Dynamical Decoupling leads to improved scaling in noisy metrology

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Motivation & Layout

- Quantum control beneficial when evolution described by master equation (P. Sekatski)
- In this talk, interjecting pulses much much faster than system-bath coherence time

$$H = H_S + H_{SE} = \omega\, \sigma_3 \otimes 1 + \sum_{j=0}^{3} c_j\, \sigma_j \otimes A_j$$

c_j=coupling strengths A_j=bath operators

Bang-Bang control for metro

- Bang-bang control used for storage [PRL, 82, 2417] and gate design [PRL,102, 080501, 104,090501]
- In metrology bang-bang used to correct for transversal noise [PRL, 104, 020401, PRA 87, 032102]
- Take bang-bang to its full limit.

Decoupling Strategy

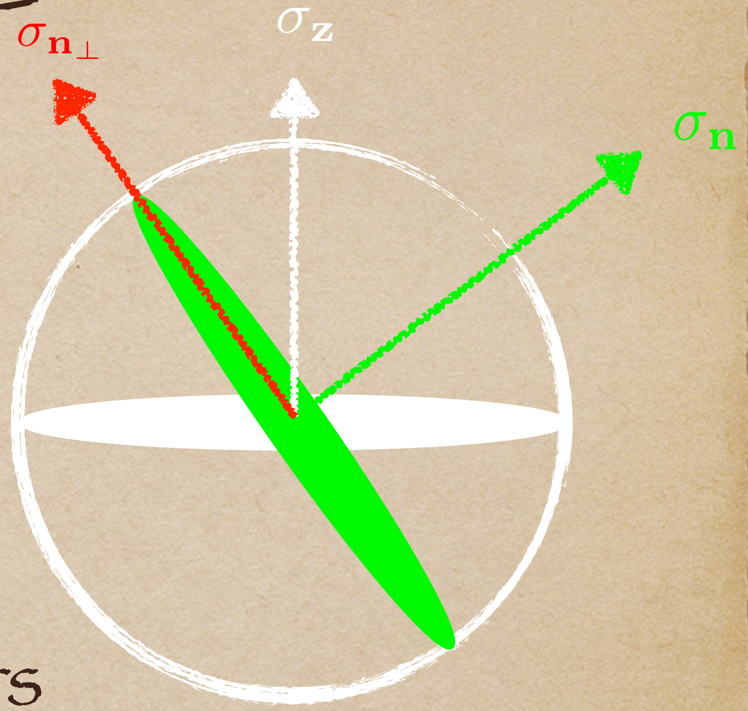
The main idea in dynamical decoupling is to modify the effective evolution of the system

$$U_i = u_i \otimes 1$$

where the U_i's act only on the system

Rank-1 noise

$$H = \omega\,\sigma_{\mathbf{z}} \otimes 1 + c_{\mathbf{n}} \sigma_{\mathbf{n}} \otimes A_{\mathbf{n}}$$



(i) Evolve system under H for half the time and under $(\sigma_{\mathbf{n}_\perp} \otimes 1)\, H\, (\sigma_{\mathbf{n}_\perp} \otimes 1)$ for the other half

(ii) Note that one has to optimise over all vectors in the perpendicular plane

(ii) Noise completely cancels

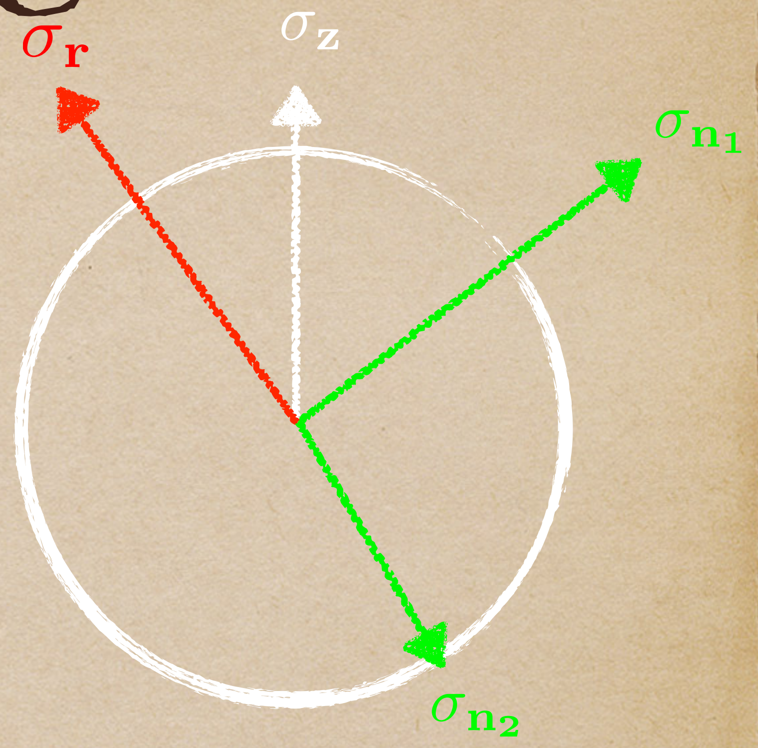
(iii) Unitary evolution becomes $(\mathbf{n}_\perp \cdot \mathbf{z})\sigma_{\mathbf{n}_\perp}$

