "sharp" measurement:

$$\widehat{A} = \sum_n \lambda_n \widehat{\Pi}_n \qquad \widehat{\Pi}_n = |\psi_n\rangle\langle\psi_n| \qquad \mathrm{Tr}[\widehat{A}\widehat{\rho}] = \sum_n \lambda_n \mathrm{Tr}[\widehat{\Pi}_n \widehat{\rho}]$$

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Wave function collapse

Non-commuting observables can’t be simultaneously measured!

$$\mathrm{Tr}\left[\widehat{\Pi}_n\left(\widehat{\Pi}_k\widehat{\rho}\widehat{\Pi}_k\right)\right] = \mathrm{Prob}(\psi_n|\psi_k)\mathrm{Prob}(\psi_k|\rho)$$

Joint and sequential weak measurements

Weak values *«challenge one of the canonical dicta of QM: that non commuting observables cannot be simultaneously measured»*

«the fact that one hardly disturbs the systems in making WM means that one can in principle measure different variables in succession» [Mitchison, Jozsa and Popescu, PRA 76 (2007)]

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Joint weak measurement

Resch et al., PRL 92, 130402 (2004)

$$\widehat{U} = \exp[-i(g_x \widehat{A} \otimes \widehat{P}_x + g_y \widehat{B} \otimes \widehat{P}_y)]$$

$$\langle \widehat{X}\widehat{Y} \rangle = \frac{1}{4} g_x g_y \mathrm{Re} \left[\langle \widehat{A}\widehat{B} + \widehat{B}\widehat{A} \rangle_w + \right.$$
$$\left. + 2 \langle \widehat{A} \rangle_w^* \langle \widehat{B} \rangle_w \right]$$

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Sequential weak measurement

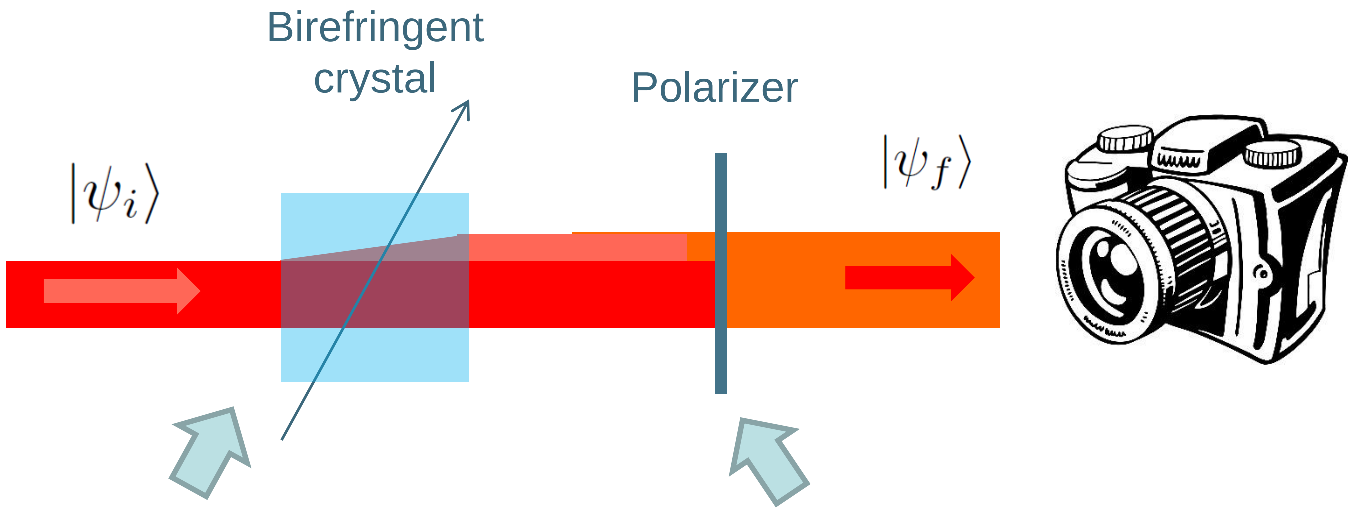
Mitchinson et al., PRA 76, 062105 (2007)

$$\widehat{U}_y = \exp(-i g_y \widehat{B} \otimes \widehat{P}_y)$$

$$\widehat{U}_x = \exp(-i g_x \widehat{A} \otimes \widehat{P}_x)$$

$$\langle \widehat{X}\widehat{Y} \rangle = \tfrac{1}{2} g_x g_y \mathrm{Re}\left[\langle \widehat{A}\widehat{B} \rangle_w + \langle \widehat{A} \rangle_w^* \langle \widehat{B} \rangle_w \right]$$

Weak measurement implementation



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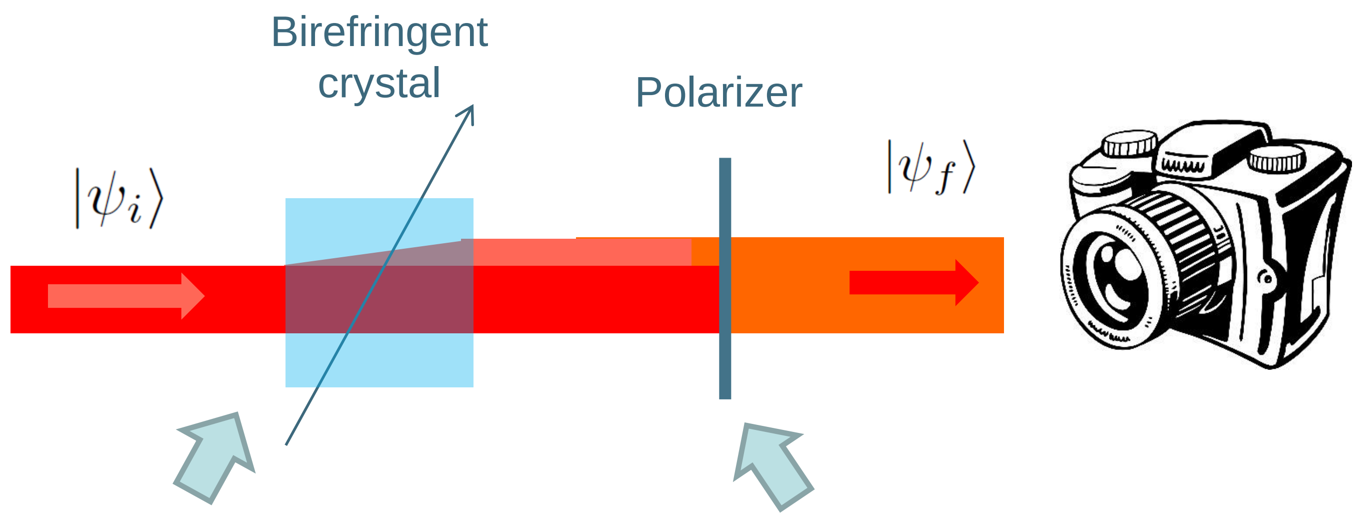

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We measure the position observable $\widehat{X}$,
canonically coniugated to the pointer observable $\widehat{P}$

$$\langle\widehat{X}\rangle = g\mathrm{Re}[\langle\widehat{\Pi}_V\rangle_w]$$

Sequential weak measurement

$$\hat{A} \longrightarrow \widehat{\Pi}_V = |V\rangle\langle V|$$

$$\hat{B} \longrightarrow \widehat{\Pi}_\psi = |\psi\rangle\langle\psi|$$

$$|\psi\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle$$

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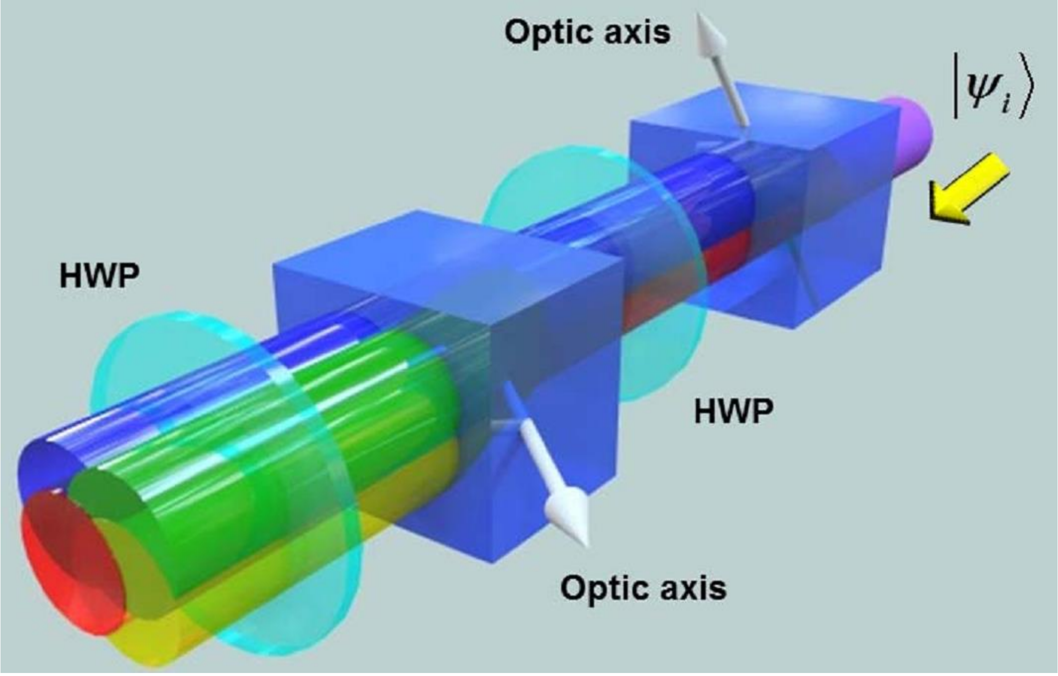

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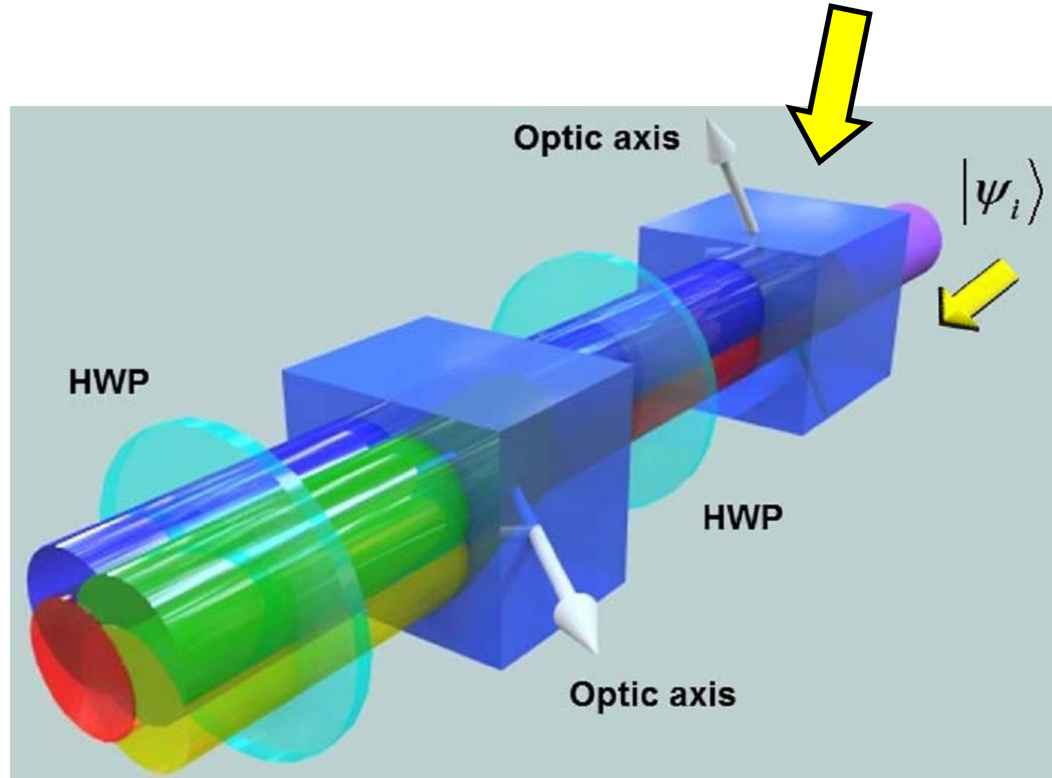

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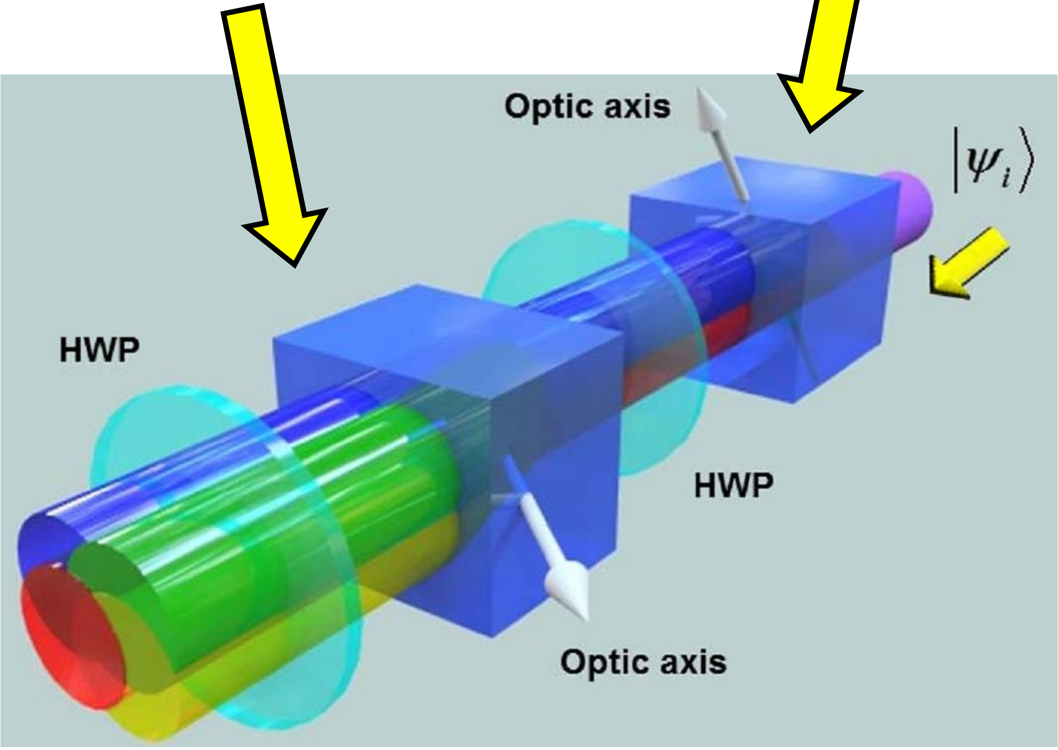

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Linearly polarized pre- and post-selection states $|\psi_i\rangle,\ |\psi_f\rangle$

$$\begin{cases} \langle\widehat{X}\rangle = g_x\langle\widehat{\Pi}_\psi\rangle_w \\ \langle\widehat{Y}\rangle = g_y\langle\widehat{\Pi}_V\rangle_w \\ \langle\widehat{X}\widehat{Y}\rangle = \frac{1}{2}g_x g_y\left(\langle\widehat{\Pi}_\psi\widehat{\Pi}_V\rangle_w + \langle\widehat{\Pi}_\psi\rangle_w\langle\widehat{\Pi}_V\rangle_w\right) \end{cases}$$

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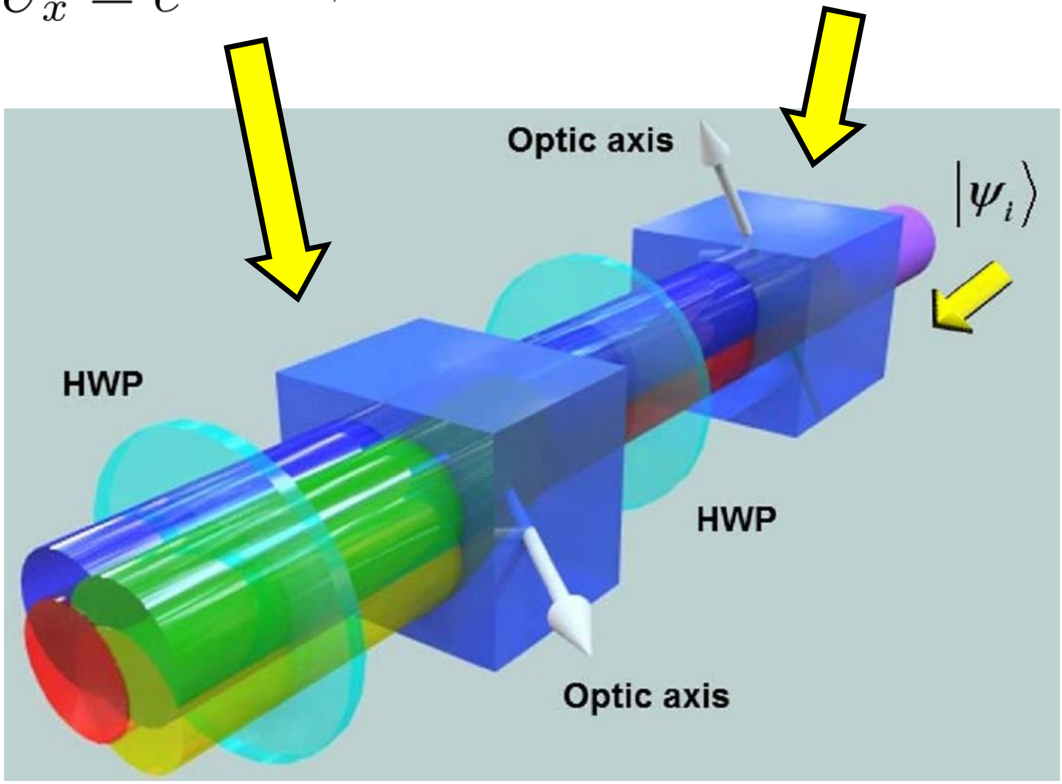
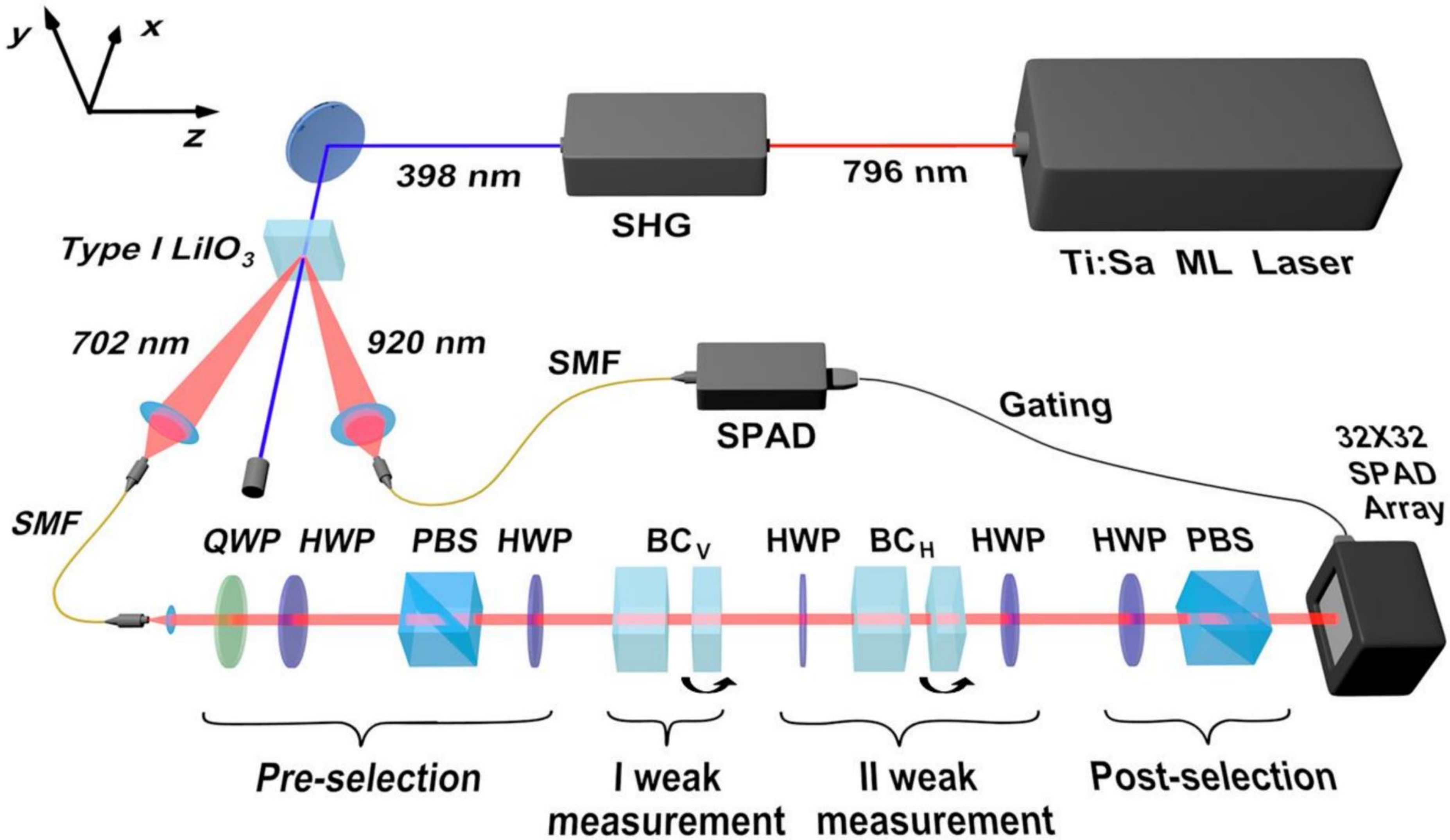


