Recent Advances in Quantum Metrology

Precision limits for frequency estimation in open quantum systems

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Introduction

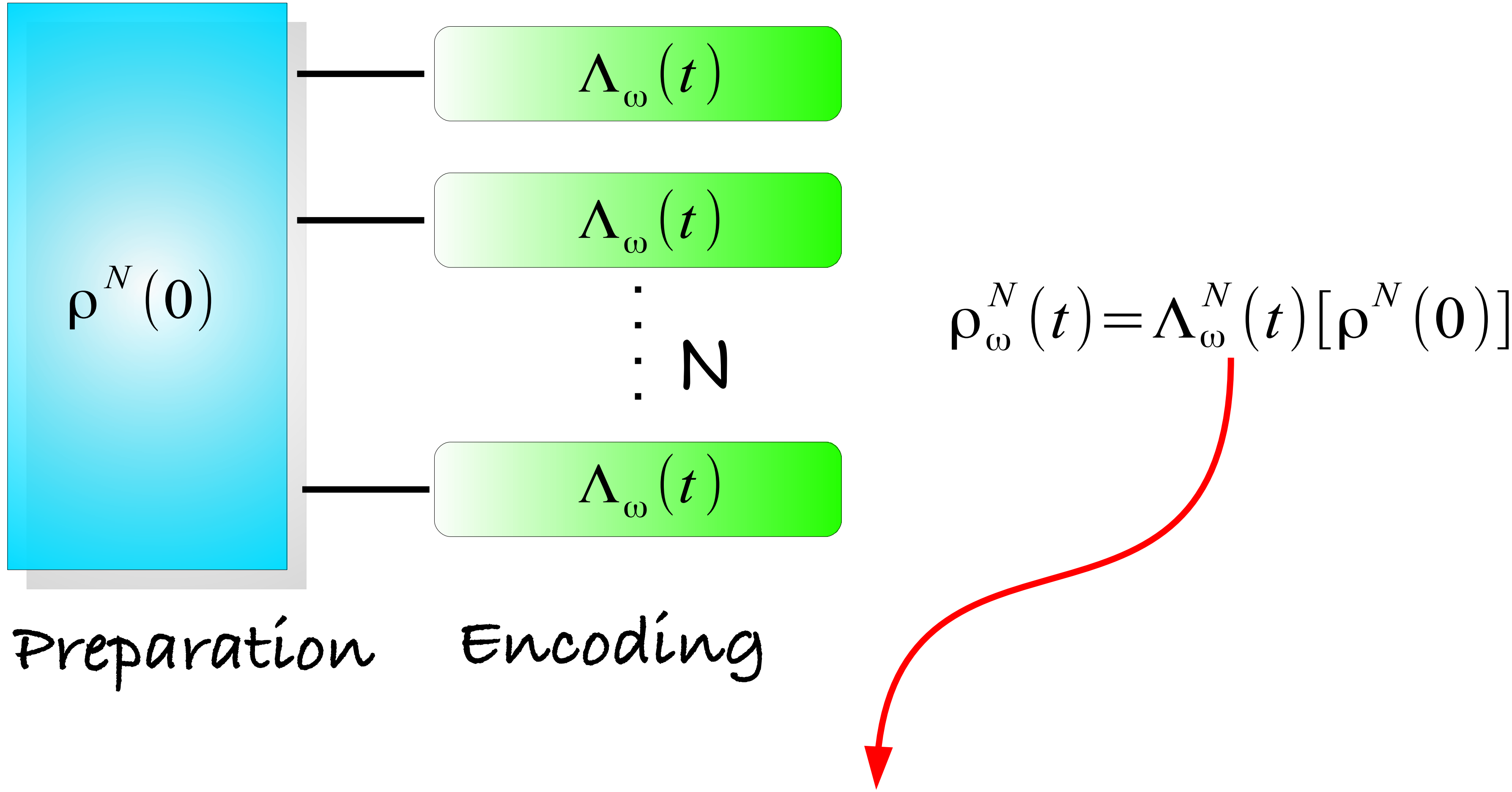
- General framework for frequency estimation
- SQL vs HL in the noiseless scenario
- Recent no-go theorems for noisy estimation

