Investigating the spatial distribution of entanglement and its role in quantum metrology

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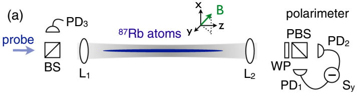
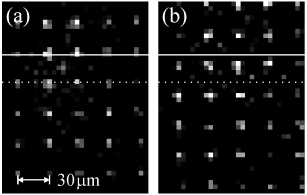
Quantum systems are growing

Cloud of cold atoms



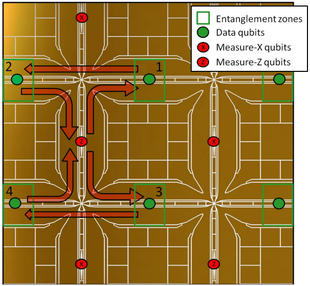
PRL**113**,093601 (2014)

Large array of cold atoms



New J. Phys. **14**, 123034 (2014)

Blueprint of a microwave ion trap quantum computer



arXiv:1508.00420

$\Longrightarrow$ Spatial distribution of fields/ entanglement matters

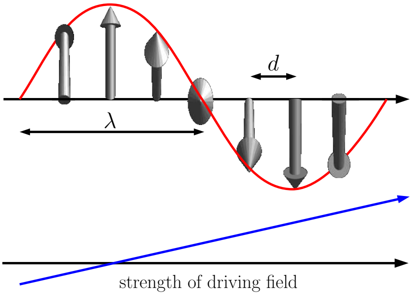
Outline

Sinusoidal observable

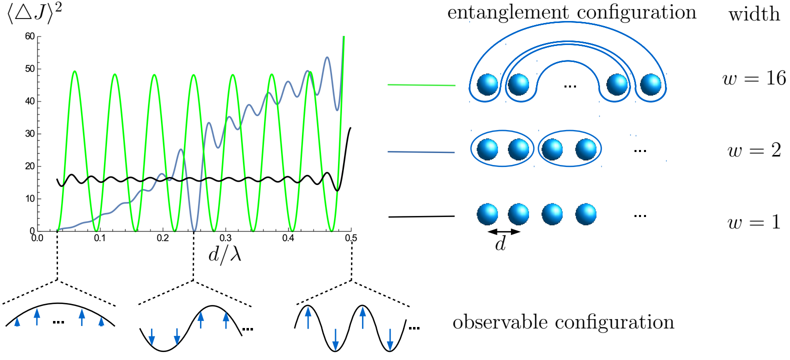
- Observable

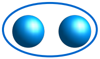
$$\vec{J} = \sum_j \sin\left(2\pi \frac{x_j}{\lambda}\right) \vec{\sigma}_j$$

- Position dependent rotation of spin



Spatial distribution matters





$$|\psi^-\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$$

S.Wölk and O.Gühne, arXiv: 1507.07226

Detecting long range entanglement

- Find the mimimum of

$$\left(\triangle\vec{J}\right)^2 = (\triangle J_x)^2 + (\triangle J_y)^2 + (\triangle J_z)^2$$

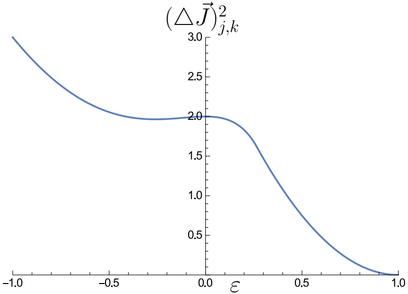
  for given entanglement distribution
- Minimum is achieved by pure states
- The variance splits into

$$\left(\triangle\vec{J}\right)^2 = \sum_{\{j,k\}} \left(\triangle\vec{J}\right)^2_{j,k}$$

  with $\{j, k\}$ denoting pairs of entangled states

Optimization

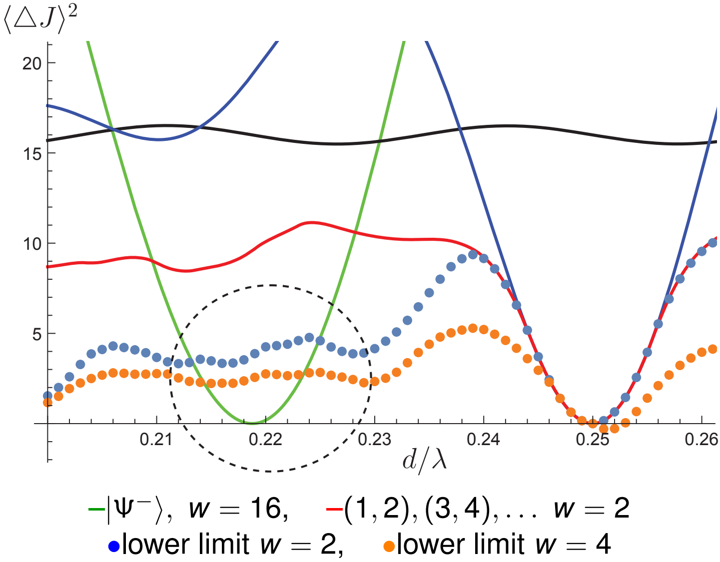
$$\vec{J} = \vec{\sigma}_j + \varepsilon \vec{\sigma}_k$$