SETUP



y
x
z
398 nm
SHG
796 nm
Ti:Sa ML Laser
Type I LiIO$_3$
702 nm
920 nm
SMF
SPAD
Gating
32X32 SPAD Array
SMF
QWP HWP PBS HWP
BC$_V$
HWP BC$_H$ HWP
HWP PBS
Pre-selection
I weak measurement
II weak measurement
Post-selection

SETUP

SPAD lab

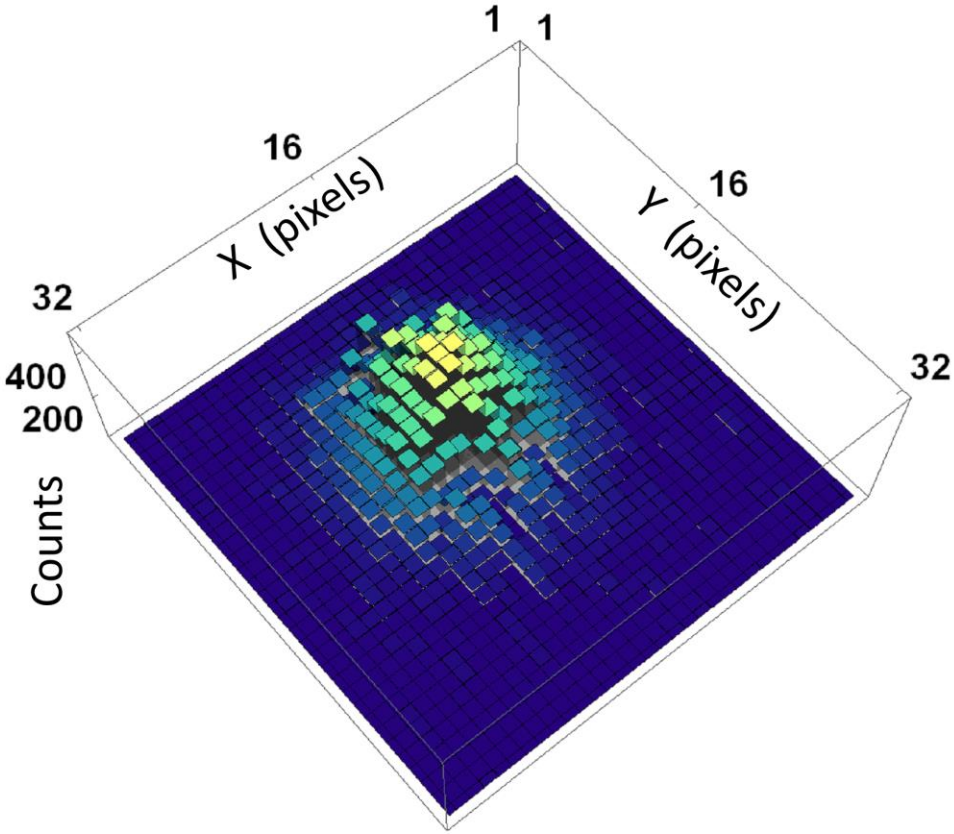
32x32 SPAD+TDC camera

Features

- Multi-modality: photon-counting, 2D imaging 3D time-of-flight ranging, TCSPC (time-correlated single-photon counting)
- Image dimension: 32x32 (1024) pixels
- In-pixel counter: 6 bit (photon-counting)
- In-pixel TDC: 10 bit (photon-timing)
- Max frame rate: 100,000 fps (burst) and 10,000 fps (continuous)
- Timing resolution: 312 ps – 0.9 ns
- Full scale range: 320 ns – 0.92 μs
- Hardware interface: USB 2.0
- Software interface: Matlab

Fig. 1: SPAD camera for 2D imaging, 3D ranging and TCSPC photon-counting.

SPAD array output VS. theoretical prediction



Typical single data acquisition obtained with our 32x32 SPAD camera (after noise subtraction)

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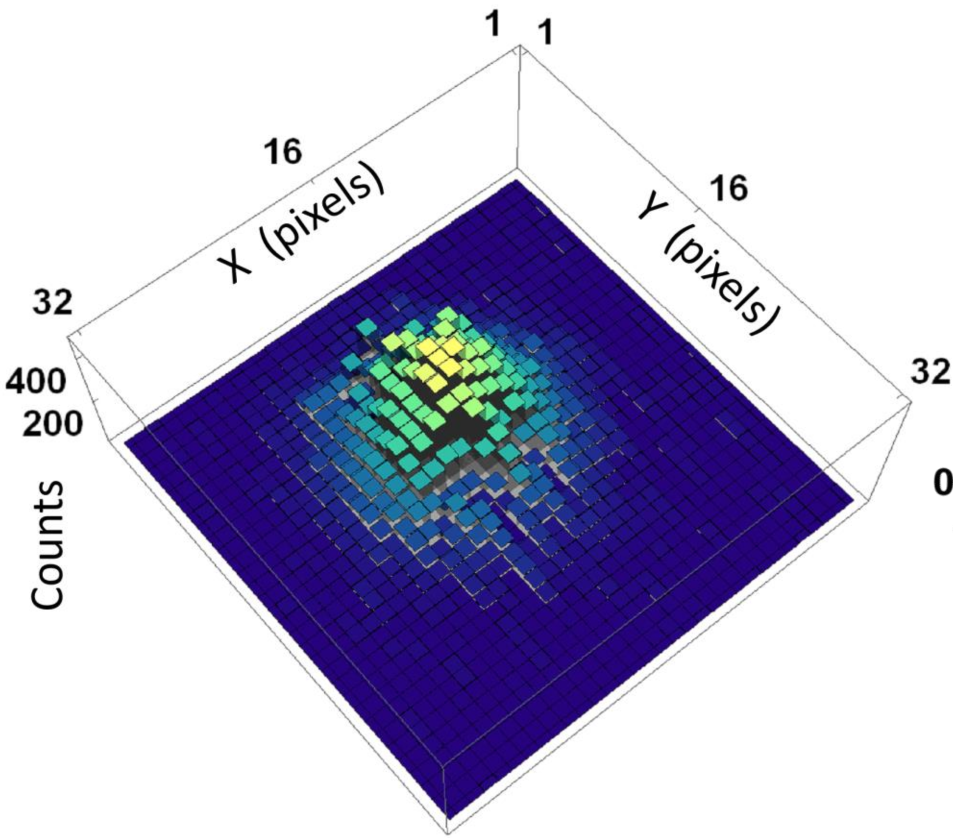

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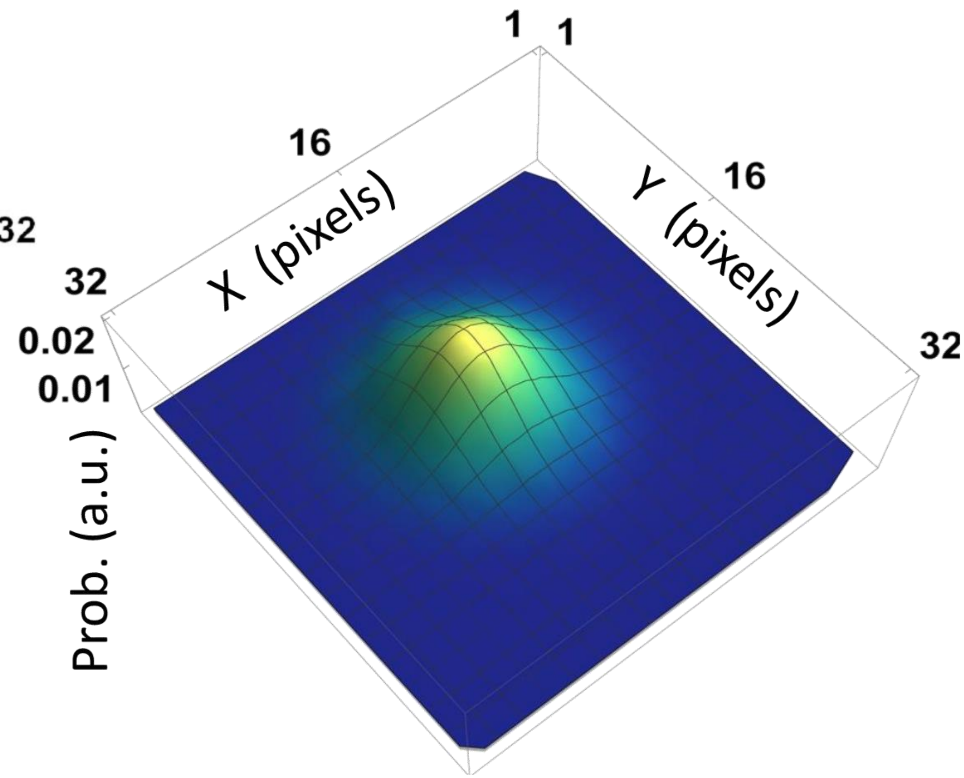

Corresponding predicted probability distribution calculated according to the theory

Results

Measured weak values (data points) compared with the theoretical predictions

$$\widehat{\Pi}_V = |V\rangle\langle V| \qquad \widehat{\Pi}_\psi = |\psi\rangle\langle\psi| \qquad (|\psi\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle)$$

$$|\psi_i\rangle = 0.588|H\rangle + 0.809|V\rangle \qquad |\psi_f\rangle = |H\rangle$$

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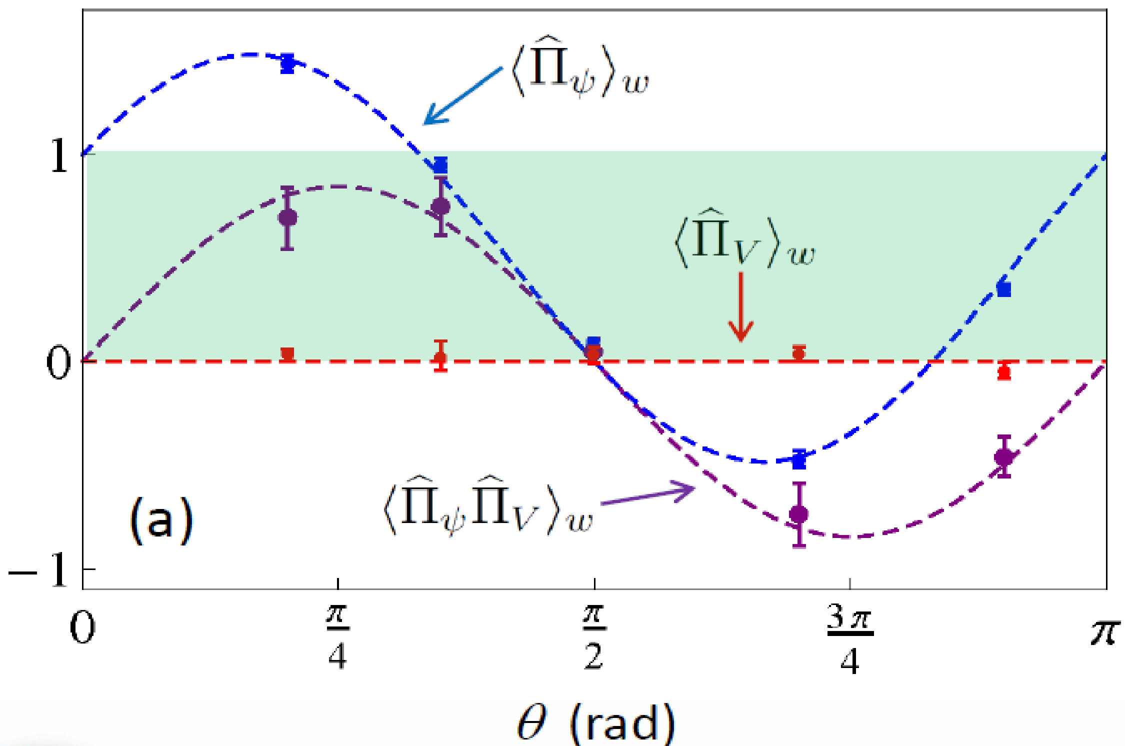

arXiv:1508.03220

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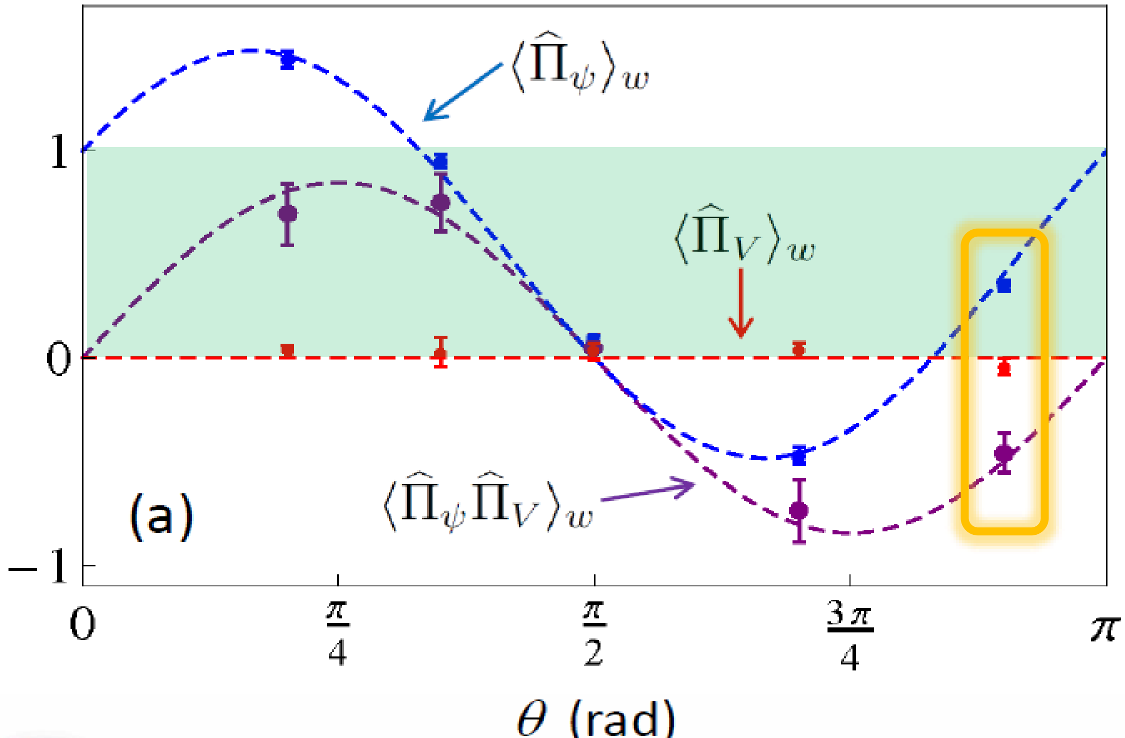

$$\langle\widehat{\Pi}_V\rangle_w = 0.04 \pm 0.03$$
$$\langle\widehat{\Pi}_\psi\rangle_w = 0.35 \pm 0.04$$
$$\langle\widehat{\Pi}_\psi\widehat{\Pi}_V\rangle_w = -0.46 \pm 0.10$$

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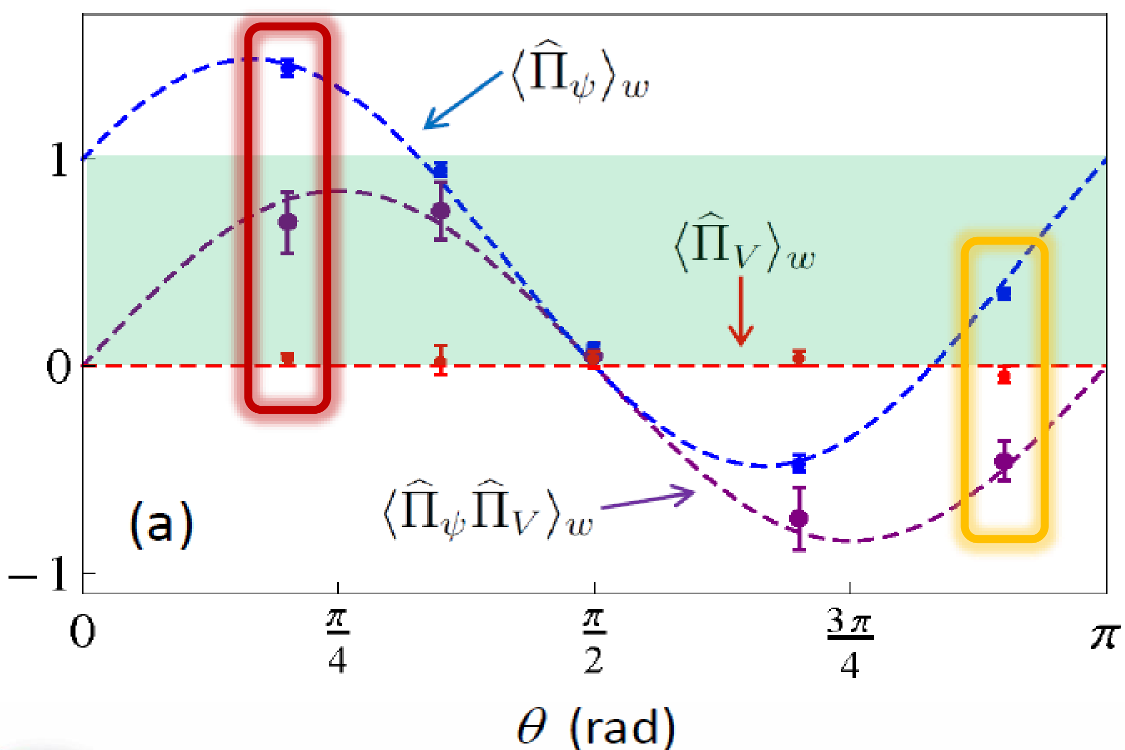
$$|\psi_i\rangle = 0.588|H\rangle + 0.809|V\rangle \qquad |\psi_f\rangle = |H\rangle$$



$$\langle\widehat{\Pi}_V\rangle_w = 0.03 \pm 0.03$$
$$\langle\widehat{\Pi}_\psi\rangle_w = 1.44 \pm 0.04$$
$$\langle\widehat{\Pi}_\psi\widehat{\Pi}_V\rangle_w = 0.69 \pm 0.15$$

$$\langle\widehat{\Pi}_V\rangle_w = 0.04 \pm 0.03$$
$$\langle\widehat{\Pi}_\psi\rangle_w = 0.35 \pm 0.04$$
$$\langle\widehat{\Pi}_\psi\widehat{\Pi}_V\rangle_w = -0.46 \pm 0.10$$

arXiv:1508.03220

Results

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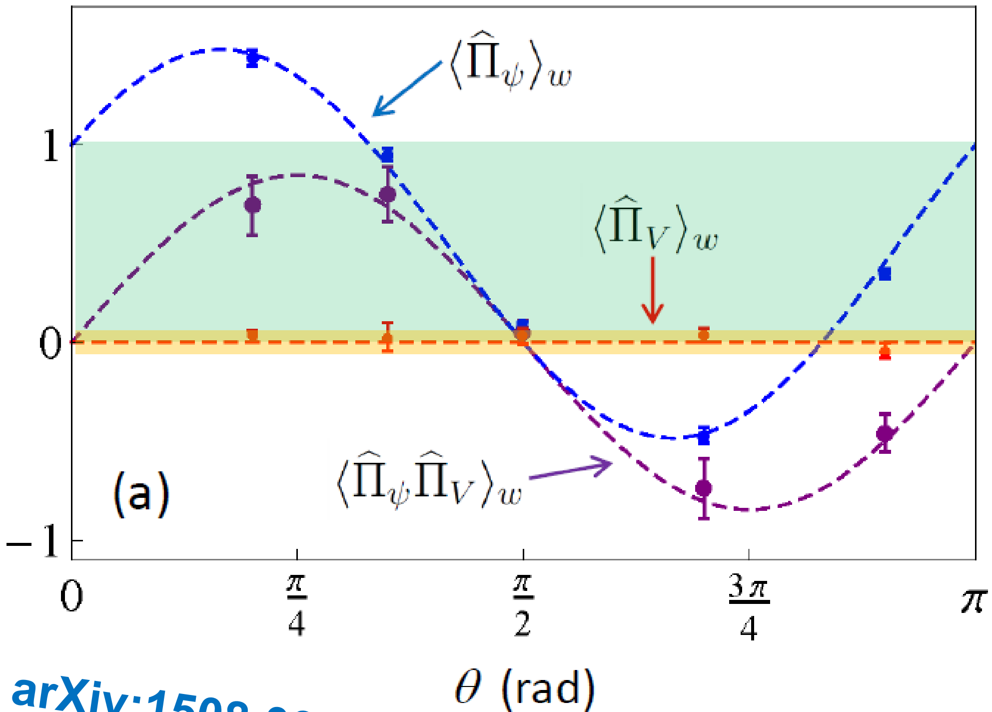