Noisy frequency estimation

$$\rho^N(0)$$

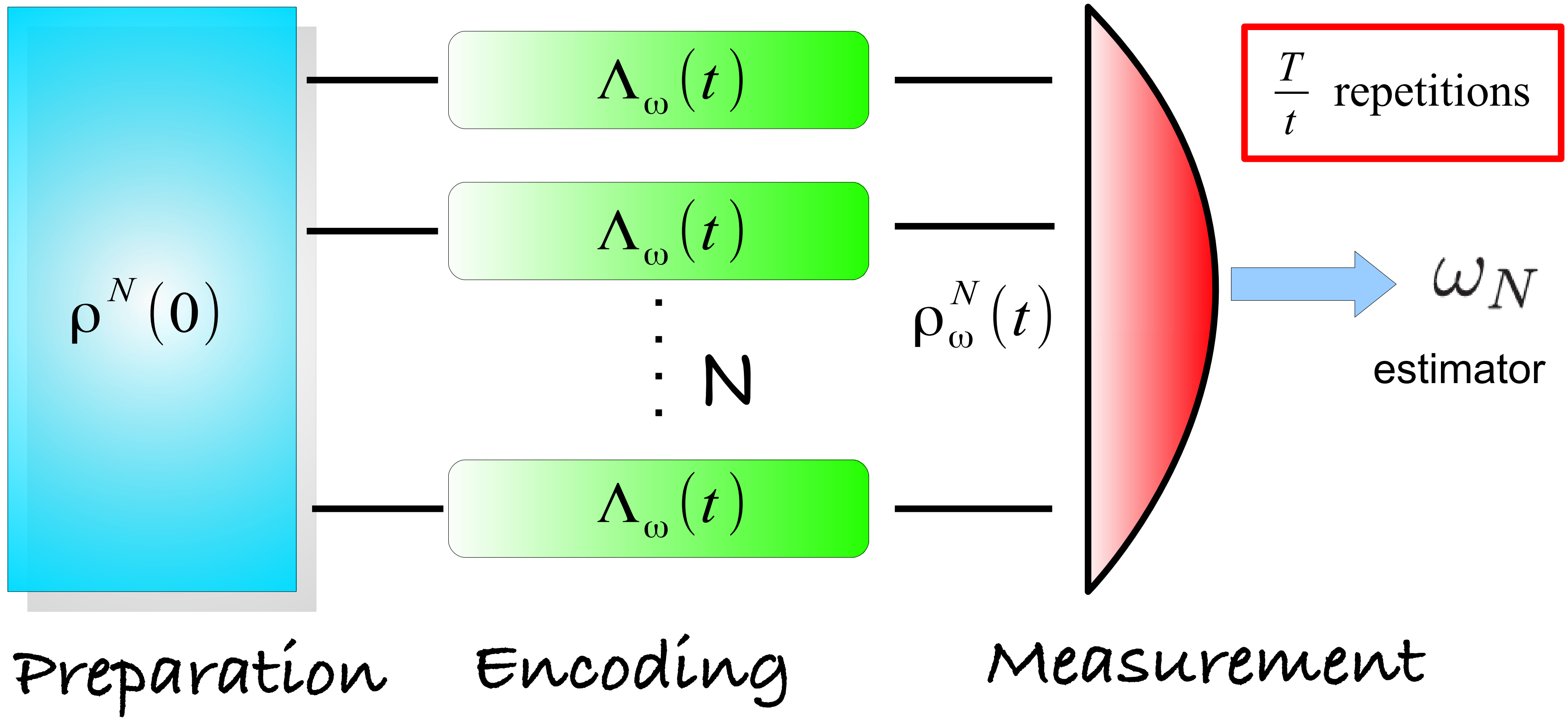
Preparation

Noisy frequency estimation



- Both the unitary encoding and the noise act identically and independently on the N probes: CPTP maps $\left\{ \Lambda_\omega^{(N)}(t) = [\Lambda_\omega(t)]^{\otimes N} \right\}_{t \geq 0}$

Noisy frequency estimation



Resources:

- Number of probes N
- Total available time T

We can control t in order to minimize the error

The optimal evaluation time $t_{\text{opt}}(N)$ will generally depend on N!!