$$\min_{|\psi\rangle} (\Delta \vec{J})^2 = 3(1-\varepsilon)^2 \qquad 0.27 \leq \varepsilon \leq 1$$

$$|\psi_{\text{opti}}\rangle = |\psi^-\rangle$$

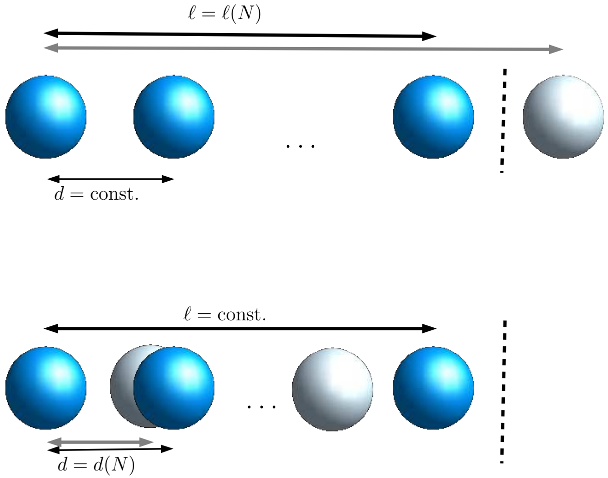
Results



$\langle \triangle J \rangle^2$
20
15
10
5
0.21
0.22
0.23
0.24
0.25
0.26
d/λ
$|\Psi^-\rangle$, $w = 16$,
$(1,2),(3,4),\ldots$ $w = 2$
lower limit $w = 2$,
lower limit $w = 4$

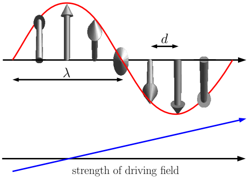
Quantum metrology

How to increase N?



Quantum Fisher information

- Evolution in time: $\hat{U}(t) = \exp\left[\mathrm{i}\,\frac{t}{T}\sum_j (2j-1)\frac{d}{\lambda}\hat{y}_j\right]$

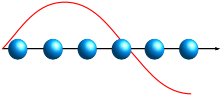


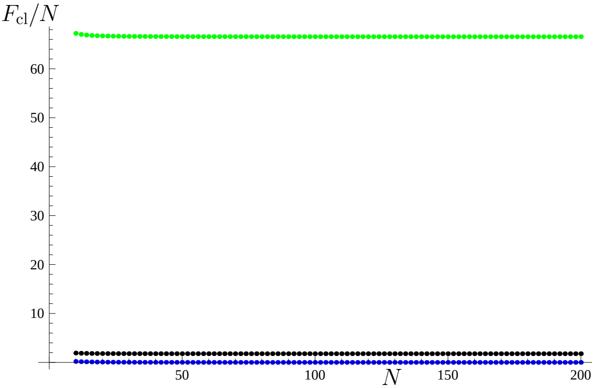
- Quantum Fisher Information

state	d =const.	$d = \ell/N$
$\|\Psi^-\rangle$, $w=2$	$4(t/T)^2N^2$	$4(t/T)^2$
$\|\Psi^-\rangle$, $w=N$	$(t/T)^2N^4$	$(t/T)^2N^2$
$\bigotimes\|0\rangle$, $w=1$	$4(t/T)^2N^4$	$4(t/T)^2N^2$

Classical Fisher Information

- Observable: $\left(\triangle\vec{J}\right)^2(d/\lambda)$
- Starting point: $\frac{\ell}{\lambda}=\frac{3}{4}$





$-|\Psi^-\rangle,\ w=16,\quad -|\Psi^-\rangle,\ w=2,\quad -\bigotimes|0\rangle,\ w=1$

Conclusion

Summary:

- The spatial distribution of entanglement matters!
- It can be characterized with the help of global observables, e.g. sinusoidal observables.
- It influences the precision achievable for measuring spatial dependent fields.

S.Wölk and O.Gühne, arXiv: 1507.07226

Outlook:

- How to characterize the width of entanglement for multipartite entanglement?
- What is the best state to distinguish different spatial variances of the field?