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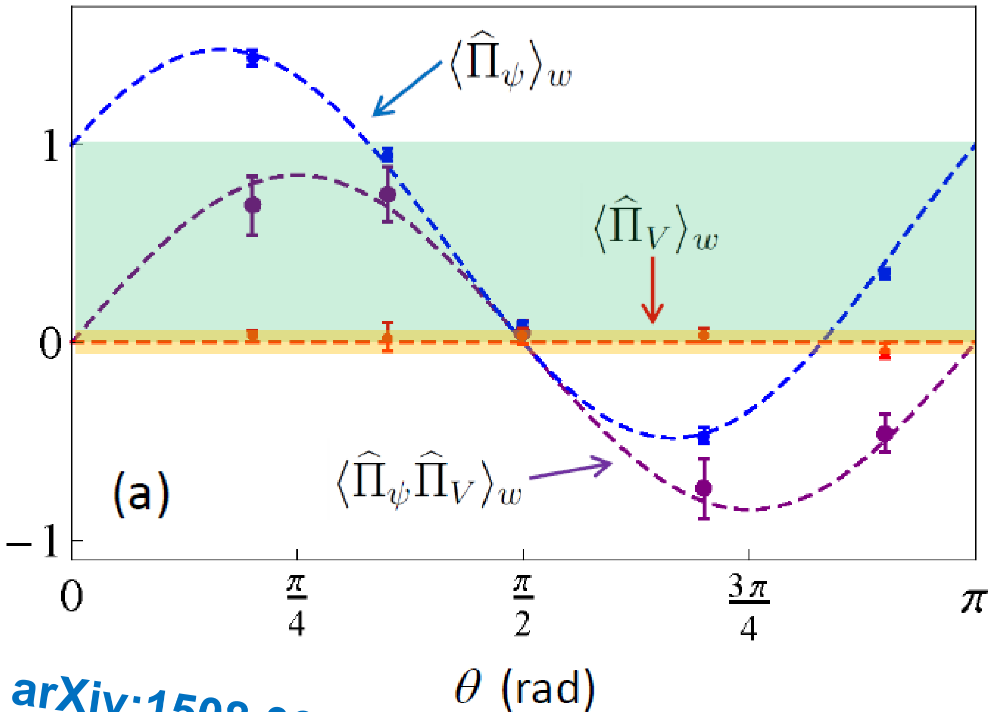
Weak values "internal consistency"



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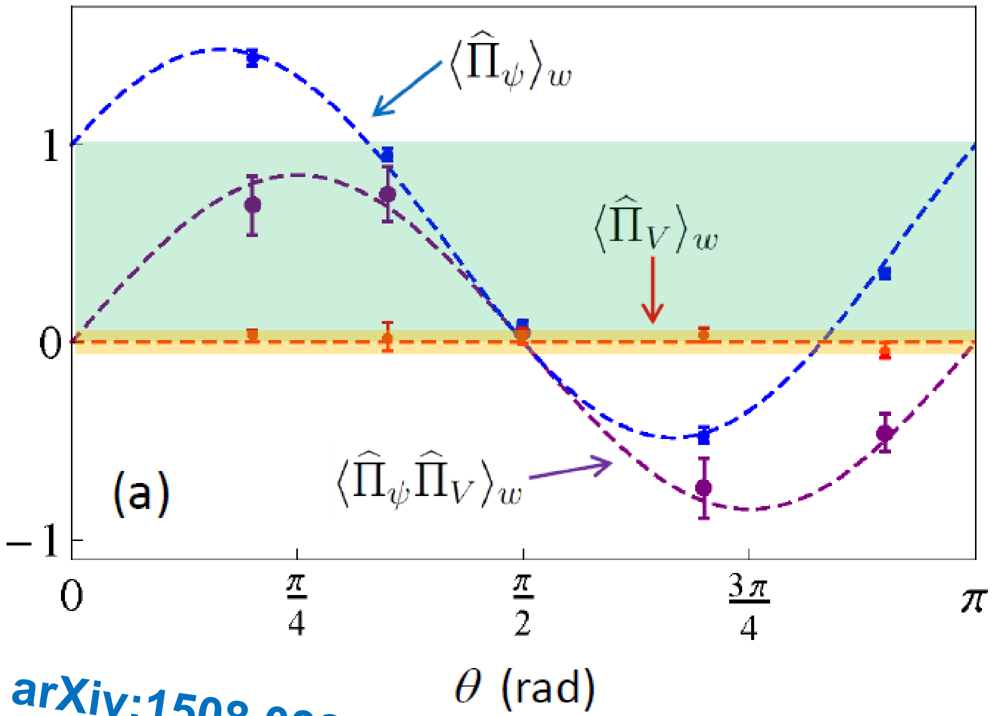

Weak values "internal consistency"

$$\langle\widehat{\Pi}_\psi\rangle_w + \langle\widehat{\Pi}_\psi^\perp\rangle_w = 1$$

arXiv:1508.03220

Results

Measured weak values (data points) compared with the theoretical predictions

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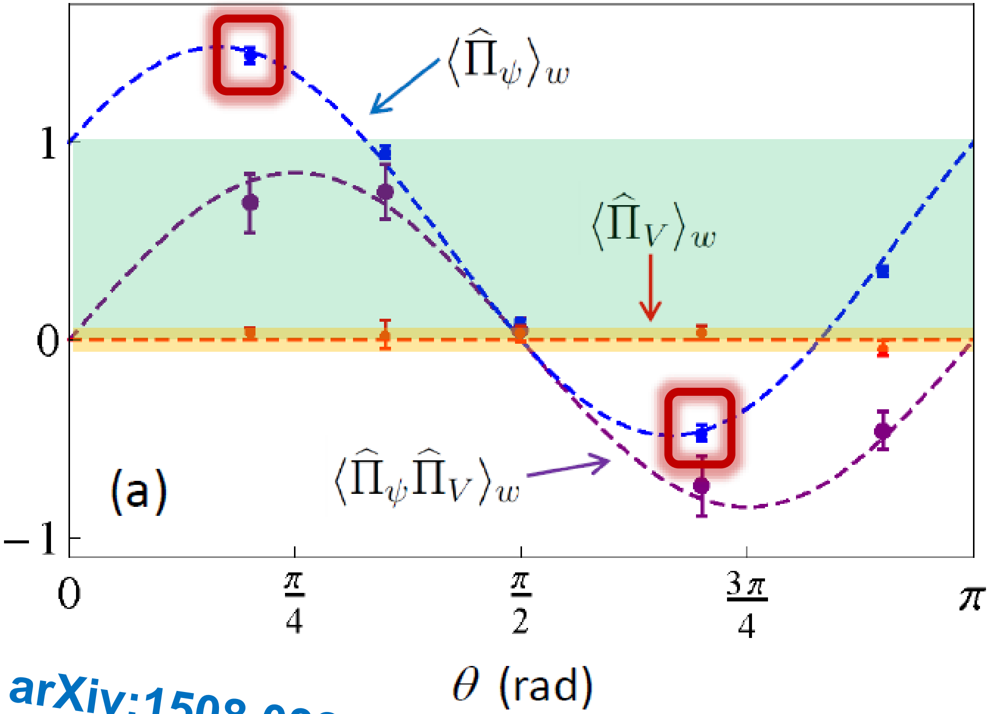

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Weak values

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$$\langle\widehat{\Pi}_\psi\rangle_w + \langle\widehat{\Pi}_\psi^\perp\rangle_w = 1$$

$$\langle\widehat{\Pi}_{\psi_0}\rangle_w + \langle\widehat{\Pi}_{\psi_0}^\perp\rangle_w = 0.97 \pm 0.06$$

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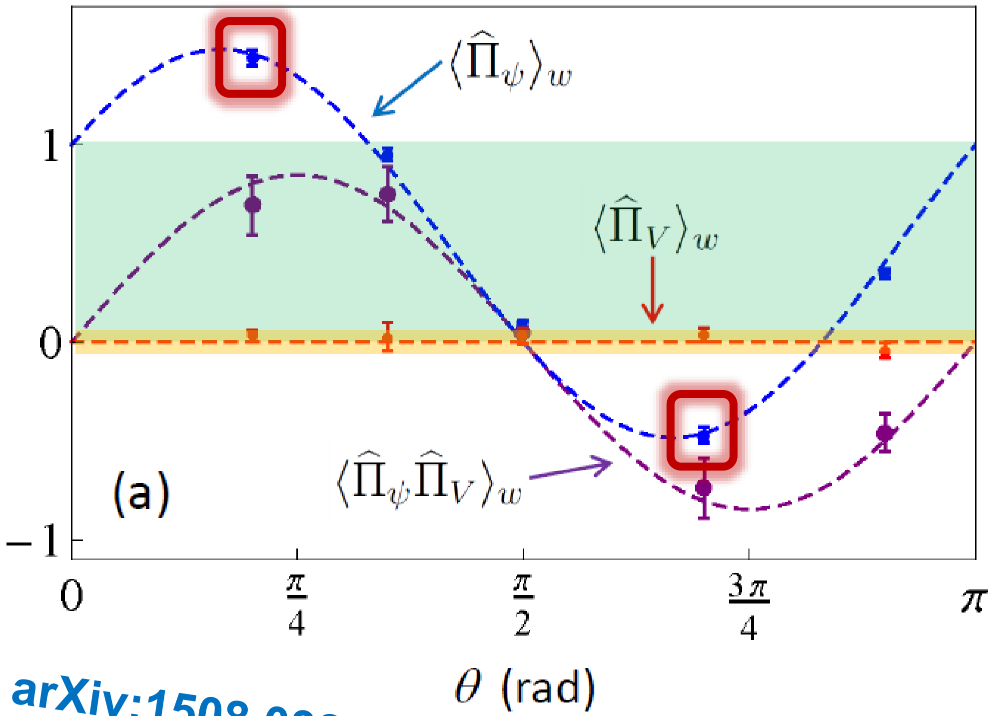

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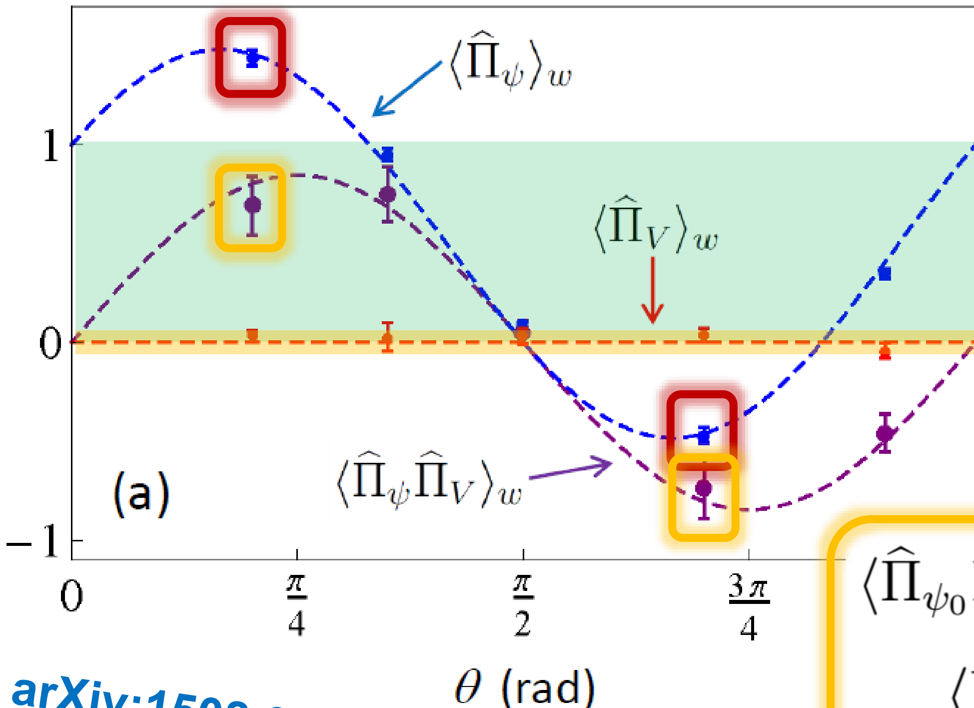

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$$\langle\widehat{\Pi}_{\psi_0}\widehat{\Pi}_V\rangle_w + \langle\widehat{\Pi}_{\psi_0}^\perp\widehat{\Pi}_V\rangle_w = -0.05 \pm 0.22$$

$$\langle\widehat{\Pi}_V\rangle_w = 0 \quad (0.02 \pm 0.06)$$

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Measured weak values (data points) compared with the theoretical predictions

$$\widehat{\Pi}_V = |V\rangle\langle V| \qquad \widehat{\Pi}_\psi = |\psi\rangle\langle\psi| \qquad (|\psi\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle)$$

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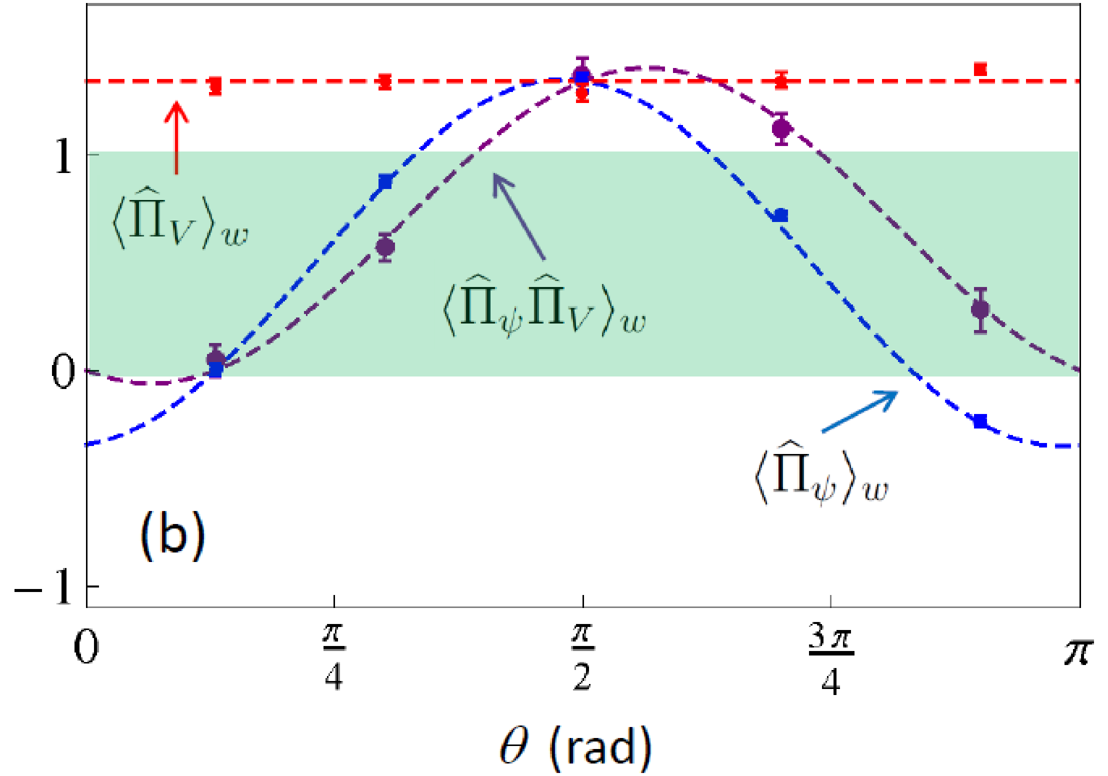

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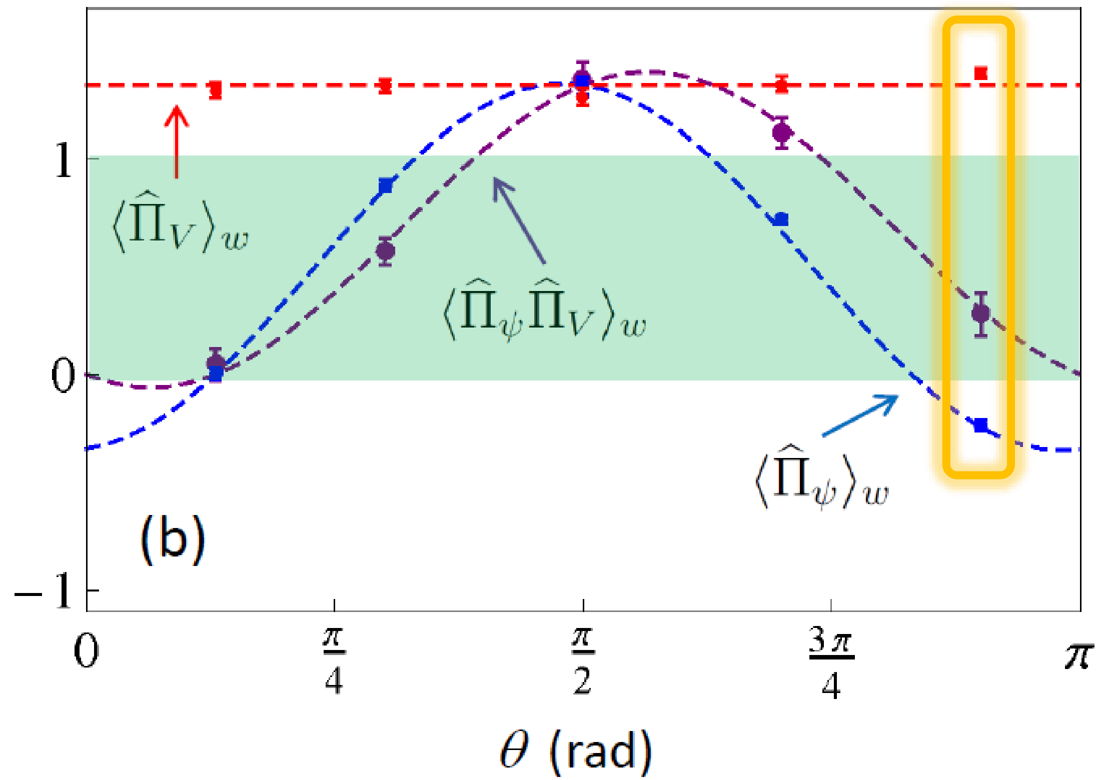
$$\langle\widehat{\Pi}_V\rangle_w = 1.40 \pm 0.04$$

$$\langle\widehat{\Pi}_\psi\rangle_w = -0.24 \pm 0.03$$

$$\langle\widehat{\Pi}_\psi\widehat{\Pi}_V\rangle_w = 0.28 \pm 0.10$$



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Results summary

Measured weak values (data points) compared with the theoretical predictions

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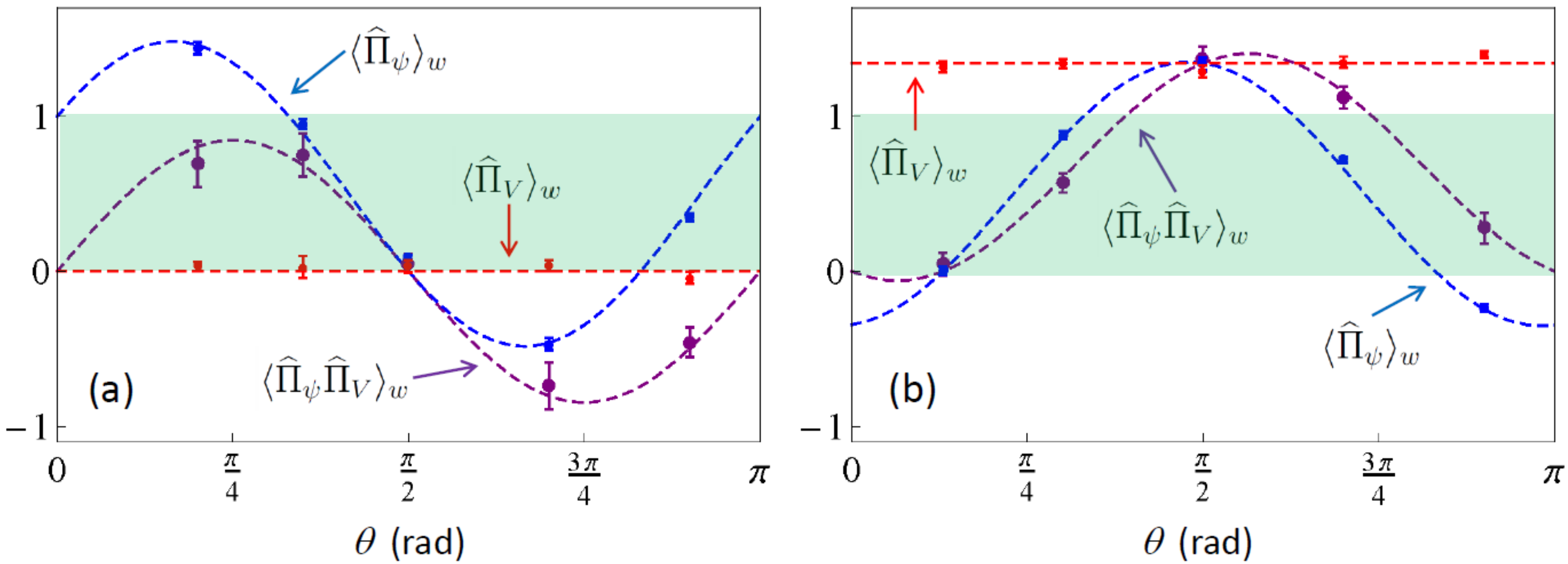

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WMs and quantum contextuality

Non-Contextual Hidden Variable Theory: ontological model of an operational theory where, if two experimental procedures are operationally equivalent, then they have equivalent representations in such model [Spekkens, PRA 71 (2005)].