Semigroup noise: back to the SQL

- Noiseless scenario $\Delta^2 \omega_N T = \frac{1}{N t}$ vs $\Delta^2 \omega_N T = \frac{1}{N^2 t}$

  SQL HL

  Giovannetti, Lloyd & Maccone
  Science 306, 1330 (2004)

- Semigroup noise $\dot{\rho}_\omega(t) = i\frac{\omega}{2}[\hat{\sigma}_z, \rho_\omega(t)] + \gamma(\hat{\sigma}_z \rho_\omega(t) \hat{\sigma}_z - \rho_\omega(t))$ Huelga, et.al PRL 79, (1997)

  - Recent techniques to evaluate QFI Demkowicz, Kolodynski & Guta Nat. Comm. 3, (2012)

QCRB → Lower bound to $\Delta^2 \omega_N$

$$\max_\rho F_Q[\Lambda_\omega(\rho)] \leq 4N \min_{\{K_i\}} (\|A\| + (N-1)\|B\|^2)$$

$A = \sum_i \dot{K}_i^\dagger \dot{K}_i$, $B = \sum_i \dot{K}_i^\dagger K_i$

Can be evaluated by means of SDP !! Kolodynski &Demkowicz NJP 15, 073043 (2013)

- *No-go theorems*

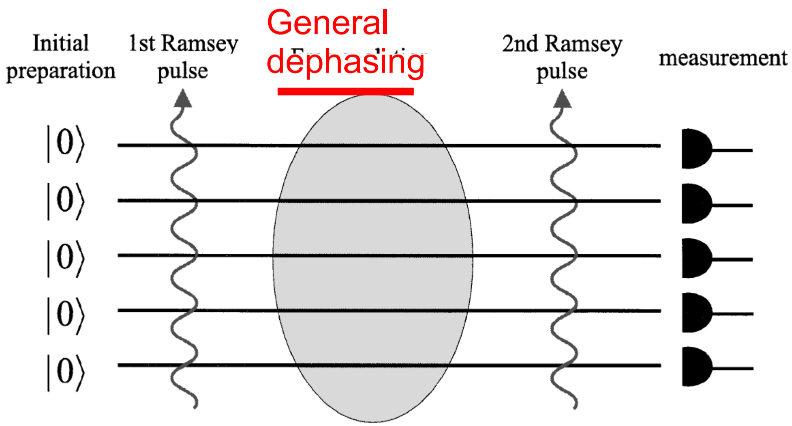
$$\frac{\Delta \omega_N|_{ent}}{\Delta \omega_N|_{sep}} = C > 1$$

← Independent from N

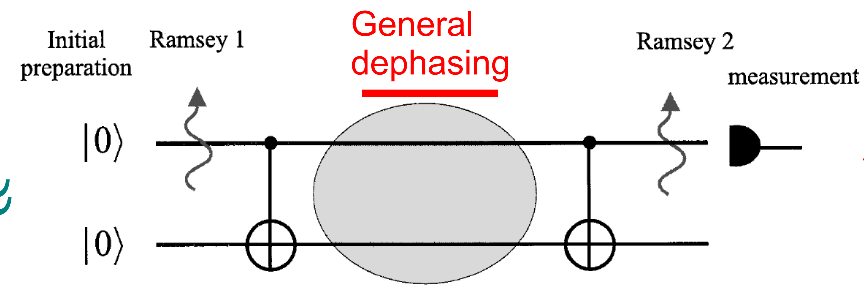
Beyond semigroup:a new limit for metrology?

- $\gamma \longrightarrow \gamma(t) = \int_0^\infty d\omega\, J(\omega) \coth\left(\frac{\beta\omega}{2}\right) \frac{\sin(\omega t)}{\omega}$

Chin, Huelga & Plenio
PRL 109, 233601 (2012)



$\not\approx$



- Short-time expansion $\gamma(t) \sim \gamma\, t$

$$\Delta^2 \omega_N T = \frac{2\sqrt{2\gamma e}}{N}$$

$$\Delta^2 \omega_N T = \frac{2\sqrt{2\gamma e}}{N^{3/2}}$$

Scaling intermediate between SQL and HL !!

How general is this limit ?
Can we go beyond it?

- Initial state preparation
- Measurement procedure
- For which open system's dynamics ?
- Beyond the short-time regime: any role of *proper* non-Markov ?