(iv) Evolution is "slowed down" by a constant factor but noiseless (QFI scales as N^2)

(v) Only exception is if $\mathbf{n} = \mathbf{z}$

Rank-2 noise

$$H = \omega\, \sigma_{\mathbf{z}} \otimes 1 + \sum_{j=1}^{2} c_j\, \sigma_{\mathbf{n_j}} \otimes A_{\mathbf{n_j}}$$



(i) Evolve system under H for half the time and under $(\sigma_{\mathbf{r}} \otimes 1)\, H\, (\sigma_{\mathbf{r}} \otimes 1)$ for the other half

(ii) Noise completely cancels

(iii) Unitary evolution becomes $(\mathbf{r} \cdot \mathbf{z})\sigma_{\mathbf{r}}$

(iv) Evolution is "slowed down" by a constant factor but noiseless (QFI scales as N^2)

(v) Only exception is if $\mathbf{r} \cdot \mathbf{z} = 0$

Rank-3 noise

- If we have full Schmidt rank then we cannot eliminate all the noise with previous procedure
- The results are the same even if we assume the environment operators to be unbounded
- However, the scaling of the QFI in the case where we cannot eliminate noise, depends on the environment operators

Multi-qubit case

- N qubits coupled to independent environments

$$H = \sum_{a=1}^{N} \left(\omega\, \sigma_3^{(a)} \otimes 1 + \sum_{j=0}^{3} c_j^{(a)}\, \sigma_j^{(a)} \otimes A_j^{(a)} \right)$$

- We can perform dynamical decoupling to each individual qubit to eliminate all rank-1 & rank-2 noise

- In addition we can perform fast random swaps (O(N) operations)

- Random permutations leave H_S unchanged, but eliminate any asymmetric noise terms, leaving only the symmetric part of the noise

$$H_{SE} = \bar{c}_3 S_3 \otimes \bar{A} \quad \bar{A} = \frac{1}{\bar{c}_3} \sum_a c_3^{(a)}\, A_3^{(a)} \quad \bar{c}_3 = \frac{1}{N} \sum_a c_3^{(a)}$$

Global fluctuating noise

- Case 1: $A_3^{(a)} = 1$ and $\bar{c}_3$ known but fluctuating with $p(\bar{c}_3)$
- $\frac{\mathcal{F}(\rho^N_{\omega,t})}{t} \leq N\sqrt{\mathcal{F}_{\text{classical}}(p(\bar{c}_3))}$

$$\mathcal{F}_{\text{classical}}(p(\bar{c}_3)) = \int \frac{\left(\frac{\mathrm{d}p(\bar{c}_3)}{\mathrm{d}\bar{c}_3}\right)^2}{p(\bar{c}_3)}\,\mathrm{d}\bar{c}_3$$

- If $p(\bar{c}_3)$ is normally distributed with width σ then

$$\frac{\mathcal{F}(\rho^N_{\omega,t})}{t} \leq \frac{N}{\sigma}$$

- A strategy using the N-qubit GHZ state yields

$$\frac{\mathcal{F}(GHZ^N_{\omega,t})}{t} \approx 0.43\frac{N}{\sigma}$$

Local fluctuating noise

- Case 2: $A_3^{(a)} = 1$ and $c_3^{(a)}$ independent and normally distributed each with width σ

- Perform fast permutations ⇒ $\bar{c}_3$ is normally distributed with width $\bar{\sigma} = \frac{\sigma}{\sqrt{N}}$

- Now prepare N qubits in GHZ state

$$\frac{\mathcal{F}(GHZ^N_{\omega,t})}{t_{\text{opt}}} = \frac{N^{\frac{3}{2}}}{\sqrt{2e\sigma}}$$

- with $t_{\text{opt}} = \frac{1}{\sqrt{N}\sigma}$

General Noise

- Case 3: $\bar{A}_3$ has discrete spectrum and $\bar{c}_3$ fixed

- Classical Fisher becomes unbounded and no general statements about scaling can be made

- Example: Consider $\bar{A}_3$ with equally gapped spectrum of gap Δ

- Then at time $t = \dfrac{2\pi}{\Delta \bar{c}_3}$ we have complete re-phasing and the QFI scales quadratically with N

Summary

- Dynamical decoupling permits us to correct for a large majority of errors in the Hamiltonian picture
- Full power of Bang-bang control, by allowing to modify the system Hamiltonian in a controlled way
- Additional techniques (symmetrization) can help boost precision beyond SQL for certain types of noise
- Ideas can be implemented in the design of high-fidelity quantum gates (e.g. DD can perfectly eliminate ALL local noise terms in designing a ZZ gate)

Dziękuję Ci

See ArXiv:1512.07476

for all the details