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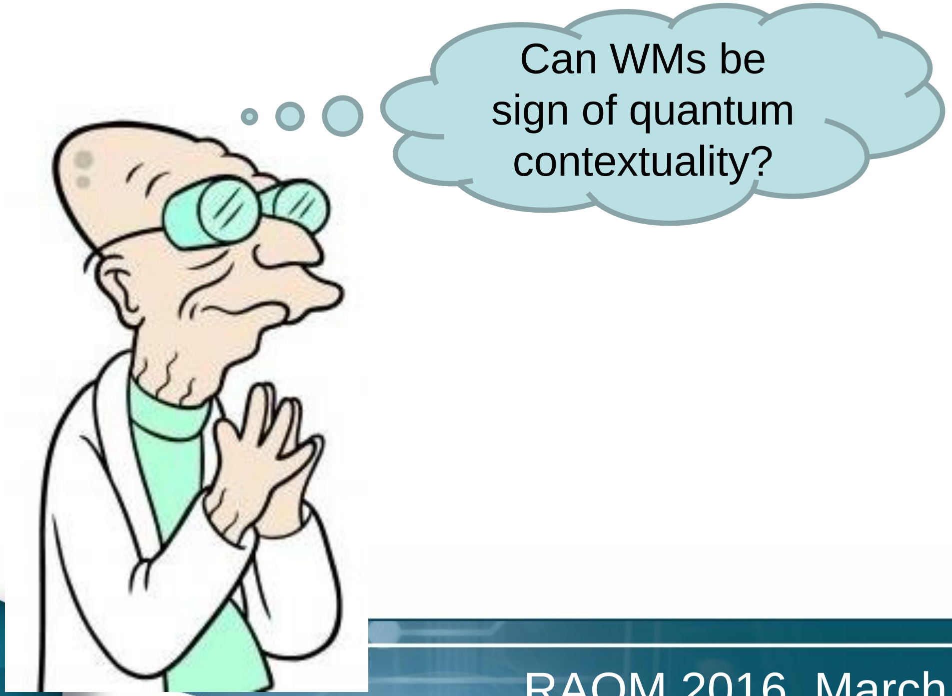

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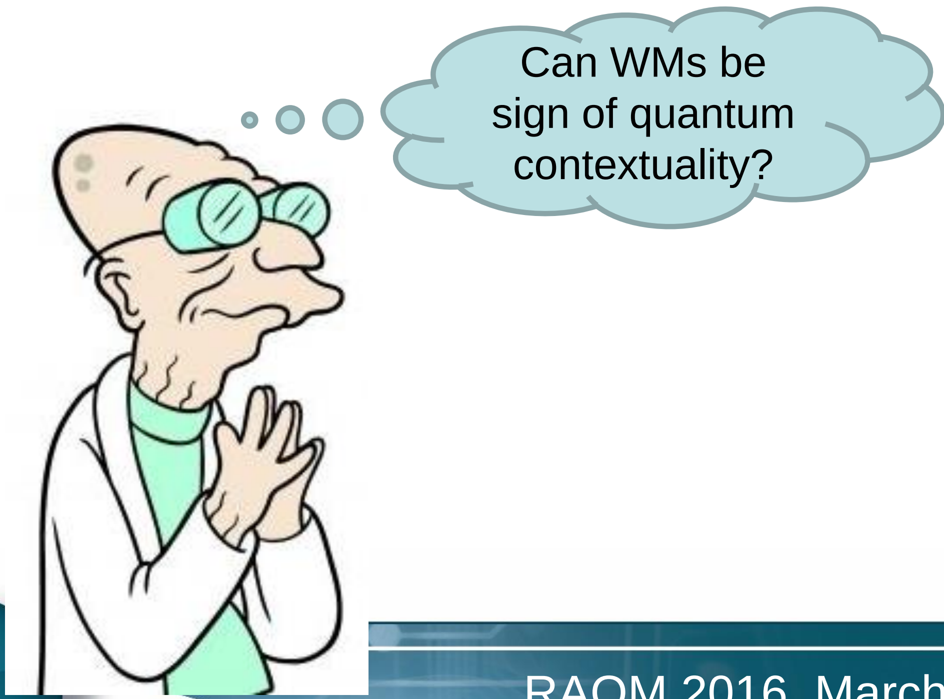
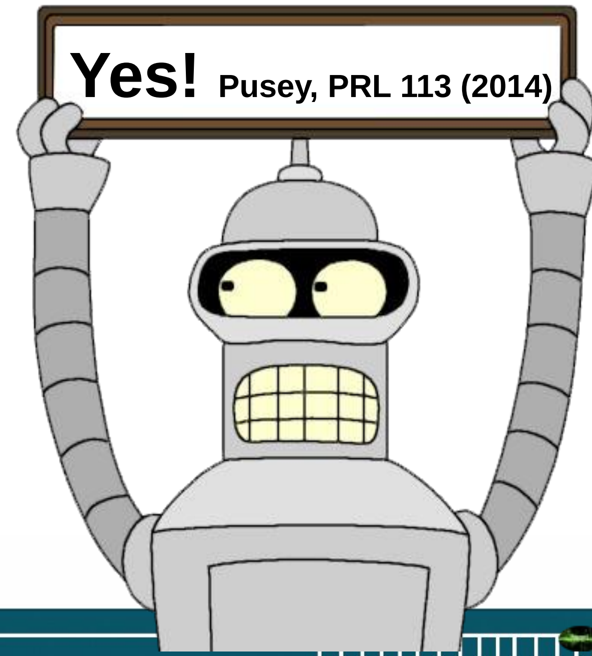
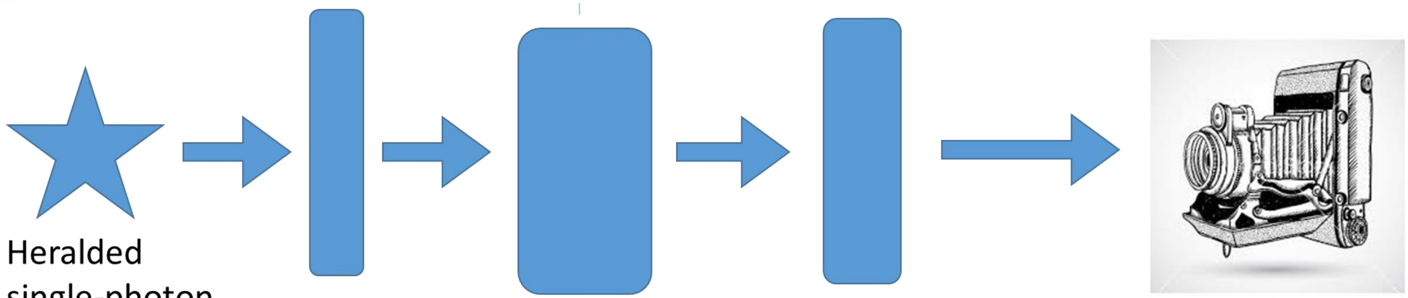
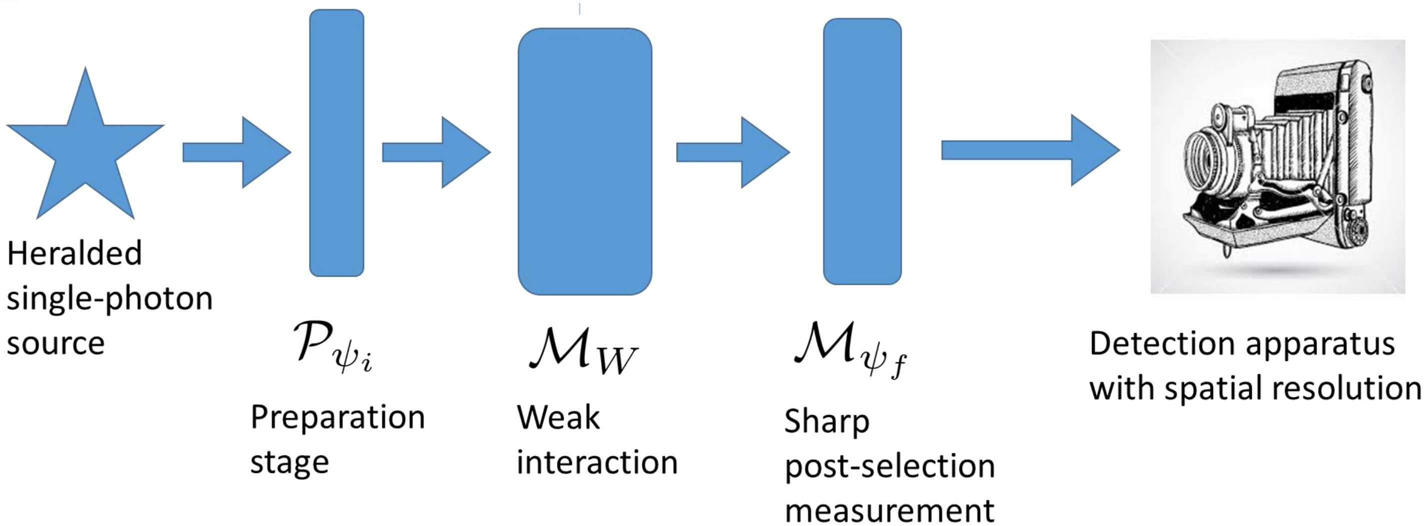




WM and quantum contextuality



WM and quantum contextuality



$$\text{Initial and final states are non-orthogonal}: \quad p_{\psi_f} := \mathbb{P}\left(\text{PASS}|\mathcal{P}_{\psi_i}, \mathcal{M}_{\psi_f}\right) > 0$$

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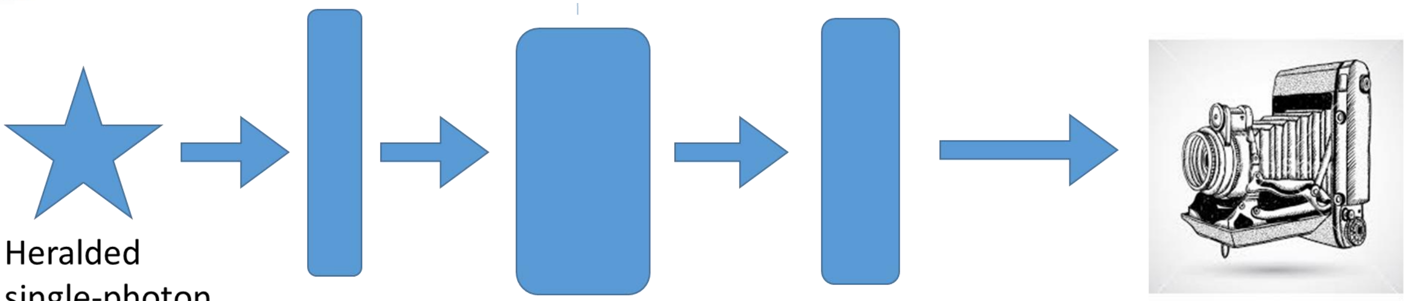

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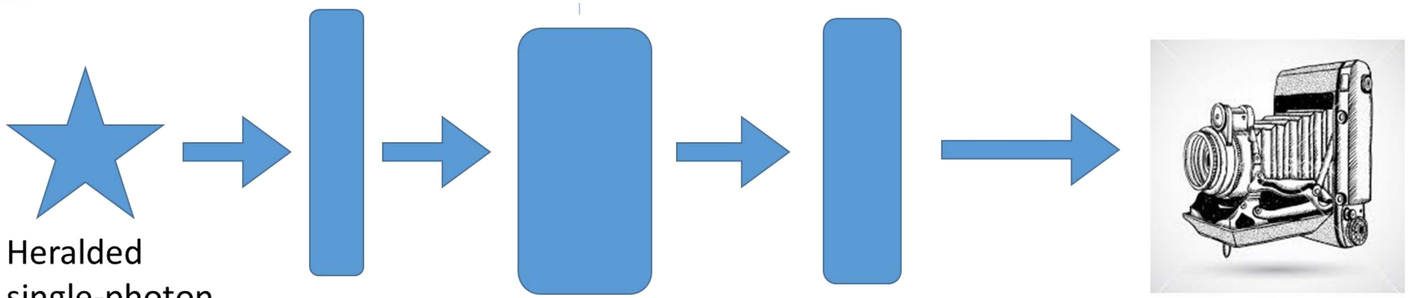

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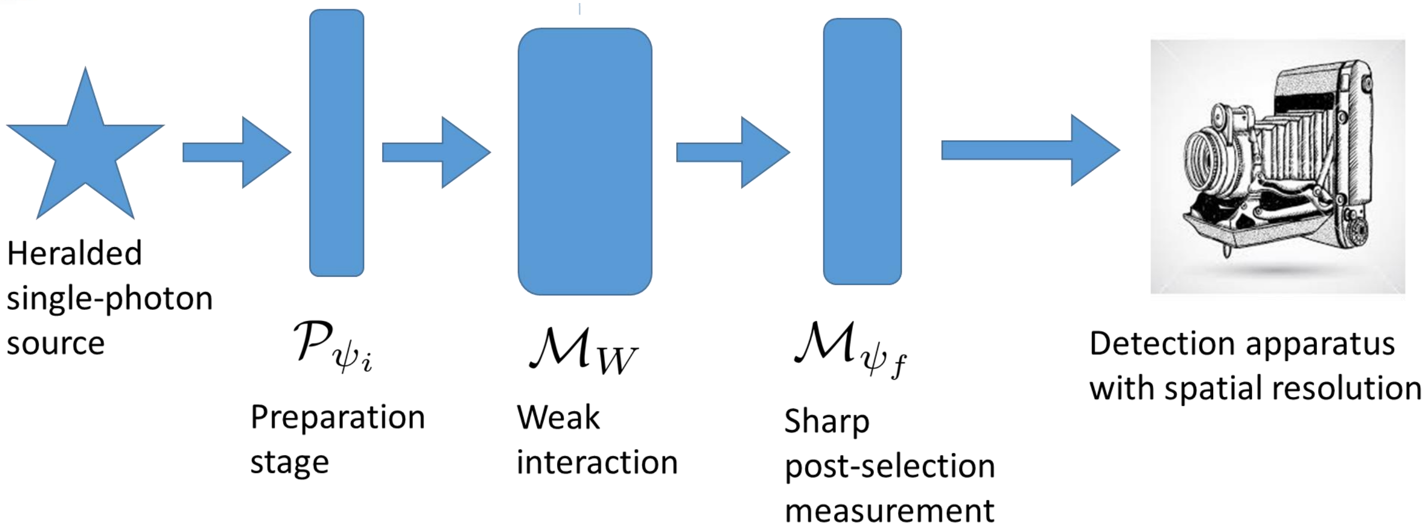
$$\exists\, p_d : \quad \mathbb{P}\left(\text{PASS}|\mathcal{P}, \mathcal{M}_W, \mathcal{M}_{\psi_f}\right) = (1-p_d)\mathbb{P}\left(\text{PASS}|\mathcal{P}, \mathcal{M}_{\psi_f}\right) + p_d\mathbb{P}\left(\text{PASS}|\mathcal{P}, \mathcal{M}_d\right) \quad \forall \mathcal{P}$$

WM and quantum contextuality



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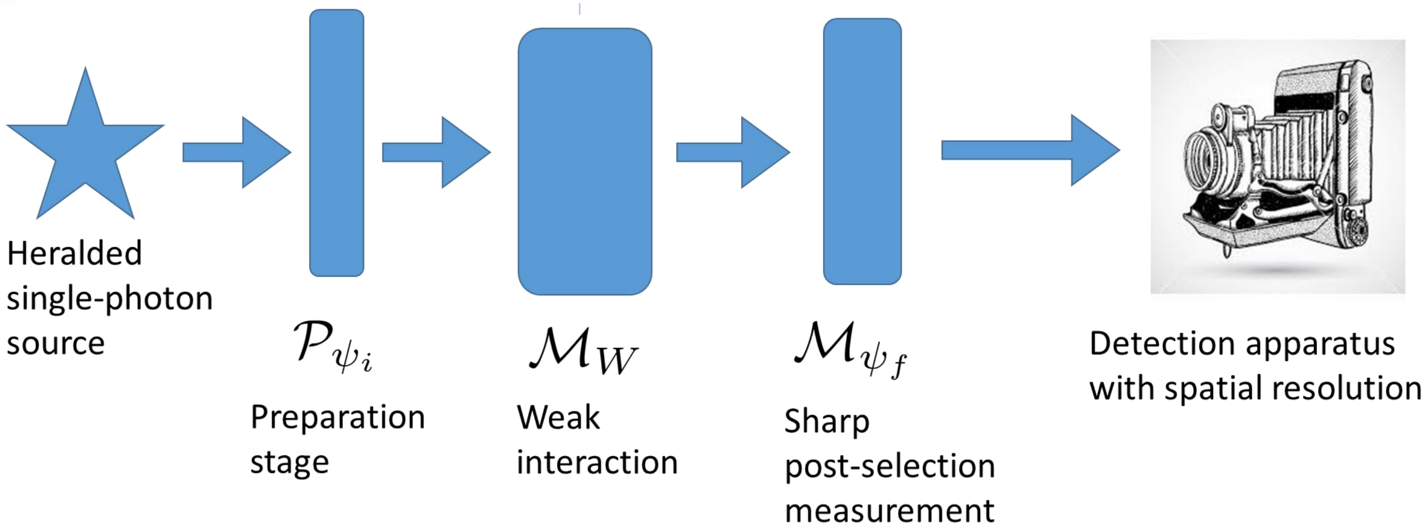
$$p_- := \left(p_{\psi_f}\right)^{-1} \int_{-\infty}^{0} \mathbb{P}\left(x, \text{PASS}|\mathcal{P}_{\psi_i}, \mathcal{M}_W, \mathcal{M}_{\psi_f}\right) \mathrm{d}x : \quad \mathcal{I} = p_- - \frac{1}{2} - \frac{p_d}{p_{\psi_f}} > 0$$