A. Smirne, J. Kolodynski, S. Huelga & R. Demkowicz-Dobrzanski
ArXiv: 1511.02708 (2015); to appear in PRL

Phase covariant dynamical maps

- **I.I.C. noise** $\Lambda^{(N)}(t) = (\mathcal{U}_\omega(t) \circ \Gamma(t))^{\otimes N}$

$$e^{-\frac{i}{2}\omega\sigma_z t} \bullet e^{\frac{i}{2}\omega\sigma_z t}$$

Unitary frequency encoding

$$\boxed{[\mathcal{U}_\omega(t), \Gamma(t)] = 0 \quad \forall t}$$

ω- independent noise map

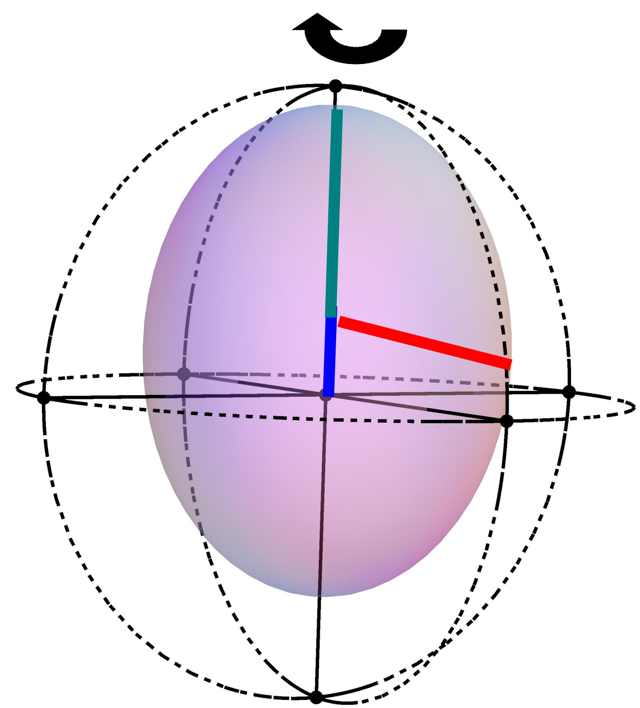
Key property to have SQL with semigroup noise

- Hilbert-Schmidt matrix representation

$$\Lambda_\omega^{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \eta_\perp \cos\psi & -\eta_\perp \sin\psi & 0 \\ 0 & \eta_\perp \sin\psi & \eta_\perp \cos\psi & 0 \\ \kappa & 0 & 0 & \eta_\parallel \end{pmatrix}$$

$\psi = \omega t + \theta$

CP
$$\eta_\parallel \pm \kappa \le 1$$
$$1 + \eta_\parallel \ge \sqrt{4\eta_\perp^2 + \kappa^2}$$



Physical meaning

Associated master equation

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) \;=\;& -\frac{i}{2}(\omega + h(t))[\sigma_z, \rho(t)] + \gamma_+(t)\left(\sigma_+\rho(t)\sigma_- - \frac{1}{2}\{\sigma_-\sigma_+, \rho(t)\}\right) \\
& + \gamma_-(t)\left(\sigma_-\rho(t)\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho(t)\}\right) + \gamma_z(t)(\sigma_z\rho(t)\sigma_z - \rho(t))
\end{aligned}
$$

Lamb shift

absorption

dephasing

emission

- Commutation of *linear maps*
- Broken by counter-rotating terms, transversal noise — Chaves & al. PRL 111, 120401 (2013)
- U(1) covariant semigroups — B. Vacchini, Lect. Notes Phys. 787, 39 (2010)

Fully general time-inhomogeneities and memory effects ***are included***

Metrological limit for IIC dynamics

Given any N-qubit IIC dynamics, if

$$\eta_\perp(t) \neq 1 \qquad \forall t > 0$$

the uncertainty on the estimated frequency satisfies

$$\lim_{N\to\infty} \frac{\Delta^2\omega_N T}{N^{-(2\beta_\perp - 1)/\beta_\perp}} \geq D$$

- Scaling fixed by the short-time expansion $\eta_\perp(t) = 1 - c_\perp t^{\beta_\perp} + o(t^{\beta_\perp})$
- From the SQL ($\beta_\perp = 1$) toward the HL with the growing of $\beta_\perp$
- Constant D > 0 fixed by the expansions of all the coefficients