WM and quantum contextuality



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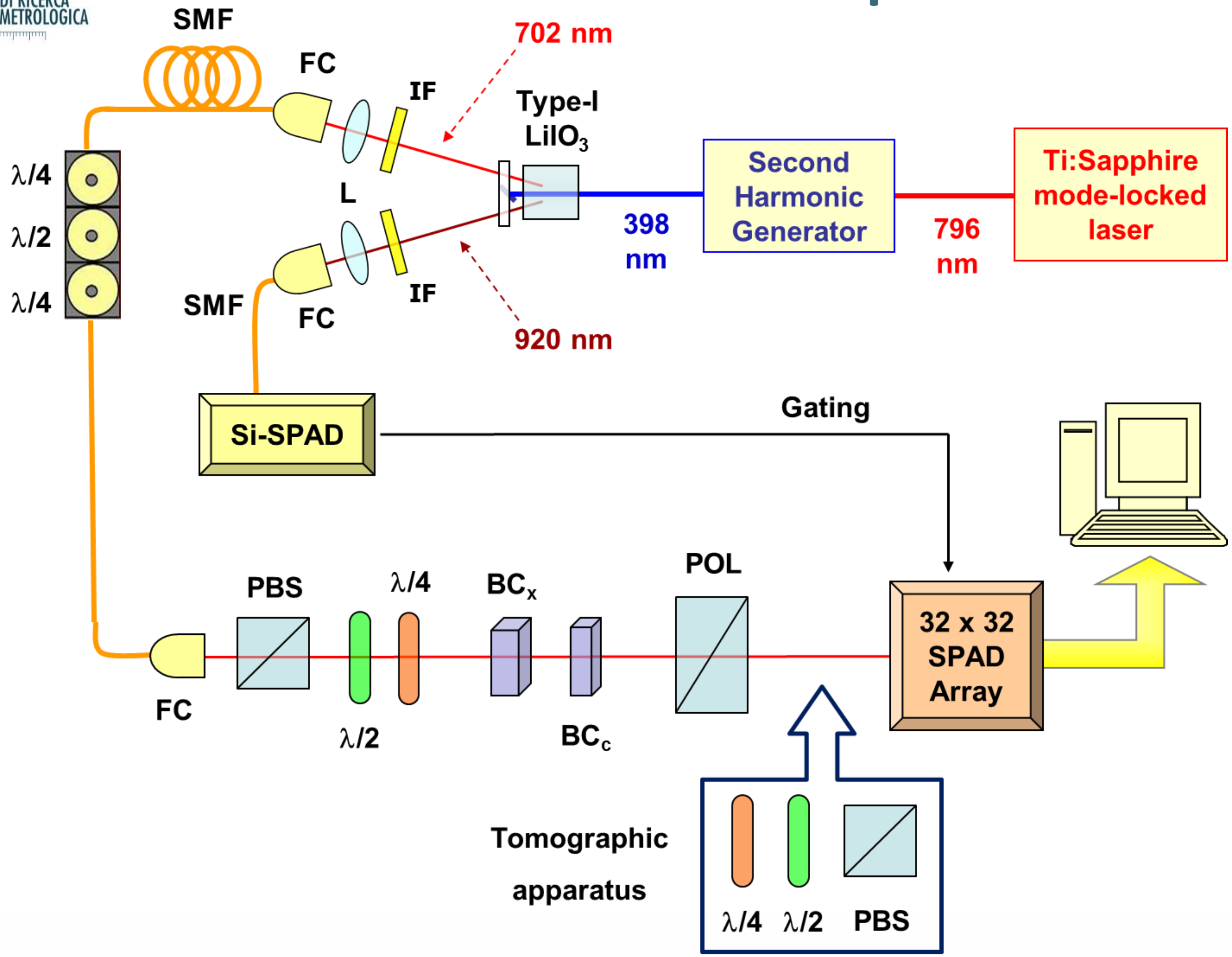
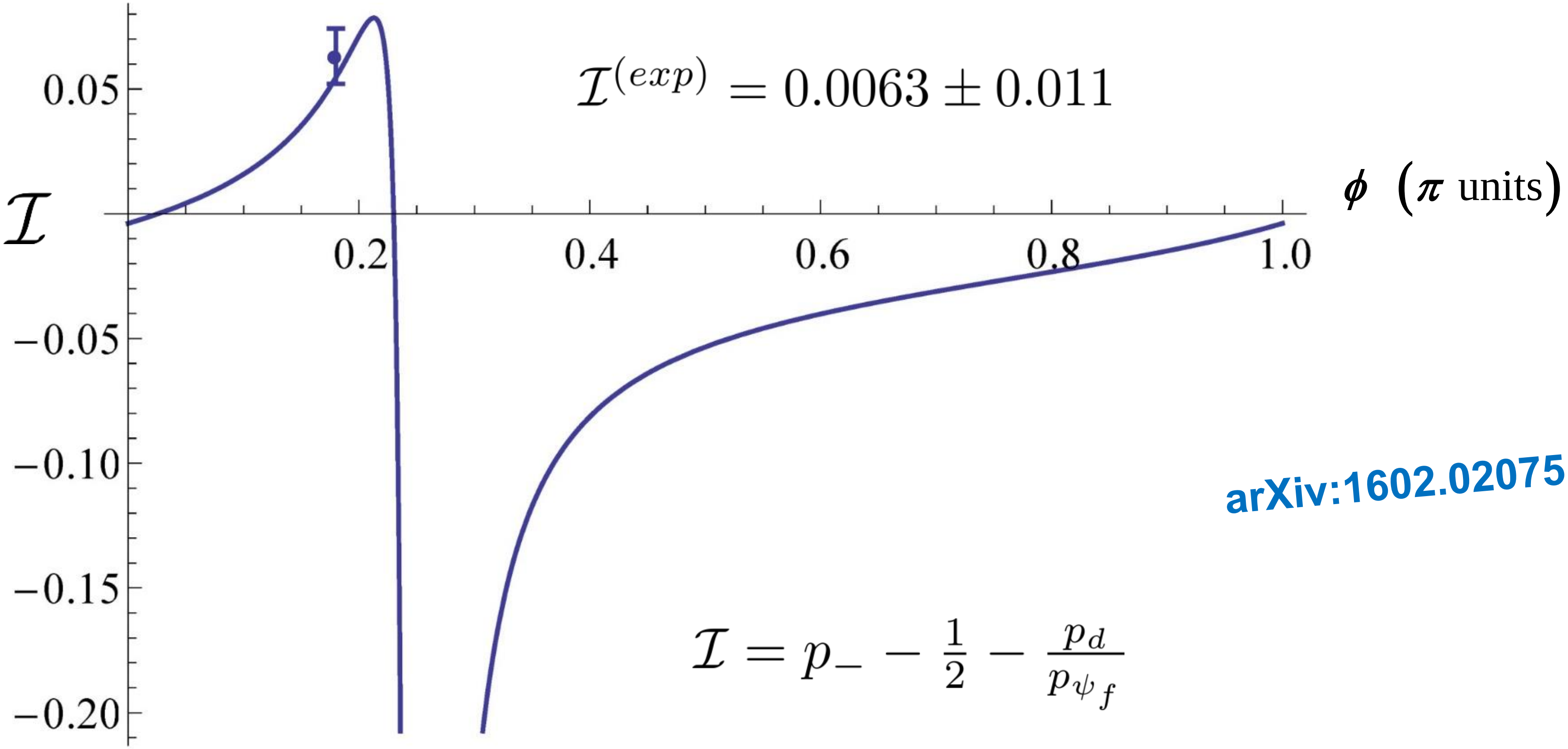
No non-contextual model satisfying outcome determinism for sharp measurements

Setup


SMF
FC
IF
702 nm
Type-I
LiIO$_3$
L
398
nm
Second
Harmonic
Generator
796
nm
Ti:Sapphire
mode-locked
laser
λ/4
λ/2
λ/4
SMF
FC
IF
920 nm
Si-SPAD
Gating
PBS
λ/4
BC$_x$
POL
32 x 32
SPAD
Array
FC
λ/2
BC$_c$
Tomographic
apparatus
λ/4
λ/2
PBS

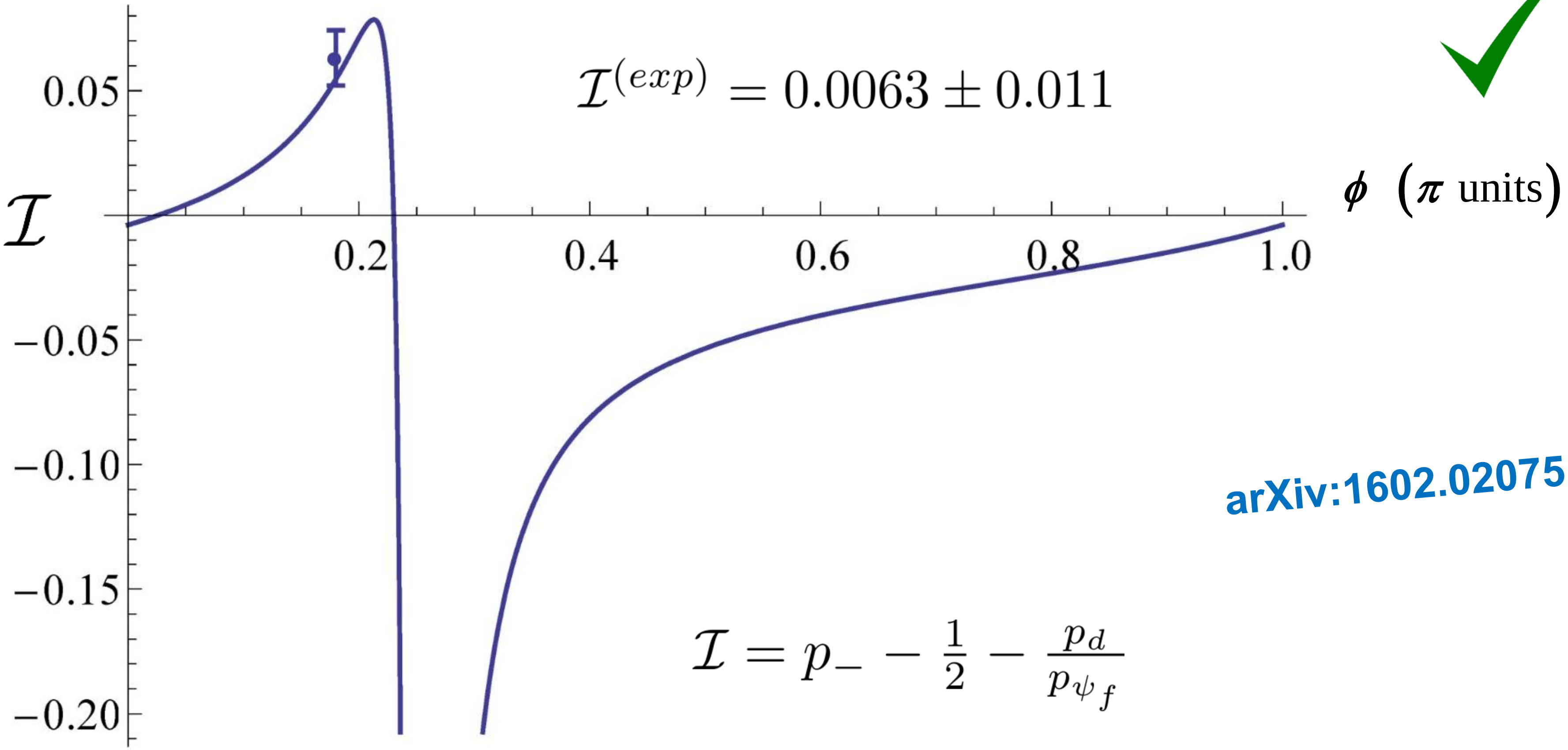
Condition 4: non-contextual bound violation



arXiv:1602.02075

- input state: $|-\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$
- post-selection state: $|\psi_f\rangle = \cos\phi|H\rangle + \sin\phi|V\rangle$ $(\mathcal{I}^{(exp)} : \ \phi = 0.18\pi)$
- From experimental parameters: $p_d = 0.0019 \pm 0.0002$

Condition 4: non-contextual bound violation



$$\mathcal{I}^{(exp)} = 0.0063 \pm 0.011$$

$$\mathcal{I} = p_- - \frac{1}{2} - \frac{p_d}{p_{\psi_f}}$$

arXiv:1602.02075

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Condition 2 verification

$$\mathbb{P}\left(x|\mathcal{P},\mathcal{M}_{W}\right) = p_n(x-g)\mathbb{P}\left(1|\mathcal{P},\mathcal{M}_{\Pi}\right) + p_n(x)\mathbb{P}\left(0|\mathcal{P},\mathcal{M}_{\Pi}\right) \quad \forall\mathcal{P}$$

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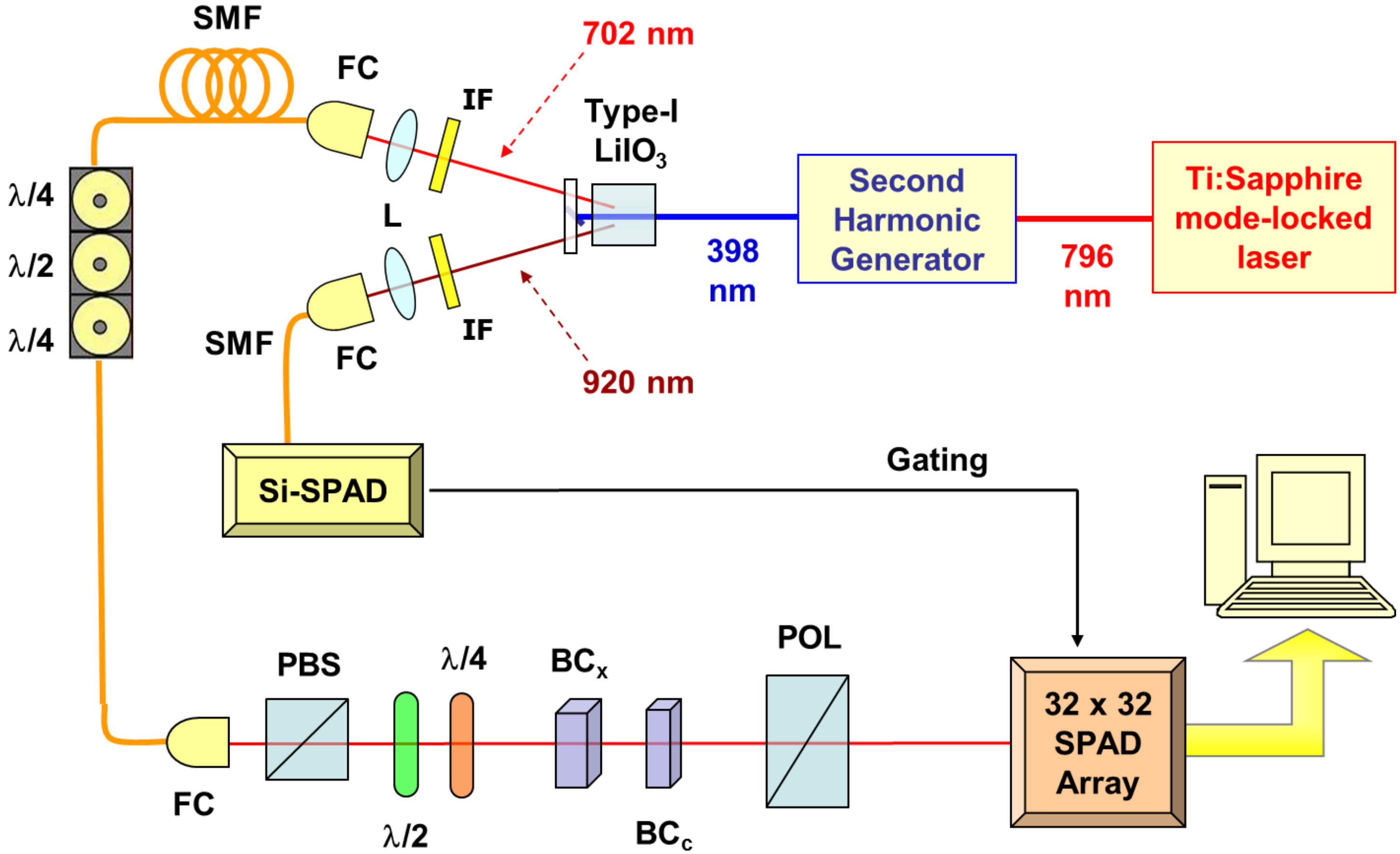

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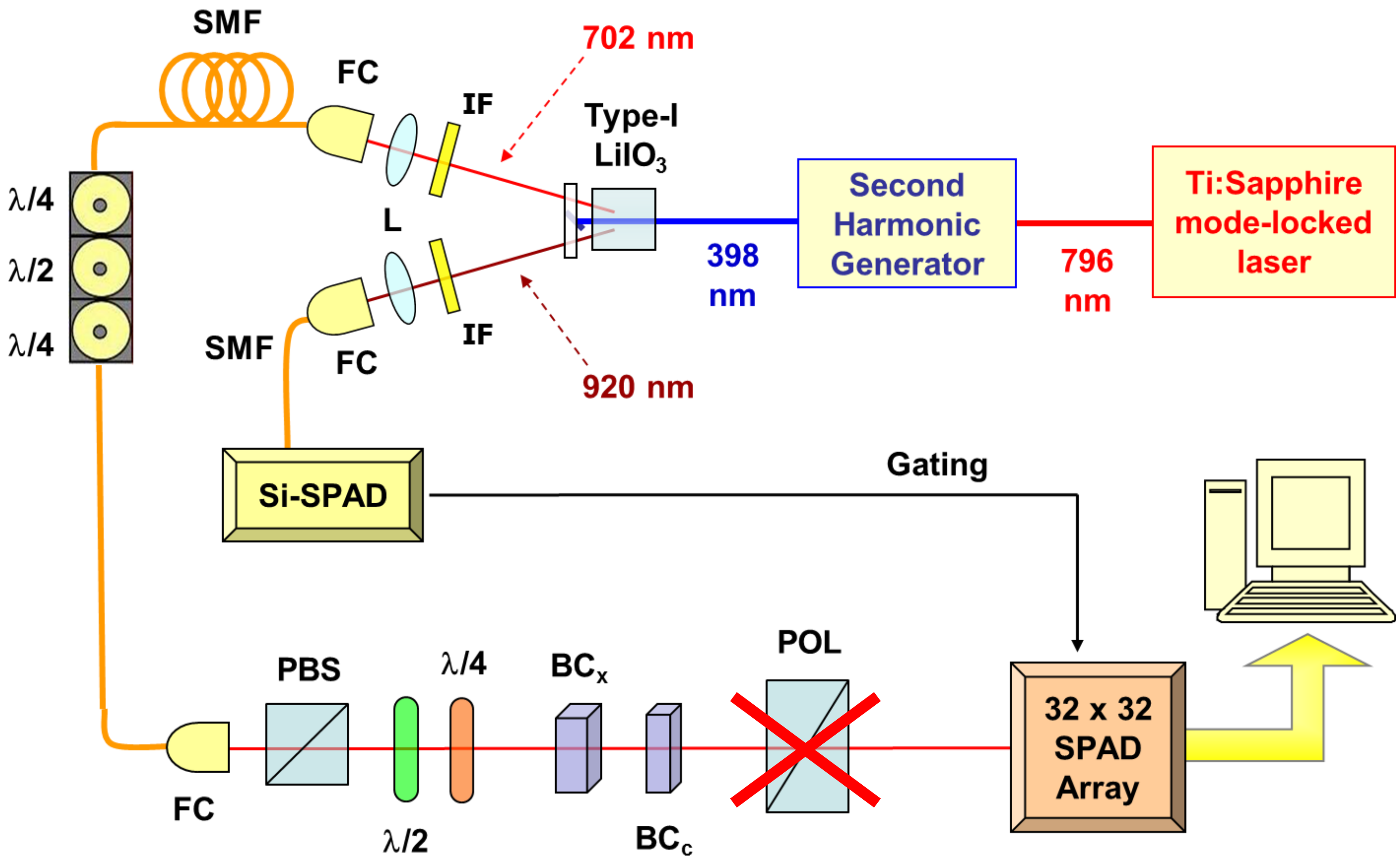

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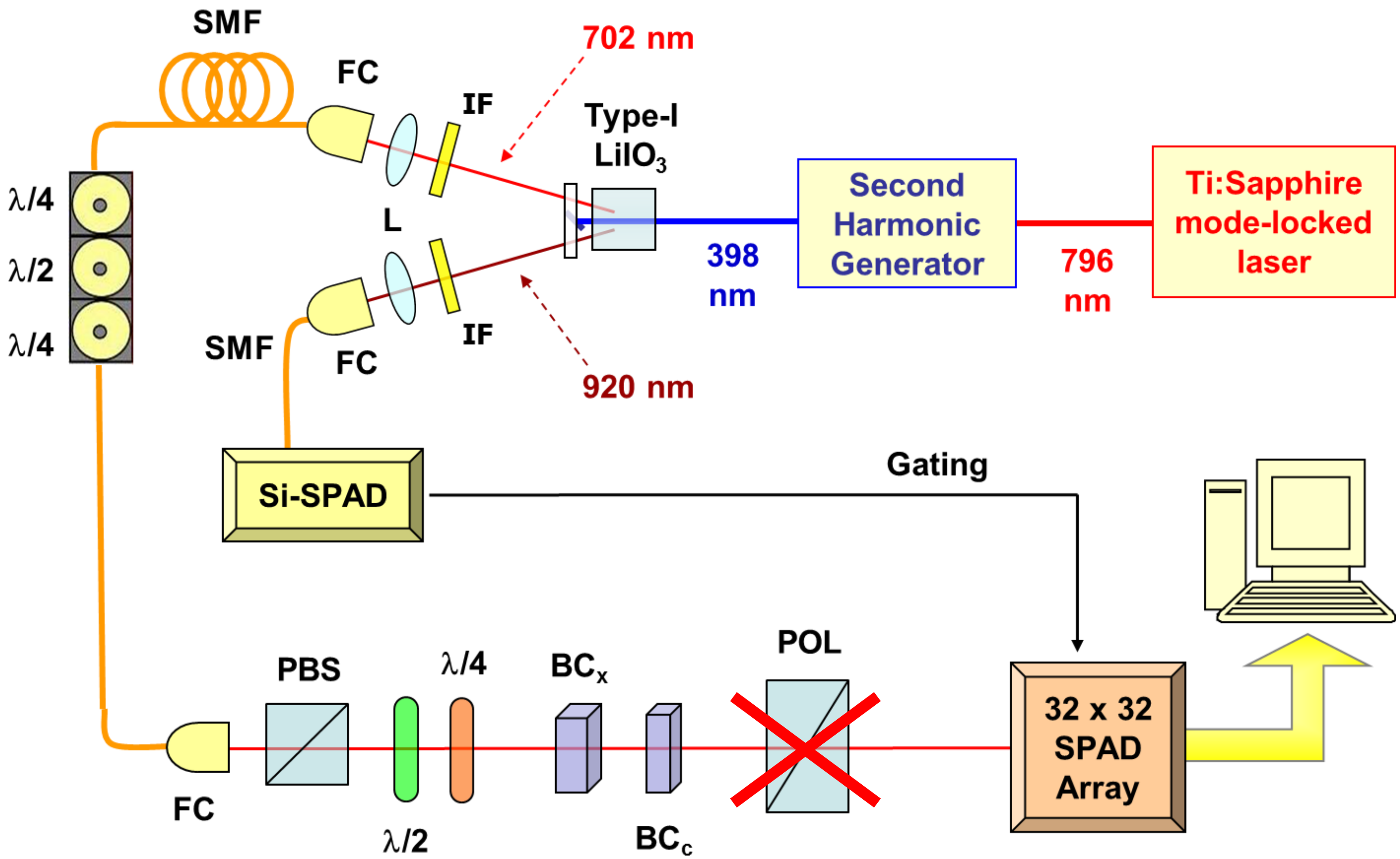
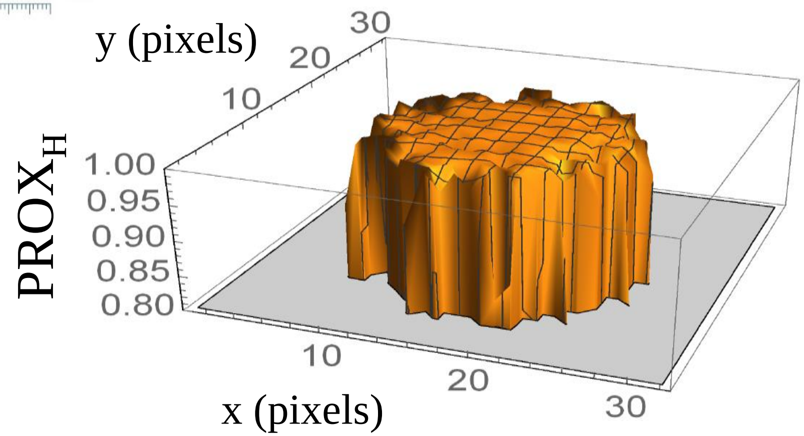
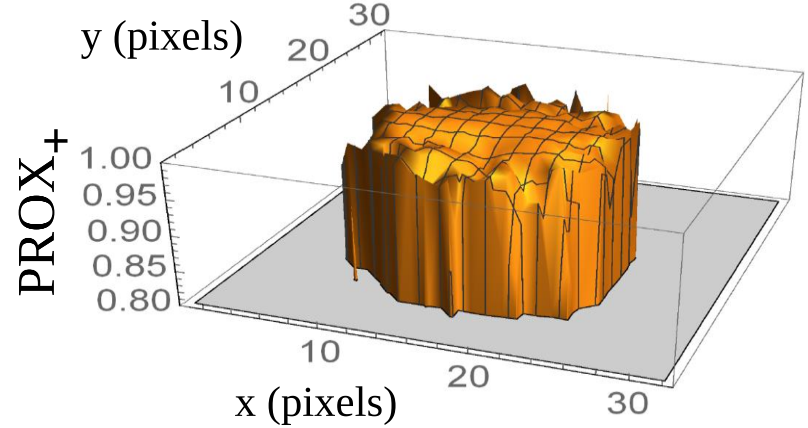


$\mathbb{P}(0|\mathcal{P},\mathcal{M}_\Pi)$: probability of not undergoing the weak interaction

$\mathbb{P}(1|\mathcal{P},\mathcal{M}_\Pi)$: probability of undergoing the weak interaction

Condition 2 verification





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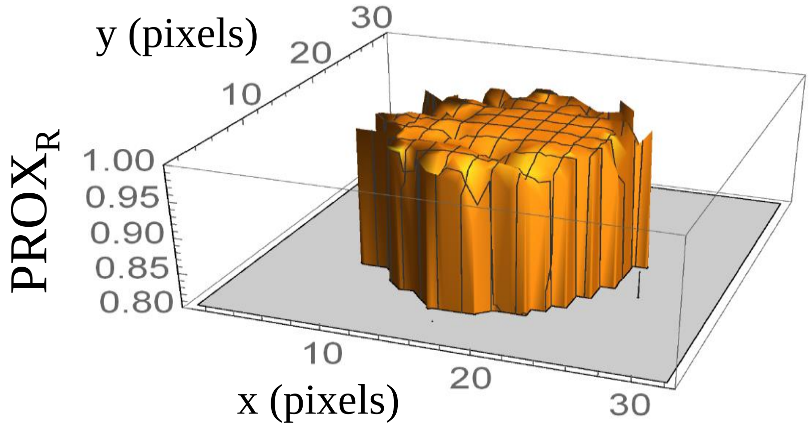
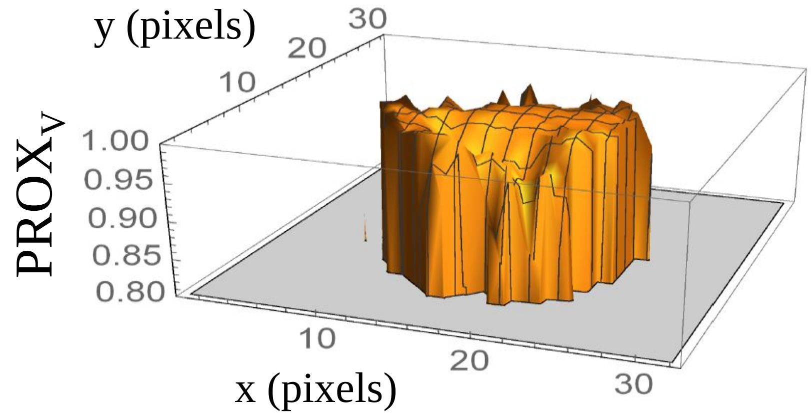
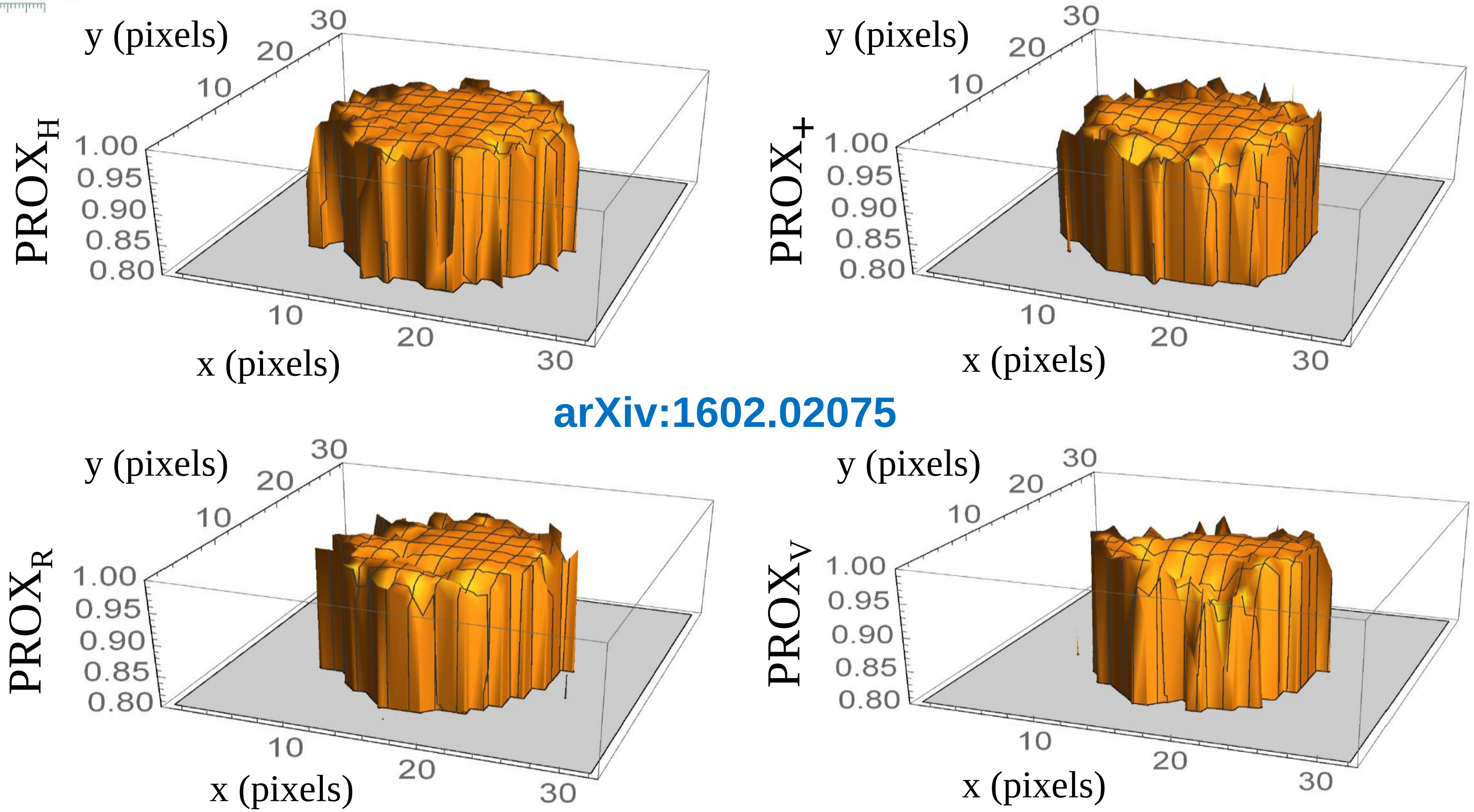




Condition 2 verification



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$$PROX_{\psi}(x) = \left[\frac{2Q(x)Q^{(e)}(x)}{(Q(x))^2+(Q^{(e)}(x))^2}\right]^{\frac{1}{2}}$$

$$Q(x) = \mathbb{P}\left(x|\mathcal{P},\mathcal{M}_W\right)$$

$$Q^{(e)}(x) = p_n(x-g)\mathbb{P}\left(1|\mathcal{P},\mathcal{M}_\Pi\right) + p_n(x)\mathbb{P}\left(0|\mathcal{P},\mathcal{M}_\Pi\right)$$

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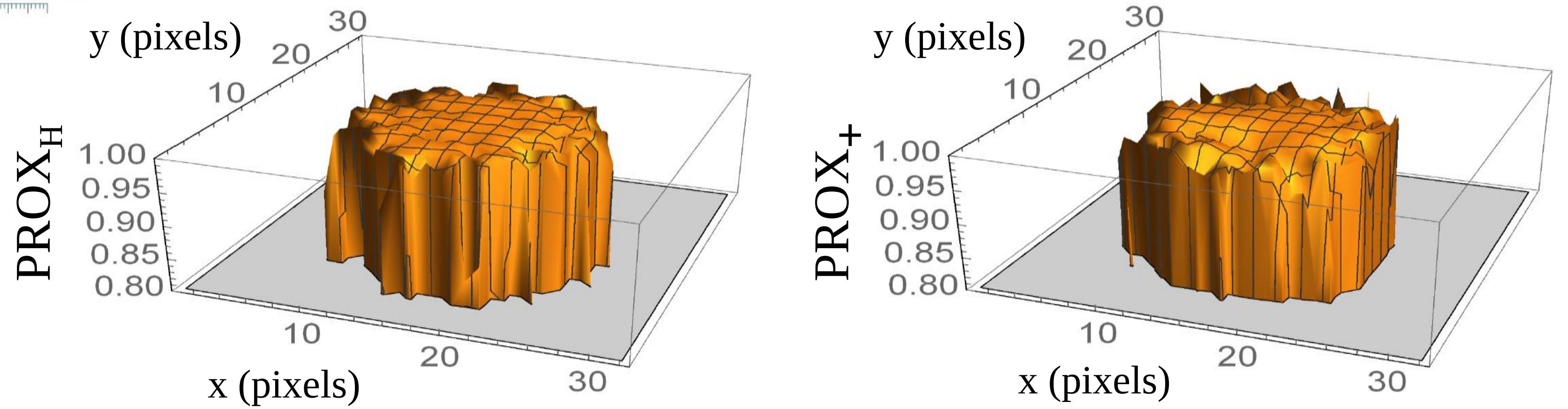
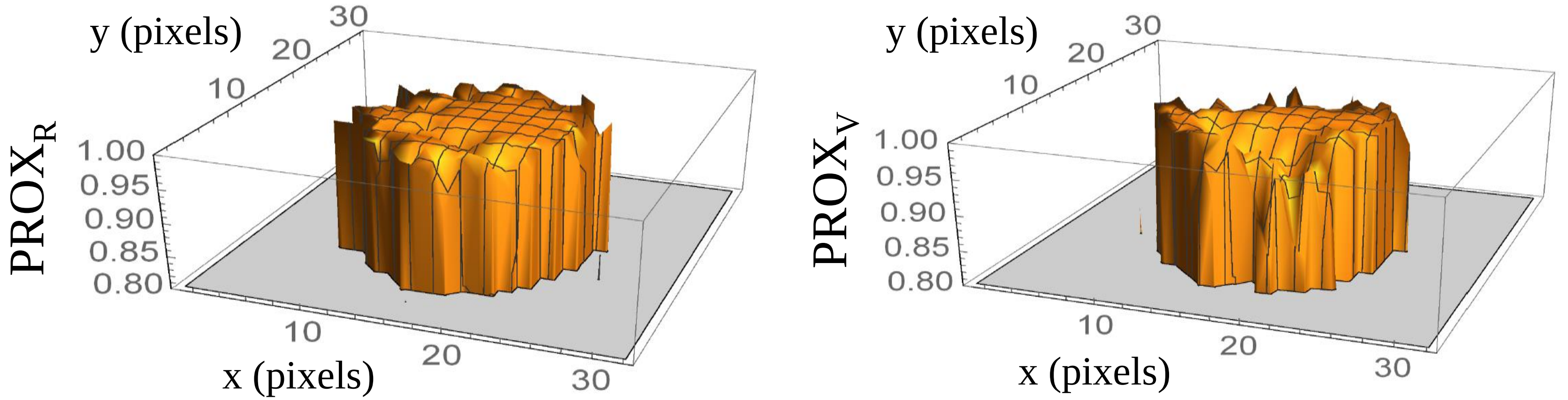
Condition 2 verification



arXiv:1602.02075



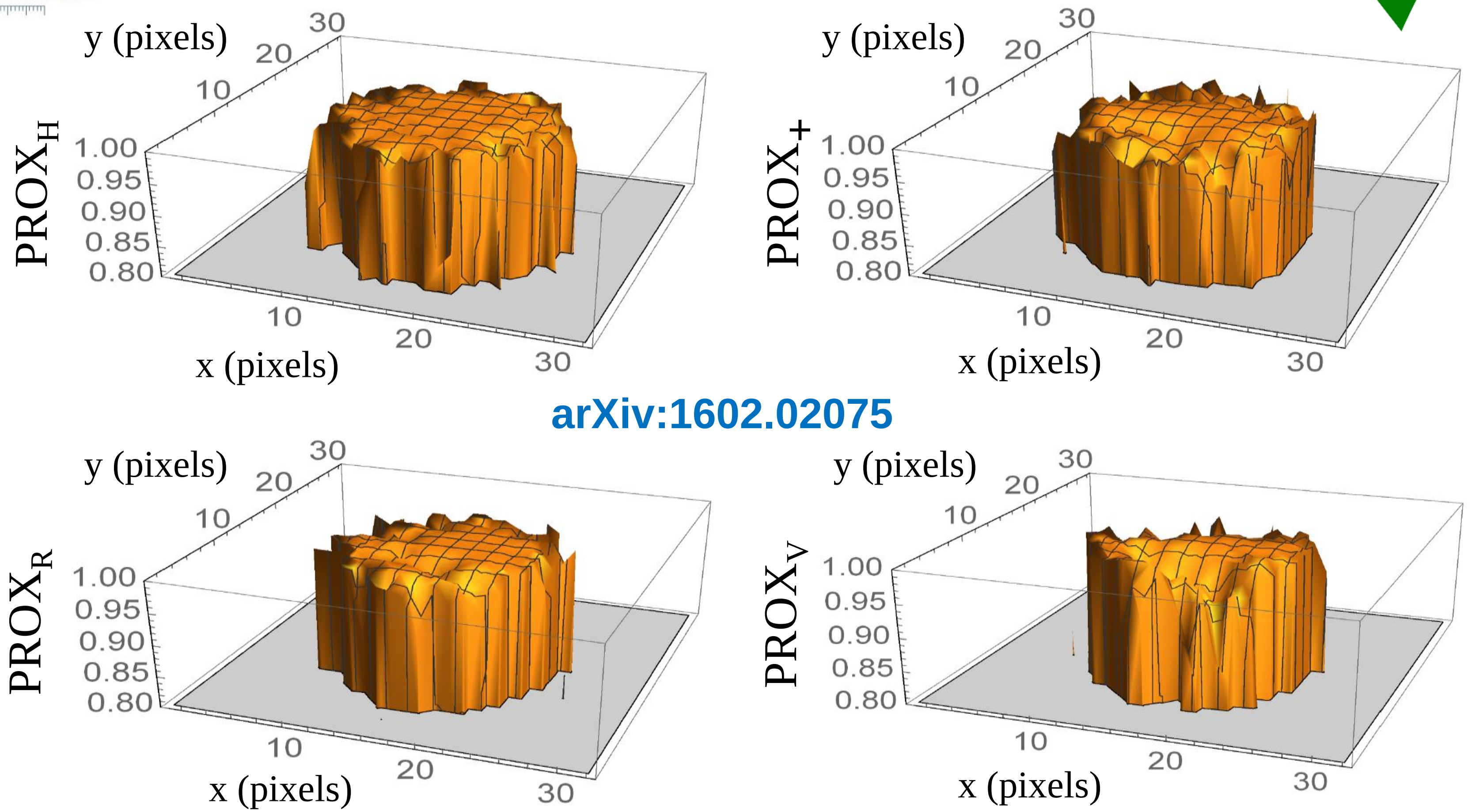
$$PROX_\psi(x) = \left[\frac{2Q(x)Q^{(e)}(x)}{(Q(x))^2+(Q^{(e)}(x))^2}\right]^{\frac{1}{2}}$$

$$Q(x) = \mathbb{P}\left(x|\mathcal{P},\mathcal{M}_W\right)$$

$$Q^{(e)}(x) = p_n(x-g)\mathbb{P}\left(1|\mathcal{P},\mathcal{M}_\Pi\right) + p_n(x)\mathbb{P}\left(0|\mathcal{P},\mathcal{M}_\Pi\right)$$

Fidelity between $Q(x)$ and $Q^{(e)}(x)$ always above 99% (sampling on >230 points)

Condition 2 verification



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Condition 3 verification

$$\exists\, p_d : \quad \mathbb{P}\left(\mathrm{PASS}|\mathcal{P}, \mathcal{M}_W, \mathcal{M}_{\psi_f}\right) = (1-p_d)\mathbb{P}\left(\mathrm{PASS}|\mathcal{P}, \mathcal{M}_{\psi_f}\right) + p_d \mathbb{P}\left(\mathrm{PASS}|\mathcal{P}, \mathcal{M}_d\right)$$

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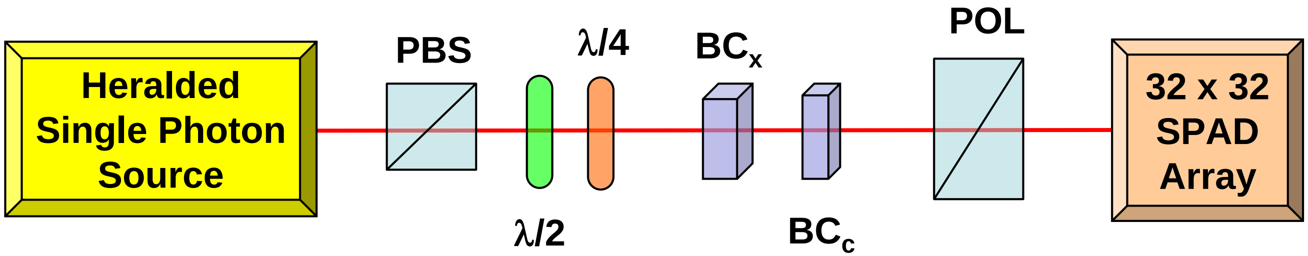

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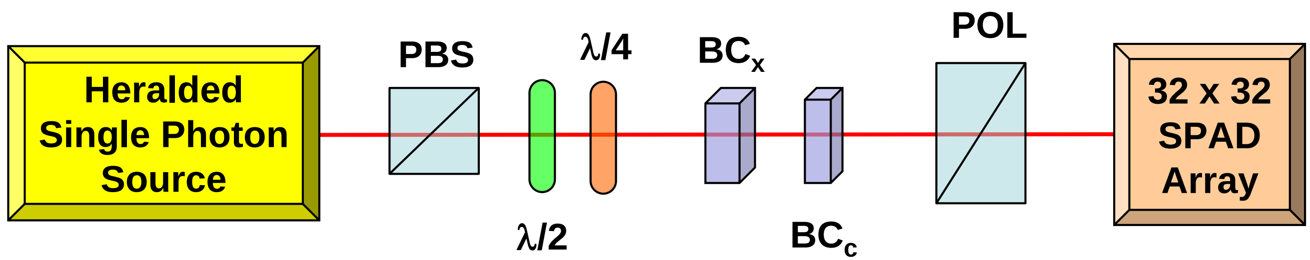

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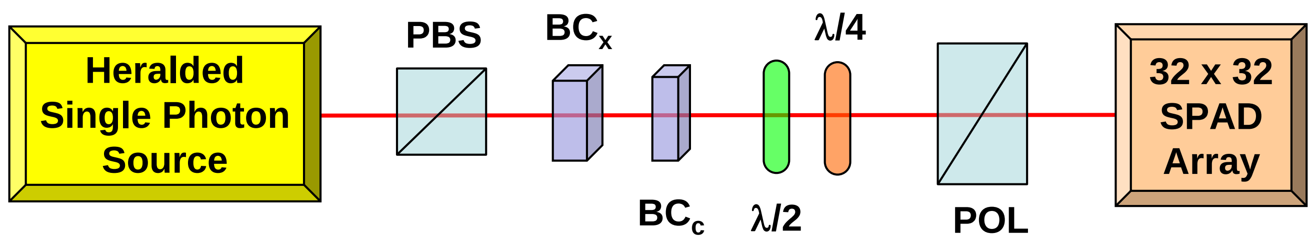

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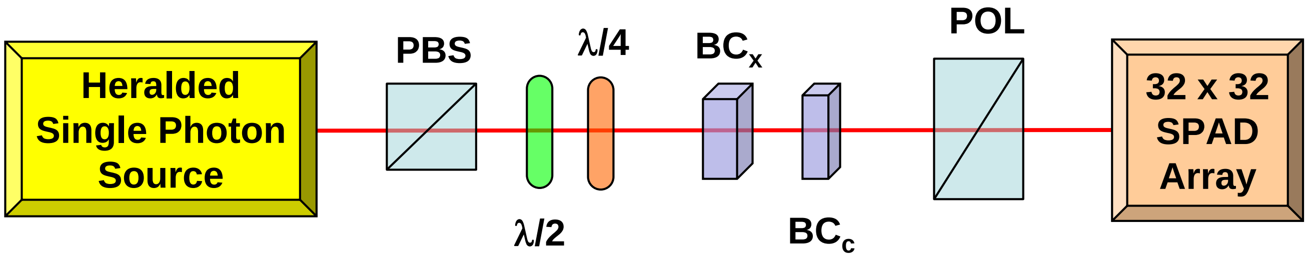