$D\,N^{-3/2}$
for $\beta_\perp = 2$

$$D = \frac{c^{1/\beta_\perp}\beta_\perp}{(\beta_\perp - 1)^{(\beta_\perp - 1)/\beta_\perp}} \qquad c = \begin{cases} 2c_\perp & \beta_\perp < \beta_\parallel; \\ 2c_\perp - \frac{c_\parallel}{2} & \beta_\perp = \beta_\parallel < \beta_\kappa; \\ \max\left\{2c_\perp - \frac{c_\parallel}{2} - \frac{|c_\kappa|}{2}, \frac{|c_\kappa|}{4}\right\} & \beta_\perp = \beta_\parallel = \beta_k. \end{cases}$$

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<u>*No full revivals*</u> !! $\eta_\perp(t) = 1$ implies (due to CP) $\eta_\parallel(t) = 1$ $\kappa(t) = 0$

Rotation by a generic angle $\psi(t)$ *about the z-axis :* ***noiseless HL!!***

Mini sketch of the proof

- **Unital** dynamics *k(t)=0,* i.e. no translation of the Bloch sphere

  - $$\Delta^2\omega_N T \geq \min_t \frac{1+N\ell(t)}{tN^2}$$ with $\ell(t) = \frac{1+\eta_{\|}(t)-2\eta_{\perp}(t)^2}{2\eta_{\perp}(t)^2}$

  - $t(N) \longrightarrow \tau > 0$ for $N \longrightarrow \infty$ ⇒ (**No full revival**) $\ell(\tau) > 0$ ⇒ $\Delta^2\omega_N T \div \frac{1}{N}$

Mini sketch of the proof

- **Unital** dynamics *k(t)=0,* i.e. no translation of the Bloch sphere

  - $$\Delta^2\omega_N T \geq \min_t \frac{1+N\ell(t)}{tN^2} \longrightarrow \frac{1+\eta_{\parallel}(t)-2\eta_{\perp}(t)^2}{2\eta_{\perp}(t)^2}$$

  - $t(N) \longrightarrow \tau > 0$ for $N \longrightarrow \infty$ **No full revival** $\Rightarrow \ell(\tau) > 0 \Rightarrow \Delta^2\omega_N T \div \frac{1}{N}$

  - We can beat the SQL *only in the short-time regime:* leading order expansion plus *CPTP constraints*

$$t_{\rm opt}(N) \;=\; \left(c\,(\beta_{\perp}-1)N\right)^{-1/\beta_{\perp}}$$

Mini sketch of the proof

- **Unital** dynamics *k(t)=0,* i.e. no translation of the Bloch sphere

  - $$\Delta^2\omega_N T \geq \min_t \frac{1+N\ell(t)}{tN^2} \longrightarrow \frac{1+\eta_{\|}(t)-2\eta_{\perp}(t)^2}{2\eta_{\perp}(t)^2}$$

  - $t(N) \longrightarrow \tau > 0$ for $N \longrightarrow \infty$ — ***No full revival*** ⇒ $\ell(\tau) > 0$ ⇒ $\Delta^2\omega_N T \div \frac{1}{N}$

  - We can beat the SNL *only in the short-time regime:* leading order expansion plus *CPTP constraints*

  $$t_{\text{opt}}(N) = (c(\beta_{\perp}-1)N)^{-1/\beta_{\perp}}$$

- General case: ***convexity of the bound*** on the QFI ***for any N***

$$\Lambda_{\eta_{\|},\eta_{\perp},\kappa} = p\Lambda_{\tilde{\eta}_{\|},\tilde{\eta}_{\perp}} + (1-p)\Lambda_{\tilde{\kappa}}$$

generic covariant — unital — amplitude damping

Attainability of the bound

- $T/t \rightarrow \infty$ ⇒ QCRB is saturable + QFI for GHZ derived

Asymptotically always attainable, at worst up to a constant factor !

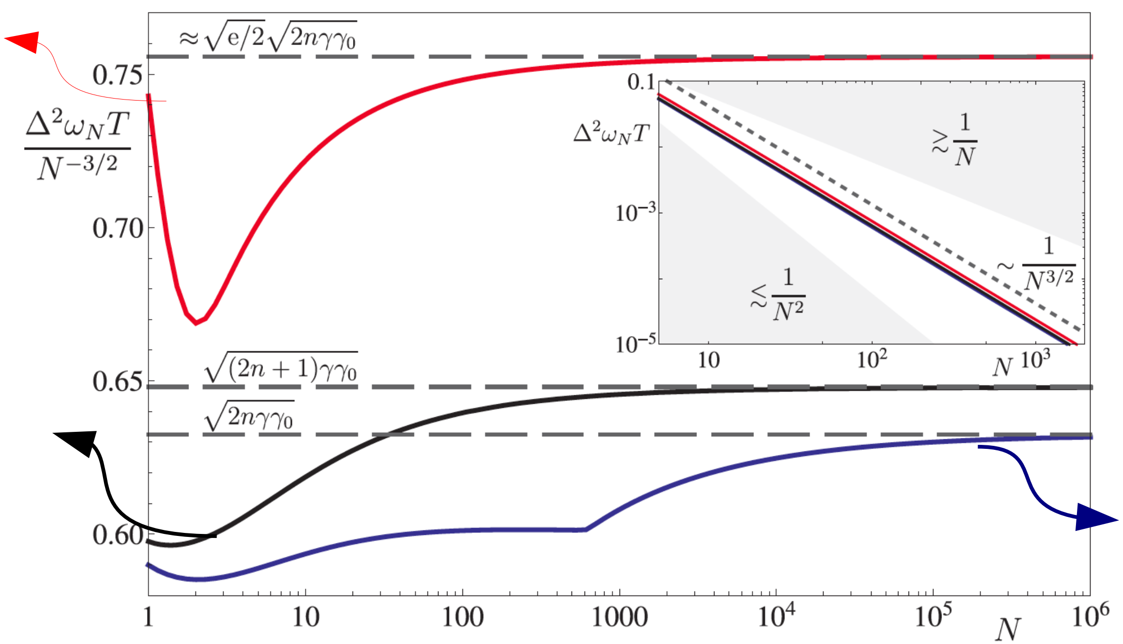
- Shabani-Lidar post-Markovian model: general covariant dynamics

  Phys. Rev. A 71, 020101 (2005)