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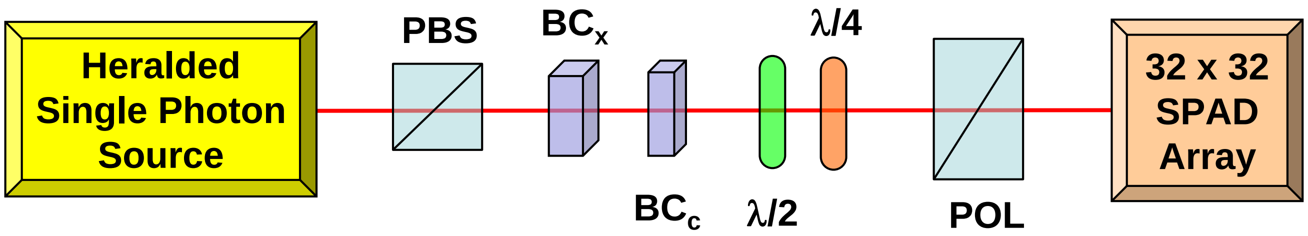

$\mathcal{M}_d$ is unknown $\quad\Longrightarrow\quad \mathbb{P}\left(\text{PASS}|\mathcal{P},\mathcal{M}_d\right) = ??$

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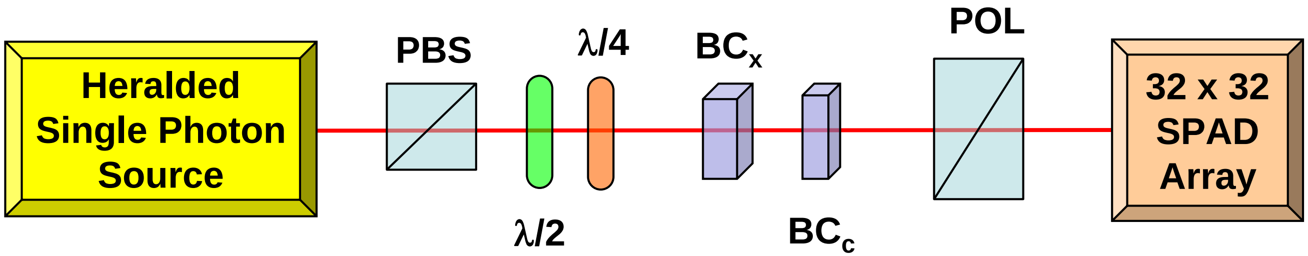

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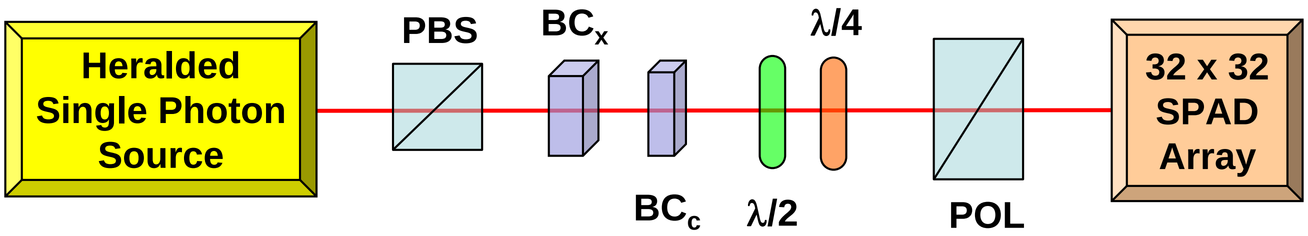

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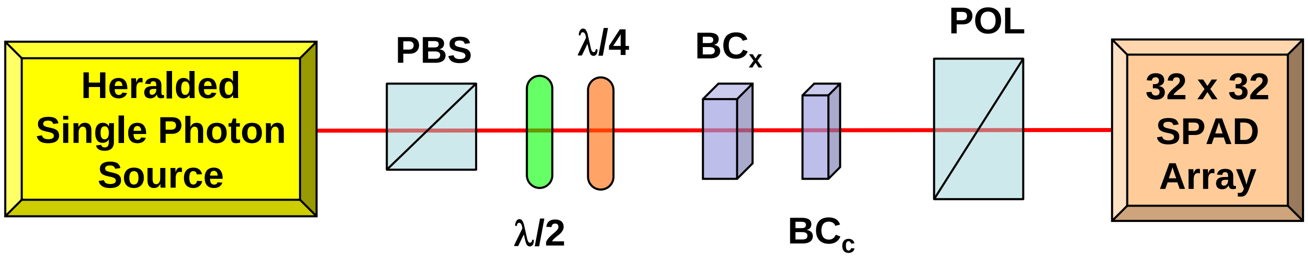

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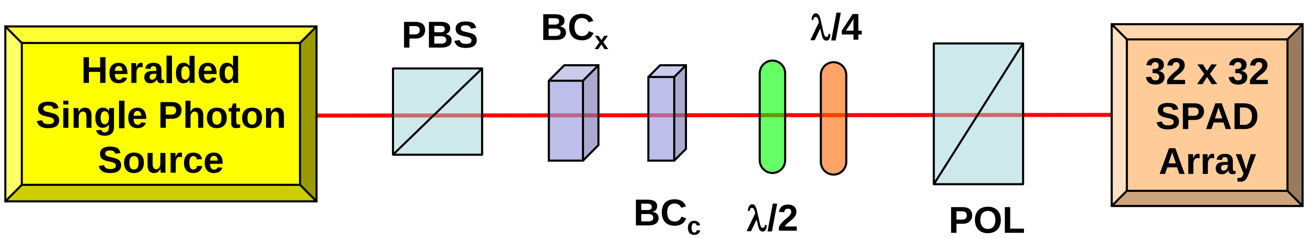

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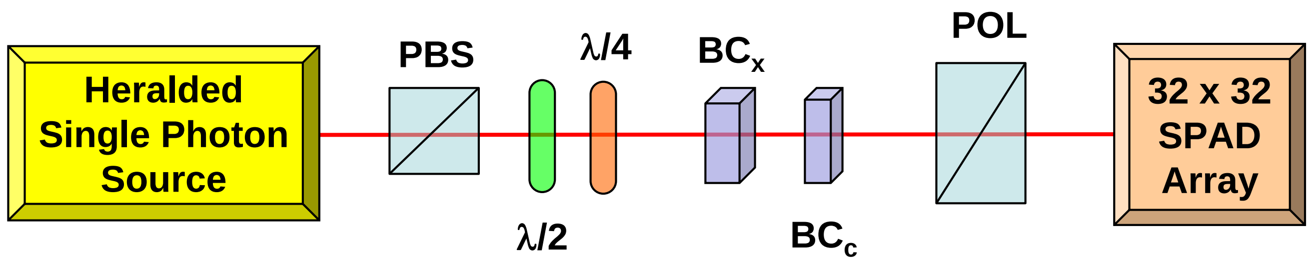

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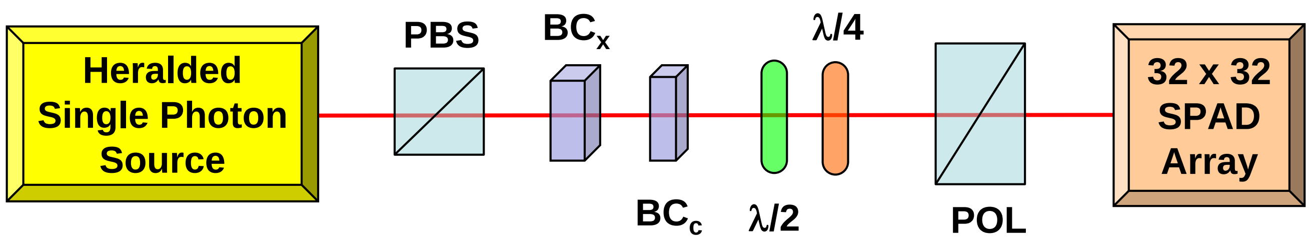

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$$(0.000021 \pm 0.000014) \leq p_d \leq (0.086 \pm 0.050)$$

arXiv:1602.02075

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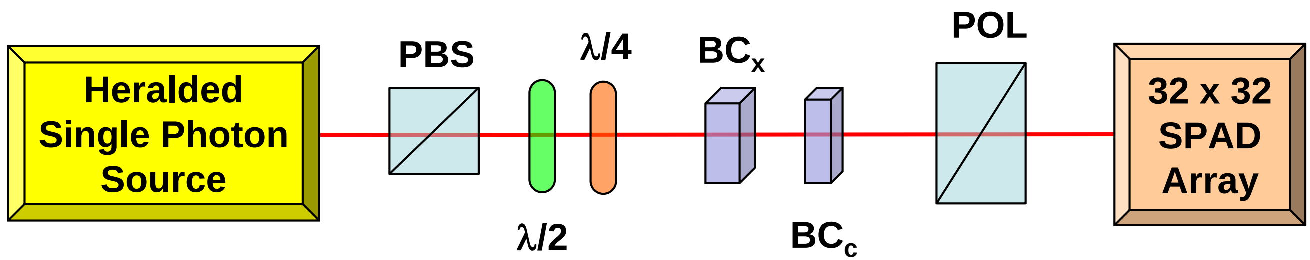

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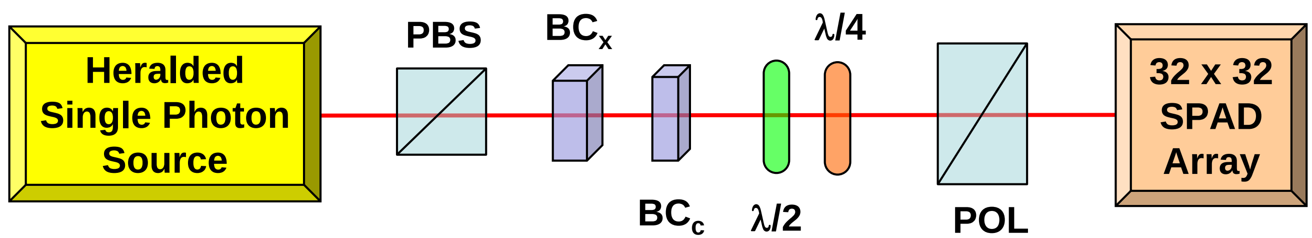

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$p_d = 0.0019 \pm 0.0002$ fits in!

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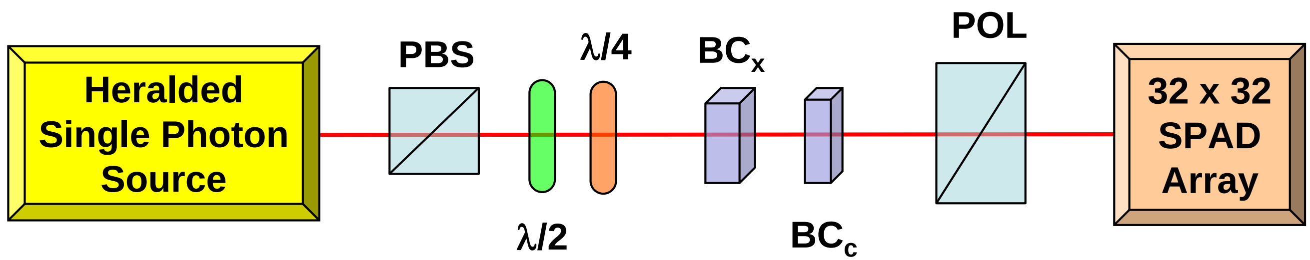

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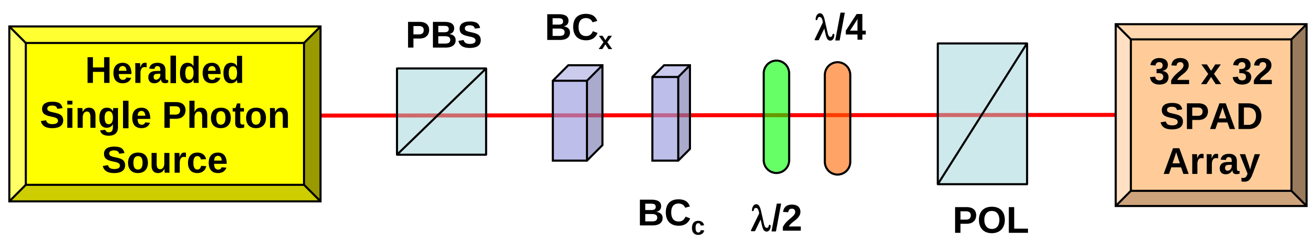

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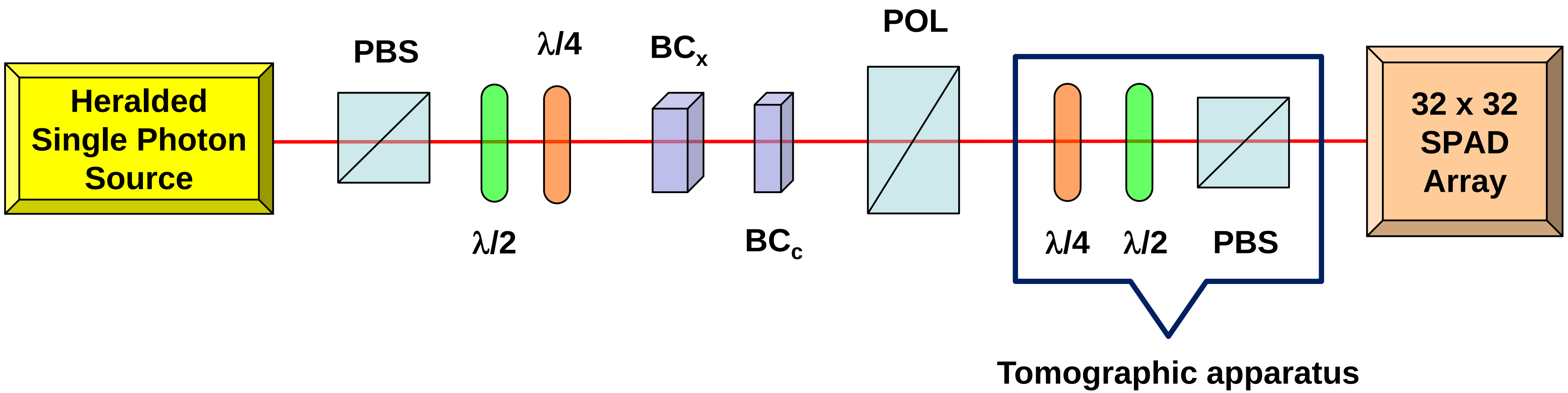
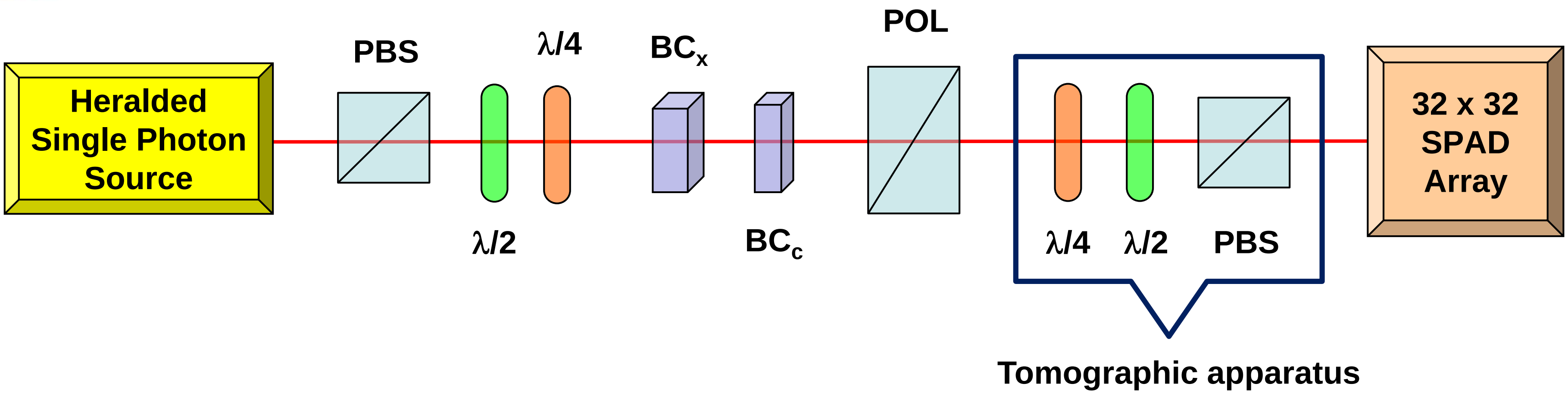
Condition 3: consistency check



Condition 3: consistency check



We produce the (tomographically complete) set of states $\{|H\rangle, |+\rangle, |L\rangle, |R\rangle\}$, and perform quantum tomography after post-selection, calculating the fidelity with respect to the post-selected state $|\psi_f\rangle = \cos(0.18\pi)|H\rangle + \sin(0.18\pi)|V\rangle$:

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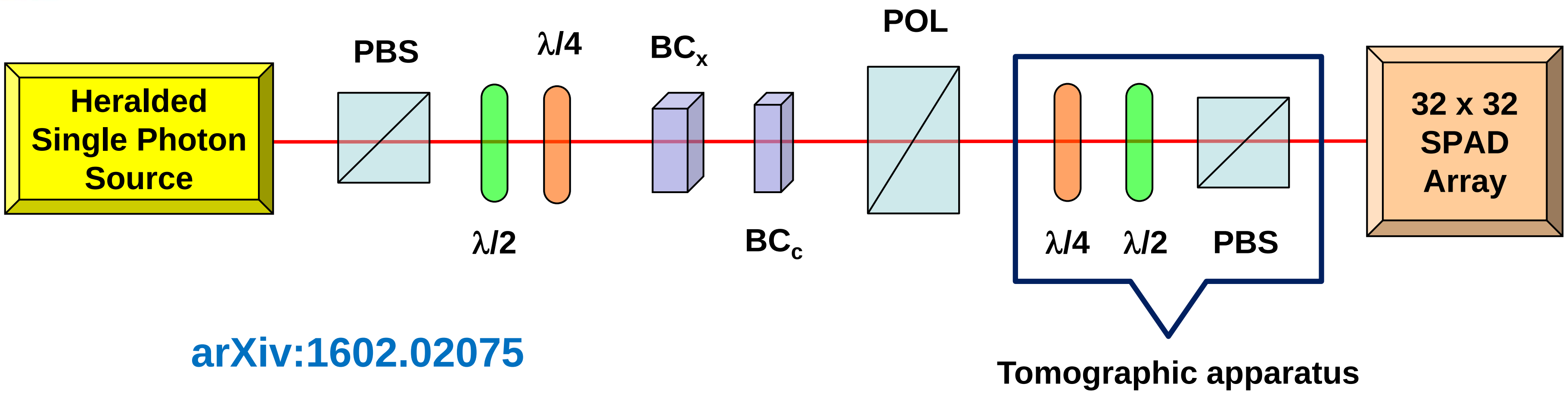

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$$\mathcal{F}_H = 0.9995$$
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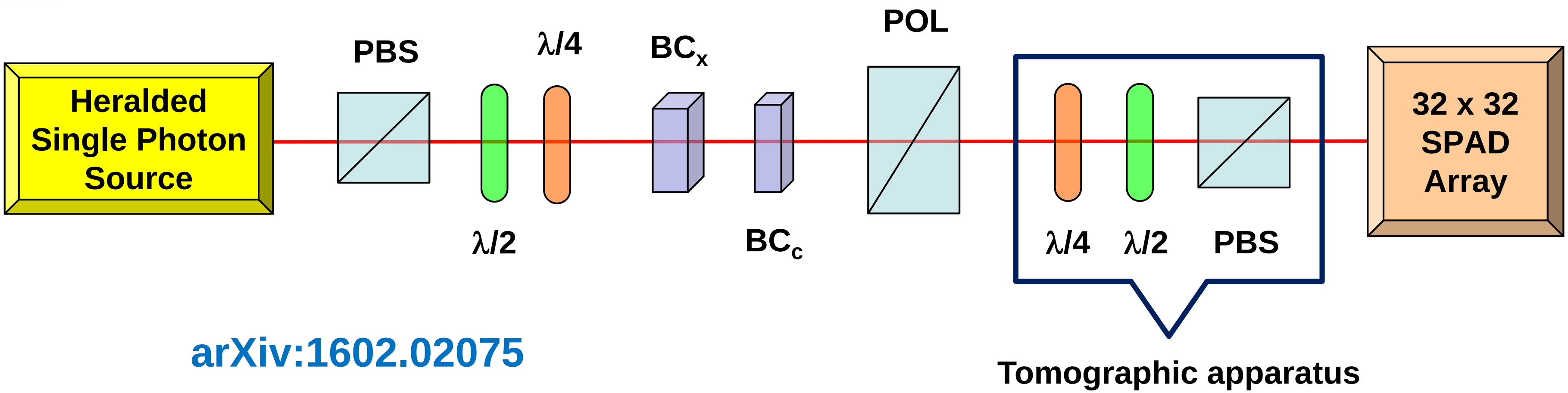