  γ dissipation constant γ_0 memory rate n excitations in E

GHZ initial state

Numerically fully optimized bound



Our bound

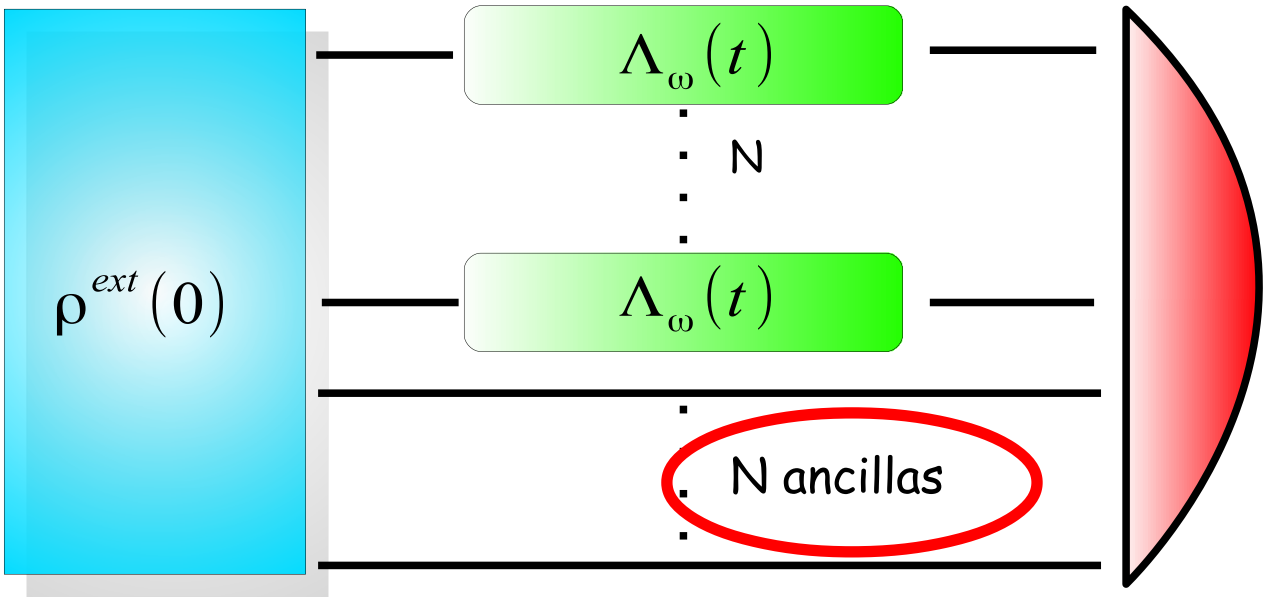
Generalization to single-step EC

- **Pure dephasing:** proof of the conjecture of Chin, Huelga & Plenio PRL 109 (2012)
  $k=0\,;\, n_{\|}=1$ recently verified in Macieszczak PRA 92, 010102 (R) (2015)

- **Single-step error correction**

$$F_{\mathrm{Q}}\left[[\Lambda_\omega \otimes \mathbb{1}]^{\otimes N}\right] \leq F^{\uparrow}[\Lambda_\omega]$$

The same bound applies !



What happens with multi-step EC?

- Semigroup: Demkowicz & Maccone PRL 113 (2014)
- Beyond: open question!

Non-Markovianity and Zeno regime

Non-Markovianity of quantum dynamics

- Several definitions, based on different ***divisibility properties***

$$\Lambda(t) = \Lambda(t,s)\Lambda(s) \qquad t \geq s \geq 0$$

propagators (in general, not even P!)

- $\Lambda(t,s)$ are CP ⟺ the dynamics is Markovian

  TLME with positive rates at any time

  Rivas, Huelga, Plenio
  PRL 105, 050403 (2010)

- $\Lambda(t,s) = \Lambda(t-s)$ ***time homogeneous dynamics***

  Semigroup composition of CP maps! Lindblad,...

- Witnesses (i.e. sufficient conditions) for non-Markovianity

  Breuer, Laine, Piilo
  PRL 103, 210401 (2009)

  NM related to a ***back-flow of information*** to S

(No) role of non-Markovianity

Can the back-flow of information about ω lead to an improved precision scaling?

NO!

$$t_{opt}(N) \to 0 \quad (\text{as } N^{-1/\beta_\perp})$$

The best strategy calls for measurement on the short-time scale

On longer times, necessarily SQL $1/N$

The crucial feature to beat the SQL with IIC dynamics is

violation of semigroup (i.e. time inhomogeneity)

$$\Lambda(t+s) \neq \Lambda(t)\Lambda(s)$$

at short time-scales

This is the case also in the *finite-N* regime

Zeno regime

- Beyond the semigroup: QZE effect $P(t) = |\langle\psi|U(t)|\psi\rangle|^2 = 1 - \lambda t^2$

$$P_K(t) = [P(\tau)]^K \sim \exp(-\lambda t \tau) \xrightarrow[\tau \to 0]{} 1 \qquad \tau = \frac{t}{K}$$

number of measurements

fixed time

interval between measurements

- Same quadratic decay for any ***exact*** reduced dynamics

$$\langle\psi_S|\,\Lambda(t)\,[|\psi_S\rangle\langle\psi_S|]\,|\psi_S\rangle \sim 1 - \lambda_S t^2 \iff \eta_\perp(t) \sim 1 - c_\perp t^2$$

For an exact reduced dynamics of a global unitary dynamics $\beta_\perp = 2$

$$\Rightarrow \quad \Delta^2 \omega_N T \geq \frac{D}{N^{3/2}}$$

Conclusions and outlooks

- We derived a ***general bound*** to the precision in the frequency estimation for a wide and well-defined class of dynamics (***I.I.C.***)
- ***Attainability***: entanglement of the probes allows to ***beat the SQL*** also in the presence of ***realistic noise***, which nevertheless limits the precision ***below the HL***
- Emergence of super-classical scaling due to ***time-inhomogeneity*** (non-exponential decays); NM does not allow further improvement
- Inclusion of NM ***non phase-covariant*** noise and ***general*** (multi-steps) ***error-correction*** techniques, where NM could play a relevant role
- Detailed analysis of the ***small-N regime***, also in view of possible experimental realizations

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PAPETS

SEVENTH FRAMEWORK PROGRAMME

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