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→

$$p_d = 0.0051 \pm 0.0046$$

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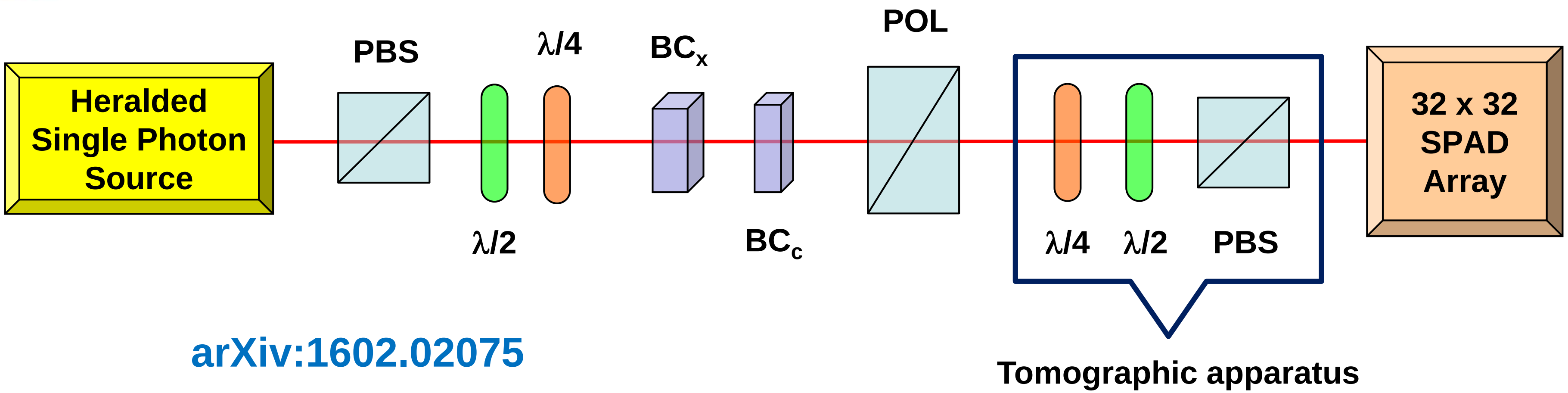

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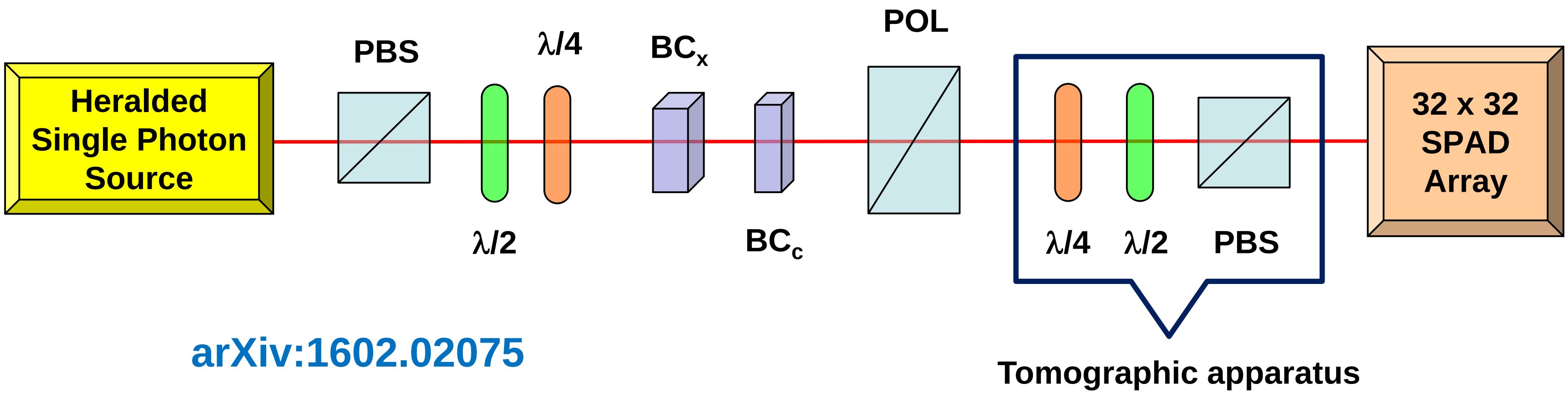

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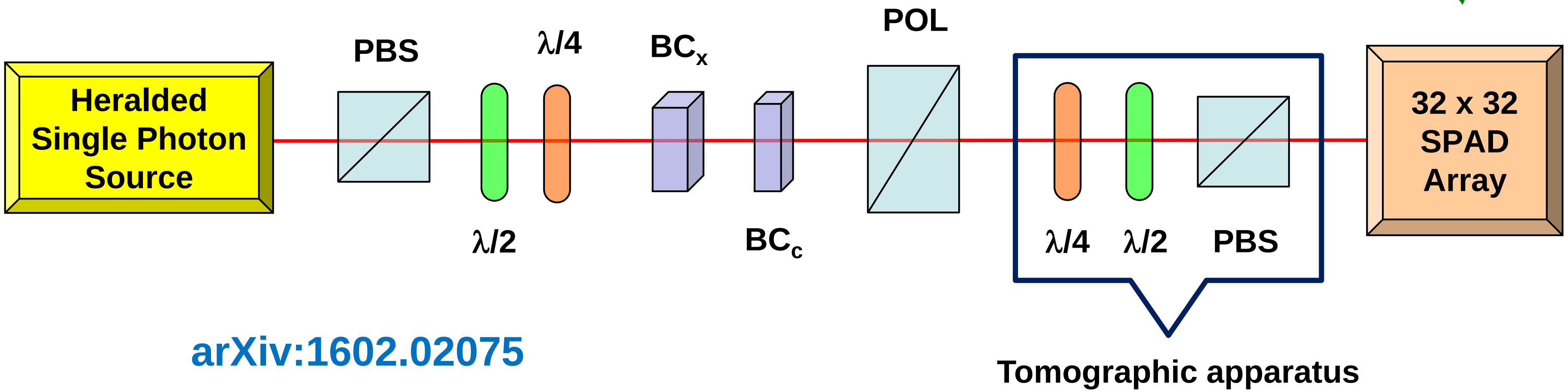

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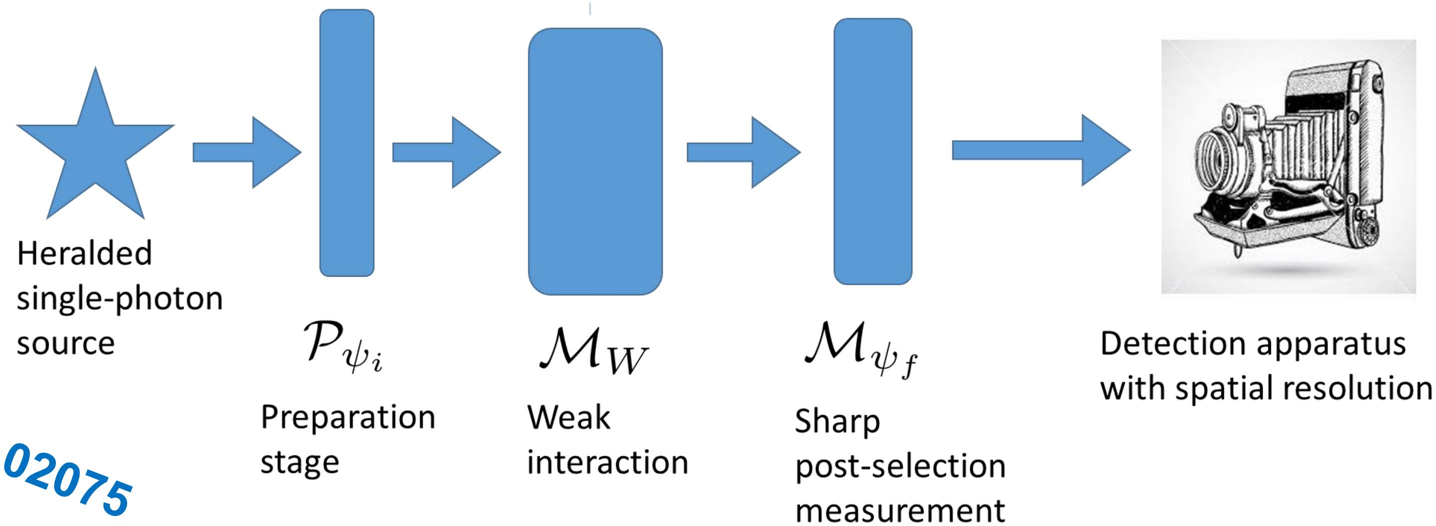
WM and contextuality: final check



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$$\text{Initial and final states are non-orthogonal}: \quad p_{\psi_f} := \mathbb{P}\left(\text{PASS}|\mathcal{P}_{\psi_i},\mathcal{M}_{\psi_f}\right) > 0$$

$$\text{Without post-selection: } \mathbb{P}\left(x|\mathcal{P},\mathcal{M}_W\right) = p_n(x-g)\mathbb{P}\left(1|\mathcal{P},\mathcal{M}_\Pi\right)+p_n(x)\mathbb{P}\left(0|\mathcal{P},\mathcal{M}_\Pi\right) \quad \forall\mathcal{P}$$

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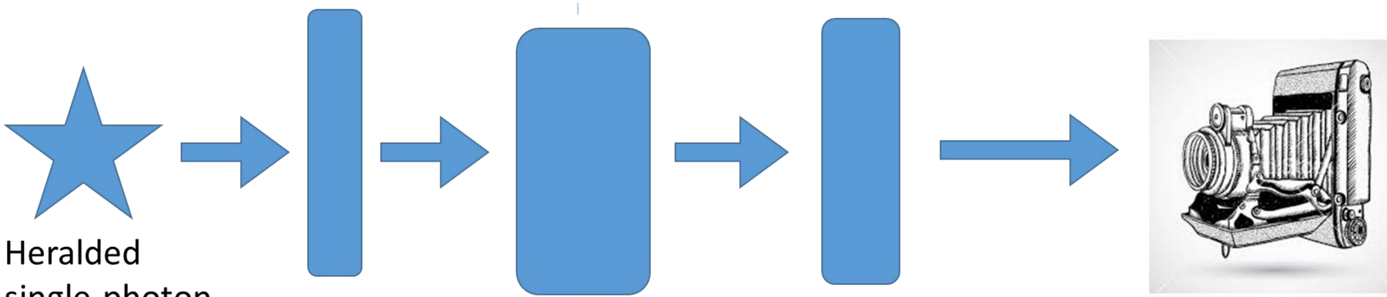
$$p_- := \left(p_{\psi_f}\right)^{-1}\int_{-\infty}^{0}\mathbb{P}\left(x,\text{PASS}|\mathcal{P}_{\psi_i},\mathcal{M}_W,\mathcal{M}_{\psi_f}\right)\mathrm{d}x : \quad \mathcal{I} = p_- - \frac{1}{2} - \frac{p_d}{p_{\psi_f}} > 0$$

WM and contextuality: final check



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Without post-selection: $\mathbb{P}\left(x|\mathcal{P}, \mathcal{M}_W\right) = p_n(x-g)\mathbb{P}\left(1|\mathcal{P}, \mathcal{M}_\Pi\right) + p_n(x)\mathbb{P}\left(0|\mathcal{P}, \mathcal{M}_\Pi\right) \quad \forall \mathcal{P}$

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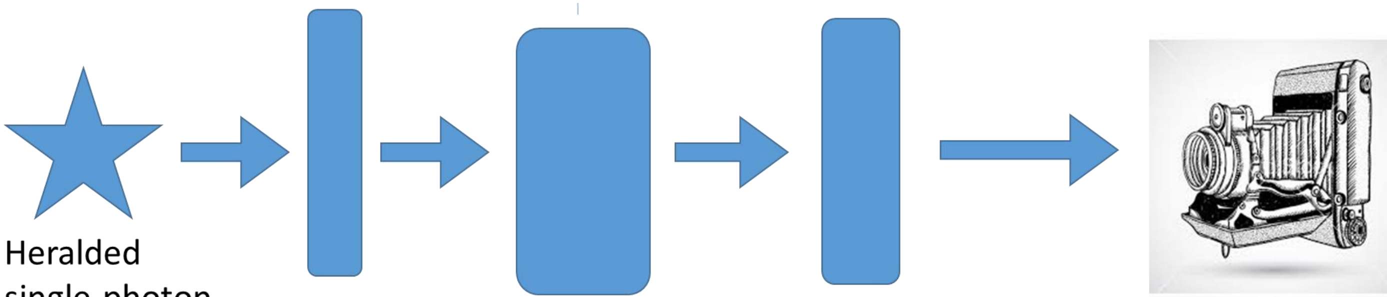

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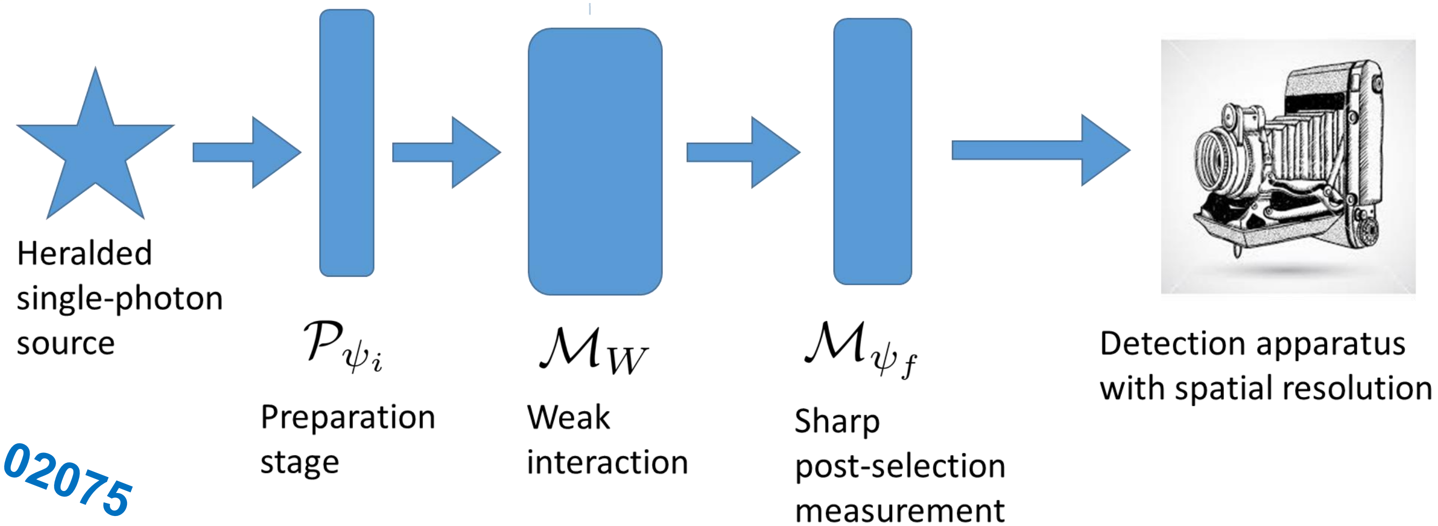
WM and contextuality: final check



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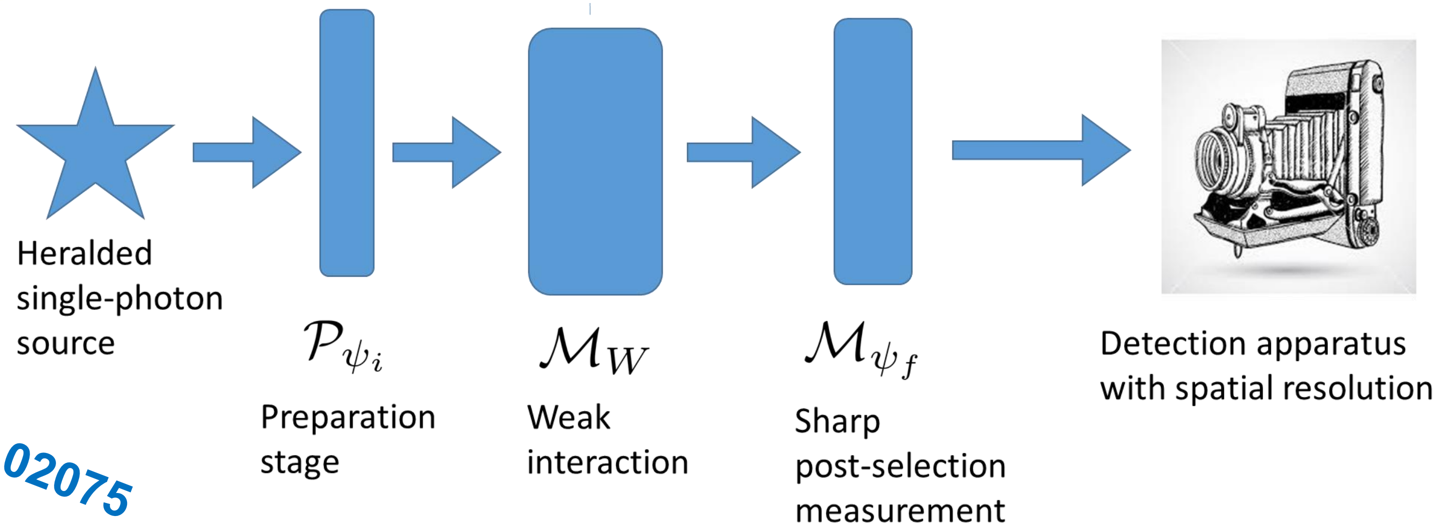
Initial and final states are non-orthogonal : $p_{\psi_f} := \mathbb{P}\left(\text{PASS}|\mathcal{P}_{\psi_i}, \mathcal{M}_{\psi_f}\right) > 0$ ✓

Without post-selection: $\mathbb{P}\left(x|\mathcal{P}, \mathcal{M}_W\right) = p_n(x-g)\mathbb{P}\left(1|\mathcal{P}, \mathcal{M}_\Pi\right) + p_n(x)\mathbb{P}\left(0|\mathcal{P}, \mathcal{M}_\Pi\right) \ \forall \mathcal{P}$ ✓

$$\exists\, p_d : \quad \mathbb{P}\left(\text{PASS}|\mathcal{P}, \mathcal{M}_W, \mathcal{M}_{\psi_f}\right) = (1-p_d)\mathbb{P}\left(\text{PASS}|\mathcal{P}, \mathcal{M}_{\psi_f}\right) + p_d\mathbb{P}\left(\text{PASS}|\mathcal{P}, \mathcal{M}_d\right) \quad \forall \mathcal{P}$$ ✓

$$p_- := \left(p_{\psi_f}\right)^{-1} \int_{-\infty}^{0} \mathbb{P}\left(x, \text{PASS}|\mathcal{P}_{\psi_i}, \mathcal{M}_W, \mathcal{M}_{\psi_f}\right) \mathrm{d}x : \quad \mathcal{I} = p_- - \frac{1}{2} - \frac{p_d}{p_{\psi_f}} > 0$$ ✓

WM and contextuality: final check



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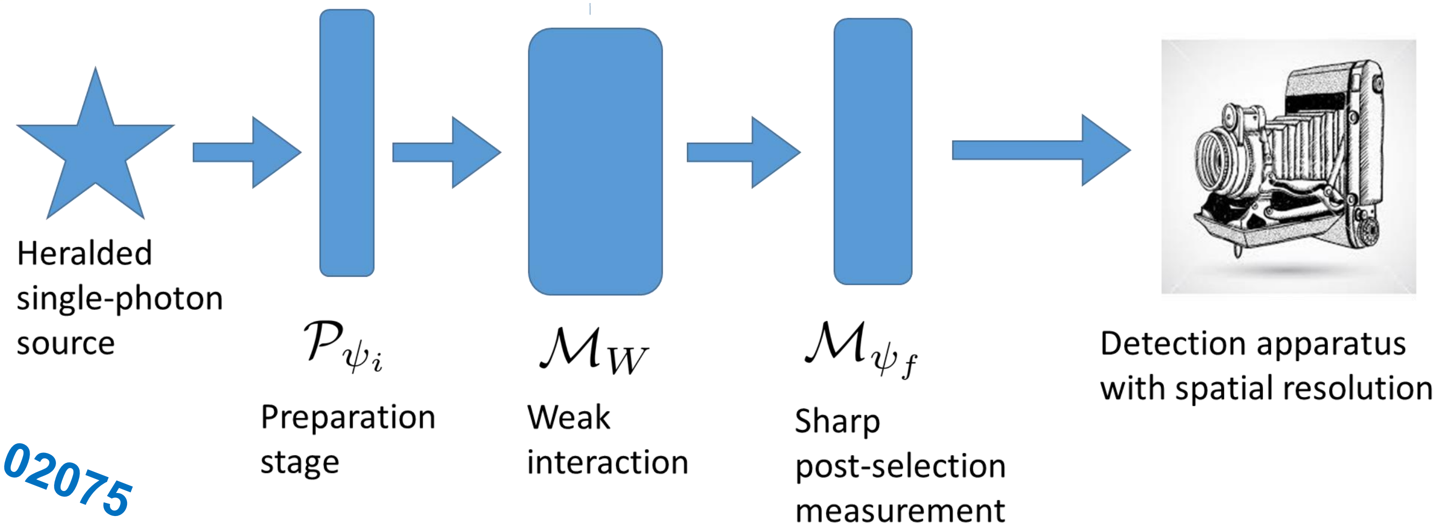
WM and contextuality: final check



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No non-contextual model allowed: weak measurements proved quantum contextuality

The weak crew

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Thanks for your attention!!